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Allowable slenderness ratio L/d and reinforcement coefficient ρ of one-way, one-span reinforced concrete slabs

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SUMMARY

This paper presents the analyses of deflections of one-way, one-span reinforced concrete slabs. All calculations have been based upon European norms for reinforced concrete structures, ENV 1992 (Eurocode 2 or EC2) and norms for actions on structures, ENV 1991 (Eurocode 1 or EC1). According to this analysis, the geometrical and material characteristics of slabs, i.e. allowable slenderness ratio L/d and reinforcement coefficient ρ *which yield deflections lesser or equal to allowables one, have been established. This analysis comprises six different category of spaces, five different service loadings q, and three coefficients of quasi-permanent values of variable actions* ψ_2. *Consequently, the data given in Chapter 4, Table 4.14, of ENV 1992 have been broadened.*

Key words*: allowable deflection, reinforced concrete slabs, reinforcement coefficient, slenderness.*

1. INTRODUCTION

Explanations, formulae and conditions of Serviceability limit states are given in EC2 [1]. The design of *Serviceability Limit States* comprises proofs that the stresses, the width of cracks and deformations of reinforced concrete structures and elements for the design loads are less than the limit values.

It should be proven that:

$$E_d \leq C_d \qquad (1)$$

where: E_d is the design effect of actions (e.g. deformation, deflection, curvature, strain, rotation), determined on the basis of one of the combinations of actions, while C_d is the nominal value or function of certain properties of material related to the design effects of certain actions considered.

The *Serviceability Limit States* include deformations or deflections that influence the appearance or use of the structure (e.g. irregular work of machines in a building) or that cause damage on plaster or on nonstructural elements. Those states also include vibrations that cause discomfort to people, damage to building or can in any way detrimentally effect the function of a building. The limit values of stresses, the width of cracks and deflections, are given in EC2.

According to EC2 the deflection of a member or a structure should not be such as to adversely affect its proper functioning or appearance. The appearance and general utility of the structure may be impaired when the calculated sag of a beam, slab or cantilever subjected to quasi-permanent loads exceeds the value of *L/250*, where *L* is the span of a member. When there is a possibility of having a pond on the flat roof during rainy period or when a slab or a beam supports partition walls, the allowable deflection is limited to *L/500*. This limit may be lowered in cases where the elements, which might suffer damage, have been designed to accommodate greater deflections or where they are known to be capable of withstanding greater deformations without damage.

The structural elements most sensitive to deflections are cantilever slabs and slabs on two supports with two opposite sides supported and the other two sides unsupported (the so called one-way, one-span slabs). These types of slabs have smaller rigidity in comparison to two-way slabs.

2. DEFLECTION CONTROL ACCORDING TO EC2

Deflection, as a part of *Serviceability Limit States*, should be controlled. For one-way, one-span slabs this control is necessary although there are cases given in EC2 where the calculation of deflections may be omitted. As EC2 has not taken into account all possible cases of loading of one-way, one-span slabs it has been decided to analyze some of such cases which will be presented subsequently.

2.1 When deflection control may be omitted

EC2 [1] gives examples when deflections control does not need to be performed. Chapter 4, Table 4.14, EC2 presents the values of the allowable ratio of span/ effective depth *(L/d)* for reinforced concrete members without axial compression. These values are given for highly and lightly stressed concrete (that corresponds to certain values of reinforcement coefficient) for the following five structural systems: (1) Simply supported beams, one-way or two-way, spanning simply supported slabs; (2) End spans of continuous beams; (3) Interior spans of beams; (4) Flat slabs; (5) Cantilevers. Table 4.14 from EC2 [1] is given here as Table 1 for case 1 only, i.e. for one-way one-span slabs, because they are dealt with in this paper. The values in this table have been derived on the assumption that stresses in reinforcing steel, under design service load at a cracked section in the concrete, in the mid portion of the span of beams or slabs, are $s_s = 250\ N/mm^2$ for reinforcement which has a characteristic yield strength $f_{y,k} = 400\ N/mm^2$. Figure 1 shows symbols of dimensions of the slab cross-section. When the stress in the reinforcement differs from $250\ N/mm^2$ or when $f_{y,k}$ differs from $400\ N/mm^2$, the values in Table 1 should be multiplied by the factor k which is given by:

$$k = \frac{250\ N/mm^2}{s_s} \qquad (2)$$

It will normally be conservative to assume that:

$$\frac{250\ N/mm^2}{s_s} = \frac{400\ N/mm^2}{f_{yk}} \frac{A_{s,prov}}{A_{s,req}} \qquad (3)$$

where:

s_s - is the stress in tension steel reinforcement calculated on the basis of a cracked section (N/mm²),

$f_{y,k}$ - is the characteristic yield stress of reinforcement (N/mm²),

$A_{s,prov}$ - is the area of steel reinforcement provided at the cracked section,

$A_{s,req}$ - is the area of steel reinforcement required at the cracked section.

Table 1. The limiting span/effective depth ratios for reinforced concrete one-way spanning simply supported slab without axial compression

Structural system	*Highly stressed concrete*	*Lightly stressed concrete*
1. One-way span simply supported slab	*18*	*25*

Interpretation of Table 1:

a) values in Table 1 are generally conservative and the calculation may frequently show that thinner members are possible,

b) members where concrete is lightly stressed correspond to $r = 0.5\%$, where $r = A_{s1}/(bd)$, and A_{s1} is the area of tensile reinforcement. It may normally be assumed that slabs are lightly stressed, with the reinforcement coefficient $r < 0.5\%$. The members where concrete is highly stressed correspond to $r = 1.5\%$,

c) if the reinforcement ratio r is known, the values intermediate between those for highly and lightly stressed cases may be obtained by a linear interpolation.

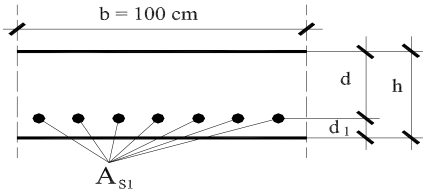

Fig. 1 Cross-section of reinforced concrete one-way slab

2.2 Values of variable actions *q*, and coefficients of quasi-permanent value of a variable action y_2

Preliminary calculations of one-way slab deflections showed that, even for a slenderness ratio span/effective depth $L/d=25$ and lightly stressed concrete $r=0.5\%$ (from Table 1), the slabs should have been loaded with an inappropriately greater loading q than those that are accustomed to ordinary slabs. Consequently, the theoretical research of deflections has continued, varying ratio L/d and coefficient r for normal values of variable loading q and coefficient y_2. The results of this research, using formulas for deflection calculations from EC2 [1], are given further in this text. Table 2 presents the categories of spaces with accompanying values of variable actions q and coefficient y_2 according to ENV 1991-1 [2] and ENV 1991-2-1 [3].

Table 2. Values of variable action q and coefficient y_2 for a quasi-permanent value of variable action

Space	*Cate-gory*	*q (kN/m²)*	y_2
Domestic, residential	*A*	*2.0*	*0.3*
Offices	*B*	*3.0*	*0.3*
Congregation areas such as class-rooms and restaurants	C_1	*3.0*	*0.6*
Congregation areas such as churches and theatres	C_2	*4.0*	*0.6*
Congregation areas such as museums, exhibitions and shopping areas	C_3,C_4, C_5,D	*5.0*	*0.6*
Storage	*E*	*6.0*	*0.8*

2.3 Calculations of deflections according to EC2

Deflections will be checked for a standard thickness of slabs: h=12 cm, 14 cm, 16 cm, 20 cm and 24 cm. Normal quality concrete will be used, i.e. concrete C25/30 and reinforcement S-400 with $f_{y,k} = 400\ N/mm^2$.

For serviceability limit states, according to EC1 [2] and EC2 [1], three representative characteristic values of variable actions Q_k, are expressed by means of factors y_i. These values are defined as:

a) combination values $y_0 Q_k$
b) frequent values $y_1 Q_k$
c) quasi-permanent values $y_2 Q_k$

A quasi-permanent combination of loads, given by the Eq. (4) will be used in this paper:

$$\sum G_{k,j} + \sum Y_{2,i} Q_{k,i} \tag{4}$$

where:

$G_{k,j}$ - characteristic value of permanent actions,
$Q_{k,j}$ - characteristic value of variable actions.

The values of coefficient of a quasi-permanent value of a variable action y_2 in Table 2 are determined under the assumption that the duration of action will be longer than half the lifetime of the structure.

The analysis of deflections is a very complex task because of the great number of material characteristics that change along the slab span, as well as through time. Consequently, it is not possible to obtain an exact algorithm for the calculation; therefore, approximate methods should be used. Those methods are the result of experimental and theoretical research conducted by scientists. The values of deflection depend on mechanical characteristics of material, geometrical values and types of loading.

The calculation of deflections according to EC2 is explained in ENV 1992-1-1 [1], and therefore will not be elaborated here. Only a principal method for the calculation of deflections will be briefly shown here.

Two extreme states exist:

- noncracked state, where concrete and reinforcement participate together in the carrying capacity (stress-state I),
- cracked state, where the participation of concrete in the tension zone is neglected (stress-state II).

The values of deformation (deflection, curvature, strain, rotation) may be obtained from the following expression:

$$a = z a_{II} + (1 - z) a_I \tag{5}$$

where:

a_I - is the deformation value in the stress-state I,
a_{II} - is the deformation value in state II,
z - is a distribution coefficient (which must be zero for uncracked section) given as:

$$z = 1 - b_1 b_2 \left(\frac{s_{sr}}{s_s} \right)^2 \tag{6}$$

where:

b_1 - is the coefficient which takes into account the bond properties of the bar,
b_2 - is the coefficient which takes into account the duration of loading,
s_{sr} - is the stress in tension reinforcement calculated on basis of a cracked section under loading which will cause cracking at the section,
s_s - is the stress in tension reinforcement calculated on the basis of a cracked section.

Total deflection, f_{tot}, could be calculated from the curvature function, $1/r_x$ that includes loading, creep and shrinkage of concrete, for both stress states, I and II, according to the next equation:

$$f_{tot} = \int \left[\int \frac{1}{r_x} dx \right] dx + C_1 x + C_2 \tag{7}$$

By satisfying the boundary conditions, the integral constants, C_1 and C_2, could be determined.

Instead of double integration, the numerical integration using Simpson's formulae is allowed. For elements of constant thickness, the simplified method is often used, according to which, the total curvature at the place of the maximum bending moment is calculated and therefore, total deflection, f_{tot}, can be calculated using the expression:

$$f_{tot} = k L^2 \frac{1}{r_{tot}} \tag{8}$$

where:

k - is the coefficient which depends on the static system ($k=5/48$ for one-way, one-span slabs),
L - is the span,
$\frac{1}{r_{tot}}$ - is the total curvature obtained from curvatures due to loading, creep and shrinkage of concrete, for both, stress states II, and I as stated in EC2.

3. DETERMINATION OF SLENDERNESS RATIO *L/d* AND REINFORCEMENT COEFFICIENT *r* FOR SATISFYING THE ALLOWABLE DEFLECTION

The research of determination of slenderness ratio *L/d* and reinforcement coefficient *r* for satisfying the allowable deflection according to EC2 was performed by numerous and systematic calculations. A special computer program was developed for the calculation of deflections. First, the deflections of slabs with a thickness of 12 cm have shown that for a slenderness ratio *L/d* from 18 to 25 and for the reinforcement coefficient *r* from 0.5% to 1.5%, the load should be much greater than usual. Therefore, the calculation has been inverted in a way that a certain reinforcement coefficient was searched for a given slenderness ratio *L/d* and the usual load on slab. Then, the deflections were calculated according to the EC2 and compared to the allowable ones. In that way, the allowable slenderness ratio *L/d* and the reinforcement coefficient *r*, for one-way, one-span slabs were determined for different slab thickness, from 12 to 24 cm.

3.1 Determination of allowable deflections for one-way, one-span slabs with a thickness h=12 cm and the reinforcement coefficient *r*, from 0.5% to 1.5%, and the slenderness ratio *L/d* from 18 to 25

A total of 75 cases of calculations of deflections have been analyzed. Examples were divided into five groups depending on the reinforcement coefficient *r(%)= 0.5, 0.75, 1.0, 1.25* and *1.5*. Inside each group of a certain reinforcement coefficient *r*, the slenderness ratio *L/d* varied, as follows: *L/d=11, 14.5, 18, 21.5* and *25*. Three groups of y_2 have also been used in the calculations given in Table 2, i.e. y_2*=0.3, 0.6* and *0.8*.

The concrete cover was taken as *c=1.5 cm*, and the diameter of reinforcing bars was taken as *f=10 mm*. Therefore, the effective slab depth was *d=12-2=10 cm*. The additional sustained load $Dg=0.5\ kN/m^2$, the final creep coefficient $j(¥,t_0)=2.71$ for dry atmospheric conditions, $t_0=28$ *days* and $2A_c/u=120$ *mm*. Final shrinkage strain for concrete members placed inside the building is $e_{cs¥}=0{,}0006$. The material characteristics are: concrete C25/30 and reinforcement S-400.

The result of this analysis showed that, for given conditions (from Table 1) the load must be abnormaly high. It can be concluded that for a usual load the slenderness ratio *L/d* should be greater and the reinforcement coefficient *r* should be smaller than that given in EC2. Therefore, a new analysis of slab deflections should be carried out in order to determine the allowable values of *L/d* and *r*.

3.2 Determination of allowable deflections and reinforcement coefficient of one-way, one-span slabs

The task of this analysis was to determine such values of *L/d* and *r*, for which deflections of one-way slabs will be less or equal to the allowable values of *L/250*. For this reason, the range of slab thickness, in comparison to Section 3.1, is increased to five different dimensions: *h=12, 14, 16, 20* and *24 cm*.

According to ENV 1991-1 [1], six categories of spaces were taken into account: 1) category *A*; 2) *B*; 3) C_1; 4) C_2; 5) C_3, C_4, C_5, *D*; and 6) *E*, with the corresponding loading *q* and coefficient y_2 (see Table 2).

For all slabs the concrete cover was taken as *c=1.5* cm, the effective slab depth was *d=h-2cm* (assuming the diameter of longitudinal bars to be 10 mm), and additional sustained load $Dg=0.5\ kN/m^2$. The statical system and materials were the same as those in Section 3.1. Creep coefficients for concrete aged 28 days and shrinkage strains were not constant, except those given in Table 3, since they depended on slab thickness.

The calculation of deflections (according to the analysis from EC2) was performed using certain span *L*, for a different slab thickness and category of spaces. The coefficient of reinforcement, *r*, was calculated as well. The calculated deflections were then compared to allowable deflections. Shaded boxes in Table 4 show greater deflections than allowable. For each space category (*A* to *E*) and each slab thickness, the ratio *L/d*, for which deflection was equal to allowable deflection *L/250*, could be roughly determined. This has been shown in Table 4 and in Figure 2 for slab thickness *h=14 cm*. Since Figure 2 shows a rough diagram with accurate calculated values of deflections only at certain points of *L/d* ratio, the real diagram between those values may differ from the one in Figure 2. Therefore, accurate values of deflections may differ from linear interpolation between two chosen *L/d* values. More accurate value of *L/d* ratios and reinforcement coefficients for which deflection is equal to allowable deflection, could then be obtained by an iterative process, increasing the value of the span for 1 cm. The results of such calculations are shown in Tables 5 and 6 as well as in Figures 3, 4, 5 and 6. Table 5 briefly presents the results of calculations of allowable spans for 14 cm slab thickness, for all five groups of space categories. The same calculation was done for slabs of thickness 12, 16, 20 and 24 cm that are not tabulated here. Table 6 contains the final results of all calculated data (allowable reinforced coefficients and L/d ratios that give allowable deflections) for given space categories, variable load and y_2 values.

Table 3. Creep coefficients and shrinkage strains according to EC2

slab thickness (cm)	*creep coefficient* $j(¥,t_0)$	*shrinkage strains* $e_{cs¥}$
12	*2.71*	*0.0006*
14	*2.64*	*0.0006*
16	*2.56*	*0.0006*
20	*2.48*	*0.000596*
24	*2.45*	*0.00059*

It is evident from Figure 2 that cracks appear at certain values of slenderness ratio *L/d* and stress state I changes into stress state II. Therefore, by creation of cracks, deflection increases drastically, as can be seen in diagrams of Figure 2. Those diagrams have a small slope in stress state I (small deflections), followed by a sudden steep slope (increase of deflections) at the moment of the creation of cracks. A less steep slope of the same diagrams in stress state II follows after that.

Table 6 contains all relevant input and calculated values for all chosen slab thicknesses h=12, 14, 16, 20 and 24 cm and space categories. The calculated values are: reinforcement coefficient *r*, *L/d* ratio, allowable deflection and maximal allowable span *L*.

From Tables 5 and 6, it is possible to make diagrams showing graphical values of allowable ratios of *L/d* as well as dependable reinforcement coefficients. Figure 3 shows how *L/d* ratios depend on the slab thickness satisfying the conditions of allowable deflection. From the same figure it is visible that for the same slab thickness a larger span is allowable for space categories with a smaller coefficient y_2 as well as a smaller load *q*. Such behavior is expectable. It is also visible that the increase of slab thickness decreases allowable *L/d* ratio for a certain category which could not be expected without such analysis. In order to be perfectly clear it should be finally stated that by increasing the slab thickness allowable span increases, while the ratio of span and effective depth decreases, which shows that thickness increases faster than the allowable span. It should be noted that in EC2 the maximum *L/d* ratio for which deflection will not be calculated is 25; however, this analysis shows that it should not necessarily be the case for one-way, one-span slabs, since for thinner slabs the allowable *L/d* is greater than 25. Figure 3 also shows that for a slab thickness less than 17 cm and for all categories of spaces, except for category *E*, the allowable *L/d* ratio could be assumed to be greater than 25.

From Figure 3 it is not yet possible to determine the reinforcement coefficients *r* which are given in EC2. Therefore, diagrams in Figure 4 are given to show the dependence of reinforcement coefficients *r*, on the slab thickness. It is visible that for slab thicknesses greater than 20 cm, categories *A* and C_1 have the smallest reinforcement coefficient, and categories C_3, C_4, C_5 and *D* the highest. For a slab thickness less than 16 cm the smallest reinforcement coefficient is given for category C_1 and highest for the category *B*.

Figure 5 shows diagrams according to which the maximum slab span could be determined for certain one-way, one-span slabs, thickness and load *q*, so that deflection is

Table 4. Presentation of the reinforcement and deflections for slab thickness h=14 cm (d=12 cm) with prescribed spans L

Span, L [m]:	category	2.70	3.00	3.30	3.60	3.90	4.20
(L/d):	from Table 2	**22.5**	**25.0**	**27.5**	**30.0**	**32.5**	**35.0**
area of longitudinal reinforcement A_{S1} [cm^2]:	A	1.881	2.332	2.836	3.396	4.007	4.676
	B	2.226	2.757	3.356	4.023	4.752	5.552
	C_1	2.226	2.757	3.356	4.023	4.752	5.552
	C_2	2.568	3.187	3.885	4.662	5.513	6.462
	C_3,C_4,C_5,D	2.915	3.622	4.415	5.309	6.291	4.676
	E	3.268	4.064	4.970	5.965	7.083	8.341
reinforcement coefficient r [%]:	A	0.157	0.194	0.236	0.283	0.334	0.390
	B	0.185	0.230	0.280	0.335	0.396	0.463
	C_1	0.185	0.230	0.280	0.335	0.396	0.463
	C_2	0.214	0.266	0.324	0.389	0.459	0.538
	C_3,C_4,C_5,D	0.243	0.302	0.368	0.442	0.524	0.617
	E	0.272	0.339	0.414	0.497	0.590	0.695
allowable deflection, L/250 f_q [cm]:		1.08	1.20	1.32	1.44	1.56	1.68
calculated total deflection f_{tot} [cm]:	A	0.20	0.30	0.44	0.61	1.93	2.62
	B	0.22	0.33	0.47	0.66	2.00	2.64
	C_1	0.25	0.37	0.54	1.84	2.49	3.18
	C_2	0.27	0.41	1.39	1.98	2.61	3.28
	C_3,C_4,C_5,D	0.30	0.45	1.52	2.09	2.70	3.36
	E	0.36	1.37	1.92	2.52	3.17	3.89

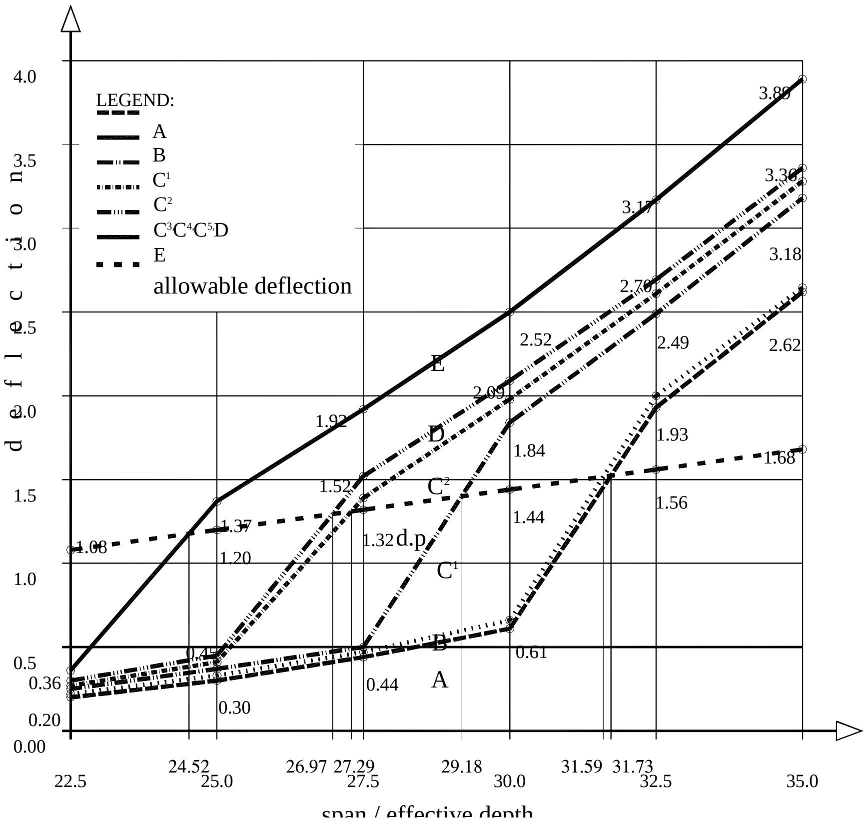

Fig. 2 Diagrams of calculated deflections for different categories of spaces with graphically determined L/d ratio for deflections equal to allowable values

not greater than *L/250*. The steps in diagrams appear because categories *B* and C_1 have the same quasi-permanent uniformly distributed load $q=3.0\ kN/m^2$ and different value of y_2, i. e. *0.3* and *0.6*, respectively.

4. CONCLUSION

From previous tables and diagrams, it can be concluded under which circumstances the deflection control need not be carried out, i.e. when deflection should be smaller than allowable. The values which must be satisfied are the reinforcement coefficients *r*, slenderness *L/d* and category of space (on which loading and coefficient y_2 depend). The conditions that are given, for a prescribed loading *q*, the geometrical and material characteristics result in a deflection equal to the allowable values.

The analysis given in this paper shows that the allowable ratio *L/d* beyond limit of 25 (see Figure 3) and the reinforcement coefficient *r* below 0.50% (see

Table 5. A short presentation of the determination of the allowable spans for slab thickness of h=14 cm by increasing span for 1 cm

category A (domestic, residential), $q = 2.00\ kN/m^2$, $y_2 = 0.3$				
span, L [m]:	*3.80*	*3.81*	*3.82*	*3.83*
area of longit. reinforcement A_{S1} [cm²]	*3.788*	*3.816*	*3.836*	*3.856*
span/effective depth (L/d):	*31.67*	*31.75*	*31.83*	*31.92*
reinforcement coefficient, r [%]:	*0.316*	*0.318*	*0.320*	*0.321*
allowable deflection, f_g [cm]:	*1.520*	*1.524*	*1.528*	*1.532*
calculated total deflection, f_{tot} [cm]:	*0.75*	*0.76*	*1.75*	*1.77*
analytical results of allowable values:				
reinforcement coefficient, r [%]:	*0.3195*			
span/effective depth, (L/d):	*31.80*			
deflection, [cm]:	*1.527*			

category B (offices), $q = 3.00\ kN/m^2$, $y_2 = 0.3$				
span, L [m]:	*3.68*	*3.69*	*3.70*	*3.71*
area of longit. reinforcement A_{S1} [cm²]	*4.213*	*4.236*	*4.259*	*4.282*
span/effective depth (L/d):	*30.67*	*30.75*	*30.83*	*30.92*
reinforcement coefficient, r [%]:	*0.351*	*0.353*	*0.355*	*0.357*
allowable deflection, f_g [cm]:	*1.472*	*1.476*	*1.480*	*1.484*
calculated total deflection, f_{tot} [cm]:	*0.71*	*0.72*	*1.58*	*1.60*
analytical results of allowable values:				
reinforcement coefficient, r [%]:	*0.355*			
span/effective depth, (L/d):	*30.82*			
deflection, [cm]:	*1.48*			

category C_1 (class rooms and restaurants), $q = 3.00\ kN/m^2$, $y_2 = 0.6$				
span, L [m]:	*3.38*	*3.39*	*3.40*	*3.41*
area of longit. reinforcement A_{S1} [cm²]	*3.528*	*3.549*	*3.570*	*3.591*
span/effective depth (L/d):	*28.17*	*28.25*	*28.33*	*28.42*
reinforcement coefficient, r [%]:	*0.294*	*0.296*	*0.298*	*0.299*
allowable deflection, f_g [cm]:	*1.352*	*1.356*	*1.360*	*1.364*
calculated total deflection, f_{tot} [cm]:	*0.59*	*0.60*	*1.42*	*1.44*
analytical results of allowable values:				
reinforcement coefficient, r [%]:	*0.298*			
span/effective depth, (L/d):	*28.33*			
deflection, [cm]:	*1.36*			

category C_2 (churches, theatres), $q = 4.00\ kN/m^2$, $y_2 = 0.6$				
span, L [m]:	*3.24*	*3.25*	*3.26*	*3.27*
area of longit. reinforcement A_{S1} [cm²]	*3.737*	*3.760*	*3.783*	*3.807*
span/effective depth (L/d):	*27.00*	*27.08*	*27.17*	*27.25*
reinforcement coefficient, r [%]:	*0.311*	*0.313*	*0.315*	*0.317*
allowable deflection, f_g [cm]:	*1.296*	*1.300*	*1.304*	*1.308*
calculated total deflection, f_{tot} [cm]:	*1.27*	*1.29*	*1.31*	*1.33*
analytical results of allowable values:				
reinforcement coefficient, r [%]:	*0.314*			
span/effective depth, (L/d):	*27.14*			
deflection, [cm]:	*1.303*			

categories C_3,C_4,C_5,D (museums, exhibitions and shopping areas), $q = 5.00\ kN/m^2$, $y_2 = 0.6$				
span, L [m]:	*3.15*	*3.16*	*3.17*	*3.18*
area of longit. reinforcement A_{S1} [cm²]	*4.014*	*4.039*	*4.065*	*4.091*
span/effective depth (L/d):	*26.25*	*26.33*	*26.42*	*26.50*
reinforcement coefficient, r [%]:	*0.334*	*0.337*	*0.339*	*0.341*
allowable deflection, f_g [cm]:	*1.260*	*1.264*	*1.268*	*1.272*
calculated total deflection, f_{tot} [cm]:	*1.24*	*1.26*	*1.28*	*1.30*
analytical results of allowable values:				
reinforcement coefficient, r [%]:	*0.338*			
span/effective depth, (L/d):	*26.35*			
deflection, [cm]:	*1.27*			

category E (storage), $q = 6.00\ kN/m^2$, $y_2 = 0.8$				
span, L [m]:	*2.86*	*2.87*	*2.88*	*2.89*
area of longit. reinforcement A_{S1} [cm²]	*3.678*	*3.704*	*3.730*	*3.756*
span/effective depth (L/d):	*23.83*	*23.92*	*24.00*	*24.08*
reinforcement coefficient, r [%]:	*0.307*	*0.309*	*0.311*	*0.313*
allowable deflection, f_g [cm]:	*1.144*	*1.148*	*1.152*	*1.156*
calculated total deflection, f_{tot} [cm]:	*1.12*	*1.14*	*1.16*	*1.18*
analytical results of allowable values:				
reinforcement coefficient, r [%]:	*0.310*			
span/effective depth, (L/d):	*23.92*			
deflection, [cm]:	*1.15*			

Table 6. Results of analysis for all chosen slab thicknesses, space categories and load q (kN/m²)

h	categories:	A	B	C_1	C_2	C_3, C_4, C_5, D	E
h	q (kN/m²)	2.0	3.0	3.0	4.0	5.0	6.0
[cm]	y_2	0.3	0.3	0.6	0.6	0.6	0.8
12	r [%]:	0.35	0.39	0.32	0.34	0.35	0.32
	L/d :	34.65	33.47	30.48	28.89	27.60	24.92
	f_g, [cm]:	1.39	1.34	1.22	1.16	1.10	1.00
	L_{max} [cm]:	346	334	304	288	276	249
14	r [%]:	0.32	0.36	0.30	0.31	0.34	0.31
	L/d :	31.80	30.82	28.33	27.14	26.35	23.92
	f_g, [cm]:	1.53	1.48	1.36	1.30	1.27	1.15
	L_{max} [cm]:	381	369	339	325	316	287
16	r [%]:	0.30	0.33	0.28	0.31	0.33	0.30
	L/d :	29.64	28.79	26.79	26.03	25.36	23.21
	f_g, [cm]:	1.66	1.61	1.50	1.46	1.42	1.30
	L_{max} [cm]:	414	403	375	364	355	325
20	r [%]:	0.28	0.31	0.28	0.30	0.32	0.30
	L/d :	26.67	26.44	24.86	24.31	23.80	22.05
	f_g, [cm]:	1.92	1.90	1.79	1.75	1.72	1.59
	L_{max} [cm]:	480	476	447	437	428	396
24	r [%]:	0.27	0.31	0.27	0.29	0.31	0.30
	L/d :	24.87	24.77	23.47	23.04	22.67	21.15
	f_g, [cm]:	2.18	2.18	2.07	2.03	1.99	1.86
	L_{max} [cm]:	547	545	516	506	498	465

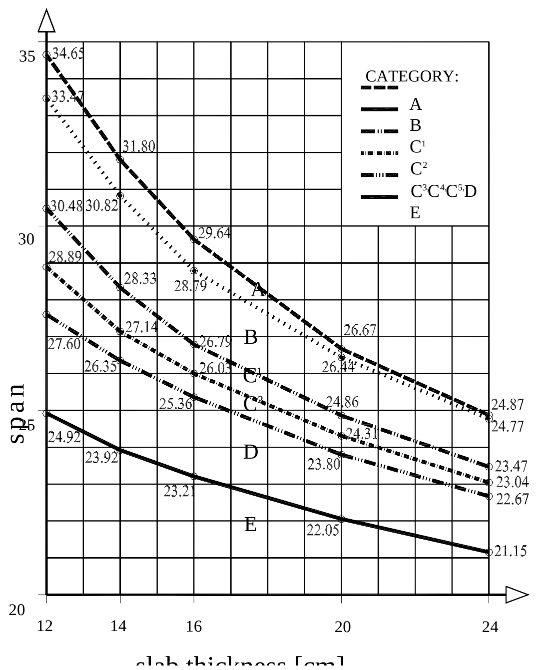

Fig. 3 Diagrams of allowable L/d (span/effective depth) ratios as a function of slab thickness for all space categories

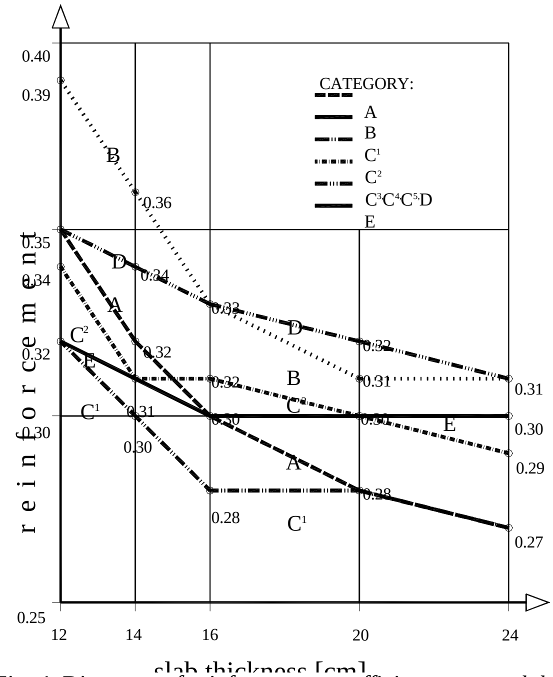

Fig. 4 Diagrams of reinforcement coefficients versus slab thickness for different space categories

Figure 4) exist for one-way, one-span slabs. Table 7 shows this range too.

It can be concluded that the range of slenderness and the reinforcement coefficients recommended in EC2 for simply supported one- and two-way slabs as well as beams, is not complete. This range should be further divided into separate groups. One group should be made for one-way, one-span slabs, as shown in Table 7.

From Table 7 it is visible that for all slab thicknesses, taken into account in this paper (*h=12* to *24cm*) and for all space categories (*A* to *E*), the greatest allowable ratio is *L/d=34.6* and the smallest reinforcement coefficient is *r*=0.27%. By increasing slenderness, the reinforcement coefficient is increased as well.

The analysis in this paper also shows that the statement given in EC2 is correct, i.e.: "Values in Table 1 are generally conservative and the calculation may frequently show that thinner members are possible. It may normally be assumed that slabs are lightly stressed, with the reinforcement coefficient *r<0.5%*".

Table 7. Allowable slenderness and reinforcement coefficients for one-way, one-span reinforced concrete slabs when deflection need not be calculated

	category A		B		C_1		C_2		C_3, C_4, C_5, D		E	
h(cm)	L/d	r(%)	L/d	r(%)	L/d	r(%)	L/d	r(%)	L/d	r(%)	L/d	r(%)
12	£ 34.6	0.35	£ 33.4	0.39	£ 30.5	0.32	£ 28.9	0.34	£ 27.6	0.35	£ 24.9	0.32
14	£ 31.8	0.32	£ 30.8	0.36	£ 28.3	0.30	£ 27.1	0.31	£ 26.3	0.34	£ 23.9	0.31
16	£ 29.6	0.30	£ 28.8	0.33	£ 26.8	0.28	£ 26.0	0.31	£ 25.3	0.33	£ 23.2	0.30
20	£ 26.6	0.28	£ 26.4	0.31	£ 24.8	0.28	£ 24.3	0.30	£ 23.8	0.32	£ 22.0	0.30
24	£ 24.8	0.27	£ 24.7	0.31	£ 23.4	0.27	£ 23.0	0.29	£ 22.6	0.31	£ 21.1	0.30

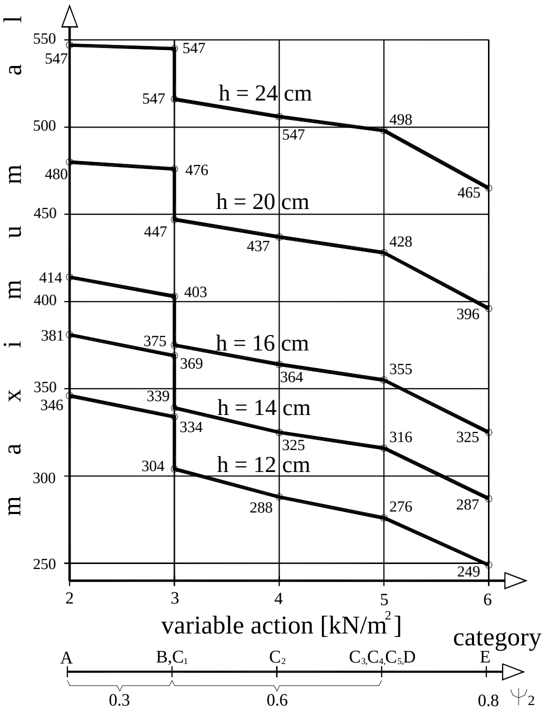

Fig. 5 Diagram of the maximum one-way slab span as the function of the uniform load q and slab thickness (for certain space categories and coefficient y_2)

5. ACKNOWLEDGEMENT

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6. LITERATURE

[1] ENV 1992-1-1: Eurocode 2: Design of concrete structures - Part 1-1: General rules for buildings, EC2, European Committee for standardization, Brussels, June 1991.

[2] ENV 1991-1: European prestandard, Eurocode 1: Basis of design and actions on structures, Part 1: Basis of design, EC1, European Committee for Standardization, Brussels, September 1994.

[3] ENV 1991-2-1: European Prestandard, Eurocode 1: Grundlagen der Tragwerksplanung und Einwirkungen auf Tragwerke - Teil 2-1: Einwirkungen auf Tragwerken - Wichten, Eigenlasten, Nutzlasten, European Committee for Standardization, Brussels, February 1995.

[4] CEB Design manual on cracking and deformations, Ecole Polytechnique Federale de Lausanne, 1985., pp. 1.1-1.9, 3.1-3.16

[5] C. Arya, Design of structural elements, Chapter 8, Design of concrete structures, An Imprint of Chapman & Hall, pp. 230-275, 1995.

[6] Z. Sori} and S. Luli}, Determining zone of allowable deflection for single span reinforced - concrete slabs, Proc. Meeting of the Croatian Constructors 2000, Cavtat, April 2000, pp. 445-453, 2000. (in Croatian)

[7] I. Tomi~i}, *Concrete Structures*, 3rd edition, DHGK, Zagreb, 1996. (in Croatian)

[8] Beton - Kalender 1994, Teil 1, s. 798-805, 1994.

[9] Z. Sori}, T. Ki{i}ek, I. Gukov and N. Bjelajac, Computer program for computing deflection of RC girders, Proc. 4th General Meeting of the Croatian Structural Engineers, Brijuni Islands, June1998, pp. 531-538, 1998. (in Croatian)

DOPU[TENA VITKOST L/d I KOEFICIJENT ARMIRANJA r JEDNORASPONSKIH ARMIRANOBETONSKIH PLO^A

SA@ETAK

U radu je analiziran progib jednorasponskih armiranobetonskih plo~a. Svi prora~uni su temeljeni na europskim normama za betonske konstrukcije, ENV 1992 (Eurocode 2) i normama za djelovanja na konstrukcije, ENV 1991 (Eurocode 1). Na temelju analize ustanovljene su geometrijske i stati~ke karakteristike plo~a, tj. dopuštena vitkost L/d i koeficijent armiranja r, koji daju progibe manje ili jednake dopuštenim. Analizom je obuhva}eno {est razli~itih kategorija prostorija, pet razli~itih uporabnih kontinuiranih optere}enja q i tri vrijednosti promjenljivih tj. nazovi-stalnih djelovanja y_2. Podaci dani u to~ki 4, u tablici 4.14, europskih prednormi ENV 1992 ovim su analizama pro{ireni.

***Klju~ne rije~i**: dopušteni progib, armiranobetonske plo~e, koeficijent armiranja, vitkost.*