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Determining vertical displacements of concrete plates using different methods

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SUMMARY

The paper presents a case study of methods for determining the vertical displacements of a prestressed concrete plate. Geodetic instruments (a level, total station and laser level) and non-geodetic instruments (inductive transducers) were used for this purpose.

Analytic and numerical methods, together with the European standards for reinforced and prestressed concrete were applied to calculate the expected theoretical displacements and the possible cracks.

Key words*: geodesy, statics, measurements, displacements, numerical analysis.*

1. INTRODUCTION

Experimental methods for investigating structures in their early phases of use were based on a theoretical calculation which describes the deformation growth and failure under load. With the development of numerical sciences and the increased surge of information science in the last 30 years, numerous national and international experts have taken up the analysis of displacements and deformations and published their results in articles and books. Most of the authors analysed displacements and deformations and fostered the upgrading of the compensation model for horizontal and vertical measurements.

Among national authors, Marjanovi~ [1] who describes non-geodetic methods for measuring linear displacements, deserves to be mentioned.

The methods for measuring displacements and deformations were also presented by Narobe in 1996 [2]. In his article, he presents measurements performed with inductive transducers, classic theodolite and precision level. All the results obtained were statistically processed, mathematical statistics thus forming a component part for the compensation of results.

The statistical processing of data based on the growth of the nil or alternative hypotheses and on different tests for checking significance as well as on the class of error values was presented by Pelzer [3].

An outstanding contribution to deformation measurement methods was made by Milev [4] in 1985.

Welsch [5] made a schematic presentation of the deformation analysis process of geodetic nets which served as a basis for the creation of computer programs.

The same topic was studied also by Kapovi}, Narobe and Masteli} [6] from the Faculty of Geodesy of the University of Zagreb, and Breznikar [7], Vodopivec and Stopar [8] from the Faculty of Civil and Geodetic Engineering of the University of Ljubljana. Their investigations focus on the calculation of precision measurements and errors in measuring structural displacements and deformations.

As evident from this survey, numerous experts have worked on the analysis of displacements and deformations. The most notable contribution to the analysis of deformations was made by Welsch and Pelzer with co-authors as they set the bases of the deformation analysis and solved the key problems in the stability of geodetic nets.

2. METHODS FOR DETERMINING DISPLACEMENTS AND DEFORMATIONS

2.1 Measuring methods

Various measuring methods differ in measurement techniques or types of instruments, with which different measurements for inspecting structures are realised. The selection of the measuring method is made for each case separately by taking into consideration numerous factors such as the type of structures, the type of materials, the required precision of measurement, duration of investigations, available financial means, qualification of available staff etc.

Each measuring method corresponds to the adequate measurement technique. The selection of the best measurement technique requires good knowledge of statical calculation of the inspected structure as well as of the measurement technique itself. The experience in the application of the measurement technique and in the inspection and investigation of structures is needed, too [2].

Two general methods are possible for determining displacements.

The first method encompasses geodetic methods for measuring the necessary elements and special processing of the obtained results. All geodetic measurement methods can be used here, provided that they assure, with respect to the used instruments, the required accuracy of the measured results. The most frequent displacements are those of the magnitude of 1 mm.

The second way to determine the magnitude of displacements is by using physical measuring methods. The measurements are performed with different instruments that directly show the magnitude of displacements or deformations. The following instruments are generally used: labels of high sensitivity, electronic labels, deformeters, tensometers, induction transducers etc.

It can be concluded from the analysis of geodetic or physical methods for determining displacements and deformations that displacements obtained by geodetic methods have the absolute character because they are determined with respect to the stability points outside the area of displacements. This, however, is not true for physical methods with which displacements and deformations are obtained as a relative value because the measuring instrument is located on the structure, i. e. in the area of displacements.

This means that geodetic measuring methods and adequate processing of results yield absolute vector values of displacements or deformations with respect to the requirement: either in plane XY or in space XYZ , see Ref. [9].

2.2 Analytical and numerical methods

2.2.1 Analytical methods

In order to enhance the measurements accuracy and to enable the verification of the results, the predicted displacements should be calculated before measurements have been taken. For an uncracked cross-section, by using analytical methods, displacements can be computed with the known methods of structural calculations. More problems are encountered at cracked cross-sections where cracks occur due to a low tensile strength of concrete. This results in the reduction of the inertial moment of the average cross-section and thus in greater deformation. As the location and the height of cracks are difficult to determine, due to concrete properties, they are stated in different national codes. Eurocode 2 (EC2) [10] has lately been applied most frequently.

2.2.2 Numerical methods

It is often the case in practical applications that the required geometry of plates cannot be obtained by analytical methods. Therefore, different numerical methods are used which are based mainly on the finite elements or finite differences. As the calculation procedure is a long one, different computer programs are used. One of them is Ocean [11]. It allows the computation of cracked cross-sections in accordance to EC2.

3. DESCRIPTION OF THE PROBLEM

The analysis and the comparison between individual methods for determining the vertical displacements of a structure in space are presented subsequently. As already stated in the introduction, the analysis of structures and deformations was made by several experts; however, the comparison of methods, has not been made yet. The comparison would give us tangible results on the adequacy of indivudiual methods with respect to the weight of the structure.

The inspection of structures and buildings is initiated due to the necessity of evaluating the condition of the structure as well as the accuracy of theoretic predictions. The inspection allows to determine the effect of such factors on the load-bearing capacity of the structure which cannot be considered in theory. It also yields necessary data to compare the structural variations, the effect of space work of the skeleton elements on the load-bearing capacity of the structure, and the effects of mutual work of two different structures. Experiments are needed to evaluate the condition of structures being either under construction, reconstruction or strenghtening.

Experimental tests are required in the following cases:

- possibility of using the structure in cases of excessive variable load;
- testing durability of structural elements of mass production with selective testing methods;
- testing the work of high and special structures under variable load;
- testing real work of sophisticated structural systems and their safety.

In practical applications, it is the vertical displacement of a structure under load which is most frequently tested. The load is computed or known in advance so that predicted vertical displacements [2] can be determined with analytical and numerical structural computations. The most relevant factors for the interpretation of the results are value, direction and character of displacements. To determine these parameters, the inspection program is selected and the suitable processing methodology of results applied. For practical applications, it is especially important and useful to evaluate correctly the character of displacements.

Considering the displacement type, the uniform and non-uniform displacements should be determined first. Leaving aside special cases, uniform displacements, as a rule, do not present any danger; they do not affect the strength, stability and safety of the structure or its parts. Non-uniform displacements present greater danger by their effects. The greater the differences in displacements at adjacent locations of a structure, the greater the danger.

Non-uniformity of displacements results in different deformations: bending, leaning, rotation, structure torsion, but failures such as cracks and breakage are also possible. This means that deformations strongly depend on the non-uniformity of displacements [2].

In geodetic practice we most frequently determine the geometric characteristics of structures and their displacements. In the narrow sense, these investigations are mostly used to determine changes in the location and shape of structures with respect to the environment and changes depending on weather conditions. Structural deformations occur due to external and internal effects such as the wind action, temperature changes, tectonic and seismic impact, changes in the underground water levels, static and dynamic loads etc. [8].

Structural deformations in space can be determined by different methods. Before determining deformations, the structure or building has to be studied in detail. This is done by making use of data from the project of the structure.

Natural and man-made structures are inspected for deformations; thus obtained data can serve as information about the characteristics of structures and materials as well as about the possibility of their failure.

4. MEASUREMENT OF DISPLACEMENTS

The maximum vertical displacements were measured in the middle of the prestressed concrete plate type PVP5, Figure 1. The data for the analysis were obtained by the uniform loading of the plate in four sequential steps. The measurements were performed by geodetic as well as by physical and electronic methods and instruments, Figure 2. In the laboratory the temperature and pressure were constant during measurement.

Fig. 1 Prestressed concrete plate of the type PVP 5

Fig. 2 Position of geodetic instruments

We took measurements solely with instruments that were up-to-date and most suitable for these types of measurements. These are, above all, the laser system and inductive transducers. Other two methods, i.e. precision levelling and measurement with the total station, are classic methods and more or less known to everyone.

4.1 Characteristics of the plate type PVP 5

The characteristics of the prestressed concrete plate of the type PVP 5 are:

- calculated static length: 418 cm
- width: 120 cm
- height: 26.5 cm
- reinforcement: 1680/1860 MPa, 10 pieces of 7×4.2 mm
- concrete quality: 60 MPa
- dead load: $3.6 \times 1.2 = 4.32$ kN/m^2
- resistance bending moment: $R_d = 293.328$ kNm
- inertial moment of the cross-section surface (without cracks) $I_c = 149.299$ cm^4

4.2 Analytical computation of predicted vertical displacements

In each step, two pallets were placed onto the plate as evident from Figure 3. The weight of a palette was P=7.895 kN.

Fig. 3 Central loading of the plate after step 4

For an uncracked cross-section, due to bending moment of the force P1 the displacement is (Figure 4):

$$v_{P1}^{(M)} = \frac{P1(L-C)}{L} \times C \times \frac{L}{4} \times L \times \frac{1}{12\beta} \times (3-4\alpha^2) \times \frac{1}{E_c I_c} \tag{1}$$

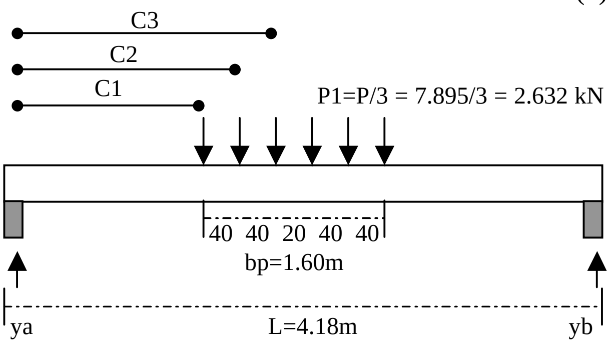


Fig. 4 Static system

1. Computation of displacement to C=C1 due to force P1 (Figure 5):

$$C_1 = \frac{L}{2} - 90cm$$
$$\alpha = \frac{C_1}{L} = 0.285 \tag{2}$$

$$v_{P1}^{(M)} = 0.00555\ cm = 0.0555\ mm$$

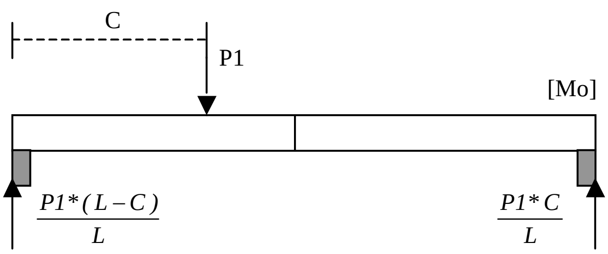


Fig. 5 Static system due to force P1

2. Computation of displacement to C=C2 due to force P1:

$$C_2 = \frac{L}{2} - 50cm$$
$$\alpha = \frac{C_2}{L} = 0.3804 \tag{3}$$

$$v_{P1}^{(M)} = 0.00672cm = 0.0672mm$$

3. Computation of displacement to C=C3 due to force P1:

$$C_3 = \frac{L}{2} - 10cm$$
$$\alpha = \frac{C_3}{L} = 0.476 \tag{4}$$

$$v_{P1}^{(M)} = 0.00727cm = 0.0727mm$$

Due to the bending moment of 2 pallets (one step), the total displacement amounts to:

$$v_{P1}^{(M)} = 2 \times (0.0555mm + 0.0672mm + 0.0727mm)$$
$$= 0.3908mm$$

To achieve accurate results, the displacement due to shear force has to be considered.

Due to force *P1*, the displacement at location *C* amounts to:

$$v_{P1}^{(Q)} = \frac{P1 \times C}{2 \times G \times As} \tag{5}$$

Total displacement, due to average force of 2 palettes (one step), thus amounts to:

$$v^{(Q)} = 2 \times (7.099 + 9.485 + 11.871) \times 10^{-4} =$$
$$= 56.91 \times 10^{-4}\ mm$$
$$v^{(Q)} = 0.005691\ mm$$

The total computed maximum vertical displacement $v^{(M)} + v^{(Q)}$ for the individual step of load at the uncracked cross-section amounts to: v = 0.39649 mm.

In loading step 4, displacements are no longer linear due to the occurrence of cracks in the tensile area.

After a detailed visual inspection of the slab in step 4, the first cracks were found in the tensile area. The results obtained with the computation according to EC2 also show the possibility of the occurrence of cracks:

f_{ctm}=4.1 MPa=0.41 kN/cm^2

E_{cm}=36.77×106 kN/m^2

$$W_I = \frac{I_I}{Z_I} = \frac{149299}{13.25} = 11267.85\ cm^3 \tag{6}$$

$$M_I \cong f_{ctm} \times W_I = 0.41 \times 11267.85 =$$
$$= 4619.82\ kNcm = 46.198\ kNm \tag{7}$$

Step 4:

$$M_p = P_1^{(4)} \times 4.77 = 50{,}22\ kNm > M_I$$
$$M_g = \frac{g\,L^2}{8} = \frac{4.32 \times 4.18^2}{8} = 9.435\ kNm \tag{8}$$
$$M_g + M_P = 9.435 + 50.22 = 59.66\ kNm > M_I$$

In this step, the computed displacement according to EC2 amounts, in the centre of the plate, to:

$$I_{II} = \frac{120 \times 4.5^3}{12} + 120 \times 4.5 \times (12.78 - 2.25)^2 +$$
$$+ 22.8 \times \frac{(12.78 - 4.5)^2}{2} + 9.70 \times \frac{20000}{3677} \times (23 - 12.78)^2$$

$$I_{II} = 67079.25\ cm^4 \tag{9}$$

where I_{II} is inertial moment of the cross-section surface with a crack,

$$I_{ef} = \xi \times I_{II} + (1-\xi) \times I_I$$

$$\xi = 1 - \beta_1 \times \beta_2 \times \left(\frac{\delta_{sr}}{\delta s}\right)^2 = 1 - \beta_1 \times \beta_2 \times \left(\frac{M_I}{M_{dej}}\right)^2$$

$$M_{dej} = M(step\,4) = 50.22\,kNm > M_I$$

$$M_I = 46.198\,kNm$$

$$\beta_1 = 1.0$$

$$\beta_2 = 1.0$$

$$\xi = 1 - 1.0 \times 1.0 \times \left(\frac{46.198}{50.22}\right)^2 = 0.1538$$

$$I_{ef} = 136653.60\,cm^4$$

Step 4:

$$v_M \approx v_{m,I_I}\ x\,\frac{I_I}{I_{ef}} = 1.5632 \times \frac{149299}{136653.60} = 1.708\,mm$$

$$v = v_M + v_Q = 1.708 + 0.0228 = 1.7307\,mm \quad (10)$$

4.3 Numerical analysis of displacements

The numerical analysis of displacements was performed with the finite element computer program OCEAN [12]. The plate was discretizated with 150 finite rectangular plate elements, which can also be seen from Figure 6. The occurrence of the crack was also considered in loading step 4.

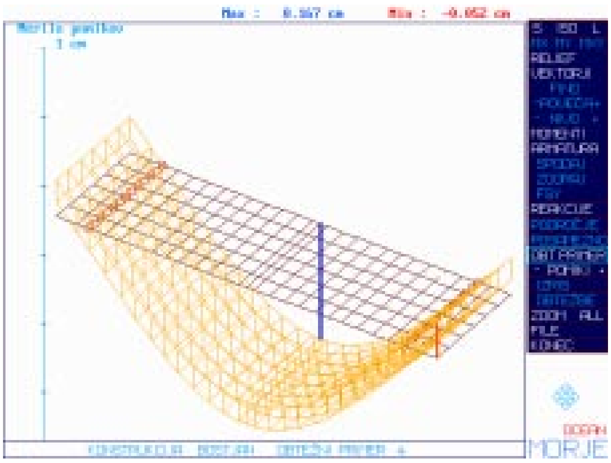


Fig. 6 The scheme of the plate discretization

4.4 Experiments

The experiment was performed in four loading steps of 1300 kg each; for each loading step the predicted displacements was computed using the analytical and numerical finite elements method. The loading steps are equal in weight and therefore it is assumed that the predicted displacements during individual steps are linear if there are no cracks. The displacements were determined with four methods; for each method, numerous readings were made. We made 14 readings for each step with the electronic tachometer and level, while 15.000 readings were made on the average with the inductive transducer and rotation level. These two methods perform measurements continuously and give, on the average, 10 readings per second. The program LASER, which supports measurements, stores data and shows them simultaneously in the form of graphs.

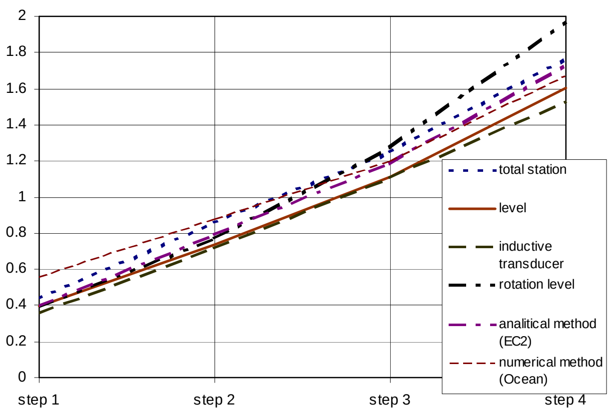
Subsequently, we present the obtained results and the deviations from the predicted displacements computed by analytical methods (including EC2 for a cracked cross-section) and by numerical methods with the help of the finite element computer programme OCEAN [12].

4.5 Results of experiments

Considering the mean values of individual instruments and assuming that computed displacements are linear until the fourth step, we obtain the following values in millimetres:

Table 1. The obtained displacements for steps 1, 2, 3 and 4

instrument/method	*step 1*	*step 2*	*step 3*	*step 4*
total station	*0.438*	*0.857*	*1.250*	*1.776*
level	*0.399*	*0.737*	*1.110*	*1.634*
inductive transducer	*0.357*	*0.719*	*1.109*	*1.535*
rotation level	*0.392*	*0.770*	*1.276*	*1.975*
analitical method (EC2)	*0.397*	*0.793*	*1.189*	*1.731*
numerical method (Ocean→)	*0.565*	*0.884*	*1.201*	*1.670*



Graph 1. Comparison of displacements

It is evident from Graph 1, that the correspondence with predicted displacements is the highest when measurements are performed with the level method and the lowest when they are performed with the method of the rotation level. It is also evident from the graph that the inductive transducer gives the most balanced results, while the least balanced ones are obtained with the rotation level.

5. ANALYSIS OF METHODS BASED ON THE COMPARISON OF PREDICTED AND OBTAINED RESULTS

The method of **classic levelling** is the most frequently used method for determining displacements. The results show that measurements were performed continuously. In any case, the accuracy of the results depends on the instruments and equipment used. For precise measurements of vertical displacements, it is suitable to use a digital level with the invar levelling rod, and take several readings in every measuring point. We can claim that this method is the most adequate of all geodetic methods for determining vertical displacements, although there is a certain time delay in taking readings with respect to other methods.

The method of determining vertical displacements with the **total station** has shown itself the least accurate. The reason may be in the error when hitting reflexive targets. The method is suitable for determining displacements of the class larger than 1 cm, where the required accuracy is 1/10 of the predicted displacement. The accuracy of the instrument is ±0.3 mm according to the producer. Thus, the results smaller than 1cm are not adequate for further processing. On the other hand, this method is suitable because it yields immediate results so that field measurements can be easily controlled. This method will also become more accurate with an improved technology.

The method of the rotation level is a simple working method although the results do not fulfil the demands for accuracy. As with the method of total station, this method is also suitable for displacements larger than 1 cm, because the accuracy, according to the producer, is ±0.5 mm at the greatest distance, and ±0.1 mm at the ideal distance of the sensor from the rotation level (up to 50 m). The problem with this method lies in the communication link, since all sensors must be linked with a communication cable in sequence; each sensor demands the voltage of 110 V which is not always available in the field.

The method of inductive transducer has given the most accurate results. Certainly, this method cannot be compared with geodetic methods which produce far more readings in each series.

We obtained approximately 15.000 readings on the average for each loading step. When the structure enables the placement of inductive transducers, the method is very suitable for determining vertical displacements; in most cases, unfortunately, the correct placement of transducers cannot be assured. The reason is that the access to central area is not possible with structures higher than 8 meters. In practical applications, therefore, inductive transducers are placed at dilatation points and possibly also in the first and last area of structure.

6. CONCLUSION

The analysis of inspections allows not only to verify static calculations and the quality of the structure but also to make their corrections by influencing the production technology. The necessity of such analysis is justified also by the fact that static calculations were based on predictions which did not represent real work of either building materials or structures. The verification of static calculations started in the thirties, especially regarding the calculations of limit conditions of structures, which required extensive experimental work.

We have been investigating and measuring vertical displacements and deformations for many years. During this time, we have performed about 50 experiments of loading structures and studied them with different methods, either geodetic or others. We have tested bridges of different dimensions, prestressed concrete plates and beams of load-bearing walls.

Several methods and instruments exist for the identification of displacements of this class.

In our experiment, we confined ourselves to available instruments and performed measurements with 4 methods. The obtained results and their analysis provided us with the knowledge about the applicability of methods for determining the so-called micro-displacements.

The determination of vertical displacements and deformations of different structures in space requires high accuracy of measurements which mostly depends on the instruments used and on conditions at work.

The vertical displacements were measured on a prestressed concrete plate of the PVP type with a uniform loading in four steps. The loading was applied in the laboratory with a constant temperature and pressure. In this way, it was possible to avoid errors that might influence the measurements.

For the precise determination of vertical displacements and deformations it is best to use several methods simultaneously. This, however, partly increases the costs and duration of measurements; these two factors, however, can be neglected in investigations of such sophistication. The advantages are the possibility of controlling measurements, comparing and analysing the results and presenting them in a clearer way.

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ODREĐIVANJE VERTIKALNIH POMAKA BETONSKIH PLO^A POMO] U RAZLI^ITIH METODA

SA@ETAK

U radu su predstavljene metode za odre|ivanje vertikalnih pomaka na konkretnom primjeru prednapete armirano betonske plo~e. U tu svrhu upotrijebili smo geodetske (nivelir, totalna stanica i laserski nivelir) i negeodetske (induktivne mjera~e) instrumente.

Predvi|eni teorijski pomaci s mogu}om pojavom naprslina izra~unati su pomo}u analiti~kih i numeri~kih metoda, uz primjenu europskih propisa za armirani i prednapeti beton.

***Klju~ne rije~i**: geodezija, statika, izmjera, pomaci, numeri~ka analiza.*