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The transformation of internal waves penetrating permeable structures

Chi-Min Liu(1), **Chin-Hwa Kong**(2) and **Chi-Kuo Lee**(3)
(1)Chien Kuo Institute of Technology, Chunghua City, Chunghua, TAIWAN, R.O.C.
(2)Department of Engineering Science and Ocean Engineering, National Taiwan University, Taipei, TAIWAN, R.O.C.
(3)Department of Marine Mechanical Engineering, Chinese Naval Academy, Kaohsiung, TAIWAN, R.O.C., e-mail: jimmyliu@ms27.url.com.tw

SUMMARY

This paper presents the interaction between the internal waves and the permeable wall. The permeable structures have been widely applied to coastal structures such as breakwaters. Not only the porous structure can reduce the energy of the reflected waves, but also it can prolong the life itself. Therefore it is important for engineers to evaluate the dynamic reaction of the permeable wall in order to design the best breakwater. In this study, we will treat the two face-flow field with the irrotational, incompressible and inviscid fluid. The perturbation method is applied to solve the problem with the small coefficient, the porous Reynolds number, which will be well defined in the following article. The permeability and the width of the wall will both deeply influence the interaction of internal waves and the wall.

Key words: *permeable structures, coastal structures, breakwaters, two face-flow field, perturbation method.*

1. FORMULATION

First of all, the flow field will be divided into three regions shown in Figure 1: permeable wall (*P*), incident region (*I*) and transmitted region (*T*). A two-layer flow exists in each region. Based on the theory derived by Biot [1], the frame of the wall is assumed to be rigid and the fluid in the wall is incompressible. The continuity equation and momentum equations in region *P* are:

$$\nabla\left(\hat{v}_p\right)_i = 0 \quad (i = 1,2) \tag{1}$$

$$\rho_i \frac{\partial\left(\hat{v}_p\right)_i}{\partial\hat{t}} = -\nabla\left(\hat{p}_p\right)_i + \rho_i g - \frac{\mu_i n_0 \hat{w}\hat{d}}{k_s \hat{A}_w}\left(\hat{v}_p\right)_i \quad (i = 1,2) \tag{2}$$

where μ_i indicates the dynamic viscosity, n_0 is the porous ratio, k_s represents the specific permeability, $\hat{d}$ is the thickness of the wall and $\hat{A}_w$ is the wall's submerged area. And the velocity potential can be assumed as:

$$\left(\hat{v}_p\right)_i = -\frac{\rho_i k_s \hat{A}_w}{\mu_i n_0 \hat{d}} \nabla\left(\phi_p\right)_i \quad (i = 1,2) \tag{3}$$

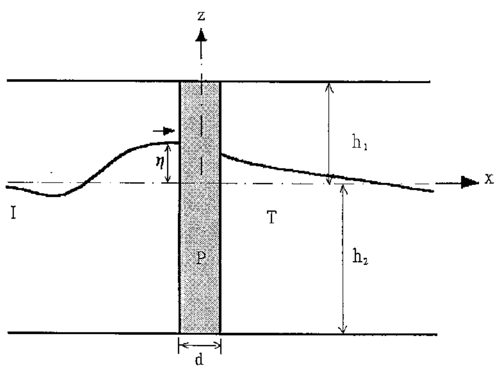

Fig. 1 Internal waves and a permeable wall

Taking Eq. (3) into Eq. (1) and Eq. (2), it yields:

$$\nabla\left(\phi_p\right)_i = 0 \quad (i = 1,2) \tag{4}$$

$$\frac{(p_p)_i}{\rho_i g} + z = \frac{w}{g}(\phi_p)_i + \frac{\rho_i k_s A_w}{\mu_i n_0 g d}\frac{\partial(\phi_p)_i}{\partial t} + h_1 \quad (i=1,2) \quad (5)$$

where h_1 indicates the undisturbed average depth of the upper layer. As for the regions I and T, we also assume the fluid in these two regions to be incompressible and inviscid. The continuity equation and momentum equations are:

$$\nabla^2(\phi_I)_i = \nabla^2(\phi_T)_i = 0 \qquad (i=1,2) \qquad (6)$$

$$\frac{(p_I)_i}{\rho_i g} + z = -\frac{1}{g}\frac{\partial(\phi_I)_i}{\partial t} + h_1 \qquad (i=1,2) \qquad (7)$$

$$\frac{(p_T)_i}{\rho_i g} + z = -\frac{1}{g}\frac{\partial(\phi_T)_i}{\partial t} + h_1 \qquad (i=1,2) \qquad (8)$$

2. BOUNDARY CONDITIONS

To simplify the problem, we assume boundaries at $z=h_1$ and $z=-h_2$ are rigid, therefore:

$$\frac{\partial(\phi_I)_1}{\partial z} = \frac{\partial(\phi_T)_1}{\partial z} = \frac{\partial(\phi_p)_1}{\partial z} = 0 \quad (z=h_1) \qquad (9)$$

$$\frac{\partial(\phi_I)_2}{\partial z} = \frac{\partial(\phi_T)_2}{\partial z} = \frac{\partial(\phi_p)_2}{\partial z} = 0 \quad (z=-h_2) \quad (10)$$

The kinematic and dynamic free surface boundary conditions in region I are:

$$\frac{\partial(\phi_I)_i}{\partial z} = \frac{\partial(\eta_I)_i}{\partial t} \qquad (i=1,2) \qquad (11)$$

$$(\eta_I)_i = -\frac{1}{g}\frac{\partial(\phi_I)_i}{\partial t} \qquad (i=1,2) \qquad (12)$$

Equations (11) and (12) can be combined as:

$$\frac{\partial^2(\phi_I)_i}{\partial t^2} + g\frac{\partial(\phi_I)_i}{\partial z} = 0 \qquad (i=1,2) \qquad (13)$$

Similarly, in region T, the boundary conditions are:

$$(\eta_T)_i = -\frac{1}{g}\frac{\partial(\phi_T)_i}{\partial t} \qquad (i=1,2) \qquad (14)$$

$$\frac{\partial^2(\phi_T)_i}{\partial t^2} + g\frac{\partial(\phi_T)_i}{\partial z} = 0 \qquad (i=1,2) \qquad (15)$$

In the permeable wall, it yields:

$$-\frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial(\phi_p)_i}{\partial z} = \frac{\partial(\eta_p)_i}{\partial t} \qquad (i=1,2) \qquad (16)$$

$$(\eta_p)_i = \frac{w}{g}(\phi_p)_i + -\frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial(\phi_p)_i}{\partial t} \qquad (i=1,2) \qquad (17)$$

Therefore:

$$\frac{\partial^2(\phi_p)_i}{\partial t^2} + \frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial(\phi_p)_i}{\partial t} + g\frac{\partial(\phi_p)_i}{\partial z} = 0 \; (i=1,2) \quad (18)$$

At each interface of each two regions it must satisfy the following conditions:

$$-\frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial(\phi_p)_i}{\partial x} = \frac{\partial(\phi_I)_i}{\partial x} \quad (i=1,2)\left(x=-\frac{d}{2}\right) \quad (19)$$

$$-\frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial(\phi_p)_i}{\partial x} = \frac{\partial(\phi_T)_i}{\partial x} \quad (i=1,2)\left(x=\frac{d}{2}\right) \quad (20)$$

$$(\phi_p)_i + \frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial(\phi_p)_i}{\partial t} = -\frac{1}{g}\frac{\partial(\phi_I)_i}{\partial t} \quad (i=1,2)\left(x=-\frac{d}{2}\right) \quad (21)$$

$$(\phi_p)_i + \frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial(\phi_p)_i}{\partial t} = -\frac{1}{g}\frac{\partial(\phi_T)_i}{\partial t} \quad (i=1,2)\left(x=\frac{d}{2}\right) \quad (22)$$

The Sommerfeld radiation boundary condition must also be satisfied:

$$(\phi_I)_i(x \to -\infty) \Rightarrow outgoing \qquad (i=1,2) \quad (23)$$

$$(\phi_T)_i(x \to \infty) \Rightarrow outgoing \qquad (i=1,2) \quad (24)$$

3. BOUNDARY VALUE PROBLEM

We assume the velocity potential to be periodic, $\phi=exp(iwt)$, and define the porous Reynold number as $R=\frac{\rho k k_s A_w}{\mu_i n_0 d}$. Taking both the velocity potential and the Reynold number into the above equations, it yields:

In region I

$$\nabla^2(\phi_I)_1 = \nabla^2(\phi_I)_2 = 0 \quad \left(-\infty < x < -\tfrac{d}{2}, -h_2 < z < h_1\right) \quad (25)$$

$$\frac{\partial(\phi_I)_1}{\partial z} = 0 \qquad \left(-\infty < x < -\frac{d}{2}, z=h_1\right) \quad (26)$$

$$\frac{\partial(\phi_I)_2}{\partial z} = 0 \qquad \left(-\infty < x < -\frac{d}{2}, z=-h_2\right) \quad (27)$$

$$(\phi_I)_i - \frac{g}{w^2}\frac{\partial(\phi_p)_i}{\partial z} = 0 \quad \left(-\infty < x < -\frac{d}{2}, z=0\right) \quad (28)$$

$$\frac{\partial(\phi_I)_i}{\partial x} = -n_0 R\frac{\partial(\phi_p)_i}{\partial x} \quad \left(x=-\frac{d}{2}, -h_2 < z < h_1\right) \quad (29)$$

$$(\phi_I)_i(x \to -\infty) \Rightarrow outgoing \qquad (i=1,2) \quad (30)$$

In region T

$$\nabla^2(\phi_T)_1 = \nabla^2(\phi_T)_2 = 0 \quad \left(\frac{d}{2} < x < \infty, -h_2 < z < h_1\right) \quad (31)$$

$$\frac{\partial(\phi_T)_1}{\partial z} = 0 \qquad \left(\frac{d}{2} < x < \infty, z=h_1\right) \quad (32)$$

$$\frac{\partial(\phi_T)_2}{\partial z} = 0 \qquad \left(\frac{d}{2} < x < \infty, z = -h_2\right) \tag{33}$$

$$(\phi_T)_i - \frac{g}{w^2}\frac{\partial(\phi_T)_i}{\partial z} = 0 \qquad \left(\frac{d}{2} < x < \infty, z = 0\right) \tag{34}$$

$$\frac{\partial(\phi_T)_i}{\partial x} = -n_0 R \frac{\partial(\phi_p)_i}{\partial x} \qquad \left(x = \frac{d}{2}, -h_2 < z < h_1\right) \tag{35}$$

$$(\phi_T)_i (x \to -\infty) \Rightarrow outgoing \qquad (i = 1,2) \tag{36}$$

In region P

$$\nabla^2(\phi_p)_1 = \nabla^2(\phi_p)_2 = 0 \qquad \left(-\frac{d}{2} < x < \frac{d}{2}, -h_2 < z < h_1\right) \tag{37}$$

$$\frac{\partial(\phi_p)_1}{\partial z} = 0 \qquad \left(-\frac{d}{2} < x < \frac{d}{2}, z = h_1\right) \tag{38}$$

$$\frac{\partial(\phi_p)_2}{\partial z} = 0 \qquad \left(-\frac{d}{2} < x < \frac{d}{2}, z = -h_2\right) \tag{39}$$

$$(\phi_p)_i - \frac{g}{w^2}\frac{\partial(\phi_p)_i}{\partial z} = 0 \qquad \left(-\frac{d}{2} < x < \frac{d}{2}, z = 0\right) \tag{40}$$

$$(\phi_p)_i + i\left[Rw(\phi_p)_i + \frac{w}{g}(\phi_I)_i\right] = 0 \qquad \left(x = -\frac{d}{2} - h_2 \le z \le h_1\right) \tag{41}$$

$$(\phi_p)_i + i\left[Rw(\phi_p)_i + \frac{w}{g}(\phi_T)_i\right] = 0 \qquad \left(x = \frac{d}{2} - h_2 \le z \le h_1\right) \tag{42}$$

4. ANALYSIS

Most of natural porous media should satisfy the following relation:

$$n_0 R = \frac{\rho k_s A_w}{\mu_i d} \sim \left(10^{-8} \sim 1\right) \tag{43}$$

We assume $R<<1$, therefore:

$$(v_I)_i \sim O[(\phi_I)_i] \tag{44}$$

$$(v_T)_i \sim O[(\phi_T)_i] \tag{45}$$

$$(v_p)_i \sim O[R(\phi_p)_i] \tag{46}$$

Now $(\phi_I)_i$, $(\phi_T)_i$ and $(\phi_p)_i$ can be expanded [2]:

$$(\phi_I)_i = (\phi_I)_i^{(0)} + (\phi_I)_i^{(00)} + R(\phi_I)_i^{(1)} + O(R^2) \tag{47}$$

$$(\phi_T)_i = R(\phi_T)_i^{(1)} + O(R^2) \tag{48}$$

$$(\phi_p)_i = (\phi_p)_i^{(1)} + R(\phi_p)_i^{(2)} + O(R^2) \tag{49}$$

where $(\phi_I)_i^{(0)}$ indicates the incident condition. We take Eq. (47) to Eq. (49) into all governing equations and assume the incident wave to be:

$$\eta_I = a_4 \exp\left[-ik_0\left(x + \frac{d}{2}\right)\right]\exp(iwt) \qquad (z = 0) \tag{50}$$

Therefore:

$$(\phi_I)_1^{(0)} = a_1^{(0)} \exp\left[-ik_0\left(x + \frac{d}{2}\right)\right]\cosh[k_o(z - h_1)] \tag{51}$$

$$(\phi_I)_2^{(0)} = a_2^{(0)} \exp\left[-ik_0\left(x + \frac{d}{2}\right)\right]\cosh[k_o(z + h_2)] \tag{52}$$

$$(\phi_I)_1^{(00)} = a_1^{(0)} \exp\left[ik_0\left(x + \frac{d}{2}\right)\right]\cosh[k_o(z - h_1)] \tag{53}$$

$$(\phi_I)_2^{(00)} = a_2^{(0)} \exp\left[ik_0\left(x + \frac{d}{2}\right)\right]\cosh[k_o(z + h_2)] \tag{54}$$

Taking Eq. (51) to Eq. (54) into Eq. (12), it yields:

$$a_1^{(0)} = \frac{ia_4 g}{w\cosh(-k_0 h_1)} = \frac{iaw}{k\cosh(-k_0 h_1)}\frac{\coth(kh_2) + r_\rho \coth(kh_1)}{1 - r_\rho} \tag{55}$$

$$a_2^{(0)} = \frac{ia_4 g}{w\cosh(k_0 h_2)} = \frac{iaw}{k\cosh(k_0 h_2)}\frac{\coth(kh_2) + r_\rho \coth(kh_1)}{1 - r_\rho} \tag{56}$$

From governing equations and boundary conditions, all velocity potentials are:

$$(\phi_p)_1^{(1)} = 8\pi a_4 \sum_{n=1}^{\infty}\left(\prod_{n0} 1\right)(-1)^{n-1} \cdot \frac{\sinh\left[(\alpha_n)_1\left(x - \frac{d}{2}\right)\right]}{\sinh[(\alpha_n)_1 d]}\cos[(\alpha_n)_1(z - h_1)] \tag{57}$$

$$(\phi_p)_2^{(1)} = 8\pi a_4 \sum_{n=1}^{\infty}\left(\prod_{n0} 2\right)(-1)^{n-1} \cdot \frac{\sinh\left[(\alpha_n)_2\left(x - \frac{d}{2}\right)\right]}{\sinh[(\alpha_2)_1 d]}\cos[(\alpha_n)_2(z + h_2)] \tag{58}$$

$$(\phi_T)_1^{(1)} = (a_T)_1^{(1)} exp\left[ik_0\left(x - \frac{d}{2}\right)\right] cosh[k_0(z - h_1)] + \sum_{j=1}^{\infty} (b_T)_{1j}^{(1)} exp\left[k_j\left(x - \frac{d}{2}\right)\right] cosh[k_j(z - h_1)] \quad (59)$$

$$(\phi_T)_2^{(1)} = (a_T)_2^{(1)} exp\left[ik_0\left(x - \frac{d}{2}\right)\right] cosh[k_0(z - h_1)] + \sum_{j=1}^{\infty} (b_T)_{2j}^{(1)} exp\left[k_j\left(x - \frac{d}{2}\right)\right] cosh[k_j(z + h_2)] \quad (60)$$

$$(\phi_I)_1^{(1)} = (a_I)_1^{(1)} exp\left[-ik_0\left(x - \frac{d}{2}\right)\right] cosh[k_0(z - h_1)] + \sum_{j=1}^{\infty} (b_I)_{1j}^{(1)} exp\left[-k_j\left(x - \frac{d}{2}\right)\right] cosh[k_j(z - h_1)] \quad (61)$$

$$(\phi_I)_2^{(1)} = (a_I)_2^{(1)} exp\left[-ik_0\left(x - \frac{d}{2}\right)\right] cosh[k_0(z + h_2)] + \sum_{j=1}^{\infty} (b_I)_{2j}^{(1)} exp\left[-k_j\left(x - \frac{d}{2}\right)\right] cosh[k_j(z - h_1)] \quad (62)$$

5. RESULTS AND DISCUSSION

Inserting Eq. (57) and Eq. (59) into Eq. (3), we get the velocity inside the wall:

$$\frac{(v_p)_1^{(1)}}{a_4 w} = 4\pi^2 C_{w1} R \sum_{n=1}^{\infty} (-1)^{n-1} \left(\prod_{n0} 1\right) \frac{cosh\left[(\alpha_n)_1\left(x + \frac{d}{2}\right)\right]}{sinh[(\alpha_n)_1 d]} cos[(\alpha_n)_1(z - h_1)] cos\, wt \quad (63)$$

$$\frac{(v_p)_2^{(1)}}{a_4 w} = 4\pi^2 C_{w2} R \sum_{n=1}^{\infty} (-1)^{n-1} \left(\prod_{n0} 2\right) \frac{cosh\left[(\alpha_n)_2\left(x + \frac{d}{2}\right)\right]}{sinh[(\alpha_n)_2 d]} cos[(\alpha_n)_2(z + h_2)] cos\, wt \quad (64)$$

with:

$$C_{w1} = \frac{g}{w^2 h_1},\ C_{w2} = \frac{g}{w^2 h_2} \quad (65)$$

The pressure inside the wall is:

$$\frac{(p_p)_1^{(1)}}{\rho g a_4} = \sum_{n=1}^{\infty} \frac{8\pi a(-1)^{n-1}}{2n-1} \left(\prod_{n0} 1\right) \frac{sinh\left[(\alpha_n)_1\left(x + \frac{d}{2}\right)\right]}{sinh[(\alpha_n)_1 d]} cos[(\alpha_n)_1(z - h_1)] cos\, wt \quad (66)$$

$$\frac{(p_p)_2^{(1)}}{\rho g a_4} = \sum_{n=1}^{\infty} \frac{8\pi a(-1)^{n-1}}{2n-1} \left(\prod_{n0} 2\right) \frac{sinh\left[(\alpha_n)_2\left(x + \frac{d}{2}\right)\right]}{sinh[(\alpha_n)_2 d]} cos[(\alpha_n)_2(z - h_1)] cos\, wt \quad (67)$$

The corresponding free surface profile is:

$$\begin{aligned}\frac{(\eta_T)_1^{(1)}}{a_4} = 8\pi^3 R n_0 C_{w1} \Bigg\{ & \frac{\sinh(k_0 h_1)}{w^2 h_1 + g \sinh^2(k_0 h_1)} \sum_{n=1}^{\infty} \left(\prod_{n0} 1\right)\left(\prod_{n0} 1\right) \frac{2n-1}{\sinh[(\alpha_n)_1 d]} \cdot \cos\left[wt + k_0\left(x - \frac{d}{2}\right)\right] \cosh[k_0(z - h_1)] + \\ & + \sum_{j=1}^{\infty} (b_T)_{1j}^{(1)} \frac{\sinh(k_j h_1)}{w^2 h_1 + g \sinh^2(k_j h_1)} \sum_{n=1}^{\infty} \left(\prod_{n0} 1\right)\left(\prod_{nj} 1\right) \frac{2n-1}{\sinh[(\alpha_n)_1 d]} \cdot exp\left[k_j\left(x - \frac{d}{2}\right)\right] \cosh(k_j h_1) \sin wt \Bigg\}\end{aligned} \quad (68)$$

$$\begin{aligned}\frac{(\eta_T)_2^{(1)}}{a_4} = 8\pi^3 R n_0 C_{w2} \Bigg\{ & \frac{\sinh(k_0 h_2)}{w^2 h_2 + g \sinh^2(k_0 h_2)} \sum_{n=1}^{\infty} \left(\prod_{n0} 2\right)\left(\prod_{n0} 2\right) \frac{2n-1}{\sinh[(\alpha_n)_2 d]} \cdot \cos\left[wt + k_0\left(x - \frac{d}{2}\right)\right] \cosh[k_0(z + h_2)] + \\ & + \sum_{j=1}^{\infty} (b_T)_{2j}^{(1)} \frac{\sinh(k_j h_2)}{w^2 h_2 + g \sinh^2(k_j h_2)} \sum_{n=1}^{\infty} \left(\prod_{n0} 2\right)\left(\prod_{nj} 2\right) \frac{2n-1}{\sinh[(\alpha_n)_2 d]} \cdot \exp\left[k_j\left(x - \frac{d}{2}\right)\right] \cosh(k_j h_2) \sin wt \Bigg\}\end{aligned} \quad (69)$$

When the porous Reynold number is small, the seapage phenomenon will occur at the interface of regions *P* and *I* or *T*, i.e. $\eta_P - \eta_I \neq 0$ and $\eta_P - \eta_T \neq 0$. From Eq. (68) and Eq. (69), there is a local disturbance due to the singularity at the interface. Because the porous Reynold number *R* is the ratio between the inertial force and the viscous force, the inertial effect may be neglected while *R* is very small. Then the permeability and the thickness of the wall will mainly affect the transmitted wave. The transmitted wave will be small while the value of *R* is small as well. However, if the thickness of the porous wall is greater than the water depth, the wall can be deemed impermeable. The permeable coefficient K_T defined as the ratio between the amplitude of the transmitted wave and that of the incident wave, is:

$$K_T = 8\pi^3 R n_0 C_{w2} \left[\frac{\sinh(k_0 h_2)}{w^2 h_2 + g \sinh^2(k_0 h_2)} \right] \cdot \sum_{n=1}^{\infty} \left(\prod_{n0}^{2} \right) \left(\prod_{n0}^{2} \right) \frac{2n-1}{\sinh[(\alpha_n)_2 d]} \cosh(k_0 h_2) \qquad (70)$$

The relations of K_T versus *R*, C_{w2} and $\frac{d}{2h_2}$ are displayed in Figures 2 to 5. Though some results are obtained, this problem is still worthy of studying for many purposes. The next goal we hope to achieve is how to simulate the practice more precisely.

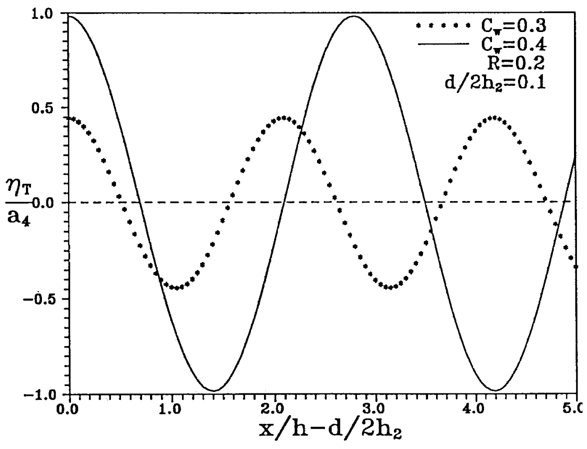

Fig. 2 The transmitted amplitude

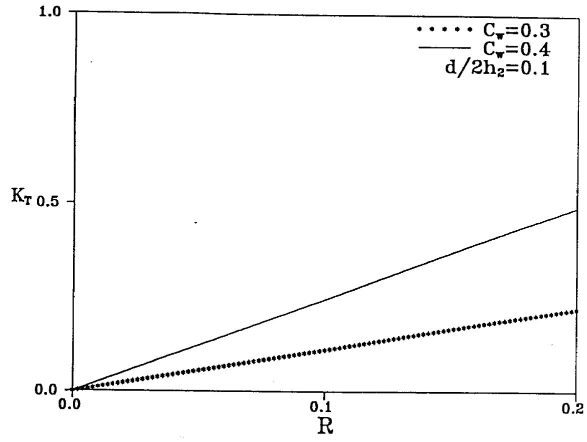

Fig. 3 The relation between K_T and R

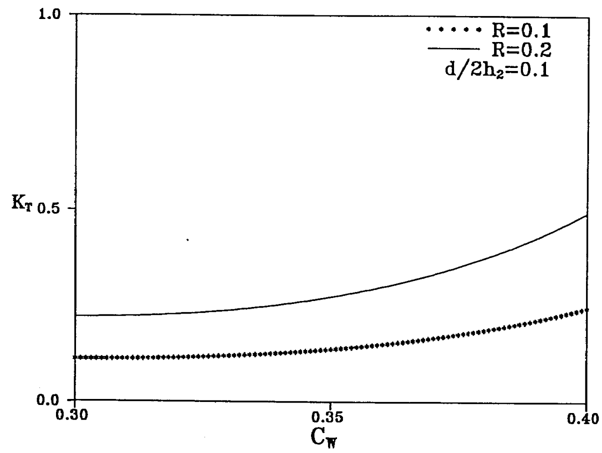

Fig. 4 The relation between K_T and C_W

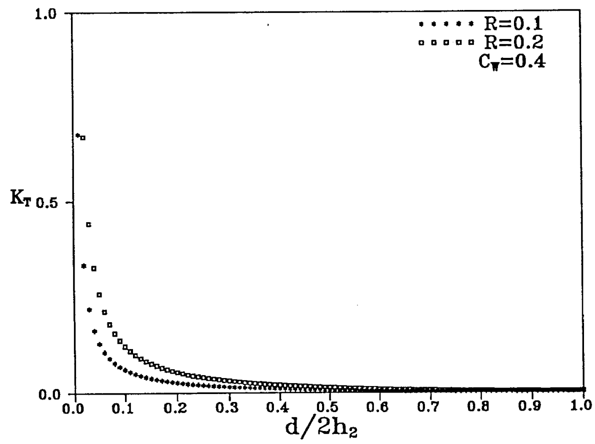

Fig. 5 The relation between K_T and $d/2h_2$

6. ACKONWLEDGEMENTS

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TRANSFORMACIJA UNUTRAŠNJIH VALOVA KOJI PRODIRU KROZ VODOPROPUSNE GRAĐEVINE

SAŽETAK

U radu se prikazuje interakcija između unutrašnjih valova i vodopropusnog zida. Propusne građevine često se koriste u obalnim predjelima za lukobrane. Ne samo što porozna građevina može smanjiti energiju odbijenih valova već može produžiti i sam vijek trajanja. Stoga je bitno da građevinari procjene dinamičku reakciju propusnog zida kako bi projektirali najbolji lukobran. U ovom radu bavimo se područjem toka s dvostrukim licem za nerotacijsku, nestišljivu i neljepljivu tekućinu. Metoda perturbacije primjenjuje se za rješavanje problema s malim koeficijentom, poroznim Reynold-ovim brojem, koji će biti precizno definiran u sljedećem radu. Propusnost i širina zida u velikoj će mjeri utjecati na interakciju unutrašnjih valova i zida.

Ključne riječi: *vodopropusna građevina, obalne građevine, lukobrani, polje toka s dvostrukim licem, metoda perturbacije.*