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# Optimization of metal sandwich plates with corrugated core

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## SUMMARY

*This paper intends to lay out a possible approach to optimizing the metal-metal sandwiche plates with corrugated core based on a number of assumptions and for certain selected cases. The objective functions and constraints are derived based on standard requirements of minimum weight and satisfaction of static, structural and design constraints.*

*The intention of the approach and optimization model presented here is to be able to define the optimum geometry (design) of the sandwich plate for given conditions. With an additional module also developed by the authors, one can design the tool (profiled rolls) that will generate the 'optimal' geometry of the corrugated core. In this sense, the approach is actually one of product development based on optimization. More particularly, starting from case-specific given load and optimality conditions one can numerically derive the optimized shape and dimensions of the sandwich and sequentially numerically derive the corresponding production tool geometry, thereby completing the optimized product development.*

***Key words***: *metal sandwich plates, optimization, objective functions, constraints.*

## 1. INTRODUCTION

Aluminum sandwich plates with corrugated aluminum core and glued bonds are used extensively in industry and construction, both as structural elements as well as cover plate elements and interior walls. The sandwich plates inheritably possess desirable properties such as high stiffness per unit mass, acceptable environmental resistance, low price (and mass) per unit area, low maintenance costs, acceptable production technology, acceptable visual properties, and other advantages over other types of plate elements.

The increasing usage of metal sandwich plates with both metal and non metal cores is demonstrated in numerous applications and design examples, for example [1, 2], also including respective design optimization [3]. The various types of sandwich cores can be manufactured at acceptable cost to provide low-weight structural elements with high structural performance, [4]. Optimization constraints for metal cores include failure modes of face plates yielding and wrinkling and core yielding and buckling. Many types of sandwich plates are considered, ranging from stiffened plates, I, V and X- shaped cores, honeycomb and corrugated cores. Applications range from shipbuilding to architecture and construction.

In this paper, design optimization of simple corrugated core aluminum sandwich plates for applications defined by respective geometric conditions and loading is considered by applying nonlinear programming methodology, [5-9]. Zero-order optimizers and gradient-based solvers can be applied with constraint penalization, efficient quasi-Newton based direct approaches are also feasible for this problem. Genetic algorithm-based optimizers [10, 11] were also applied with penalty based formulations. The standard optimization models are applied with minimum weight excellence criteria and structural constraints. More advanced formulations based on general multicriterial decision making, were the focus of many researchers in recent years, [12]. In the context of optimum design of corrugated core metal sandwich plates, the simpler approach of an a-priori compromise formulation based on total value aggregation is applied with some of the case studies.

In terms of the respective numerical optimization platform, the MATLAB optimization package [13] was applied with constraints evaluated both by using approximate simplified expressions and by coupling with external finite analysis (ADINA$^{TM}$, [14]) with data exchange via ASCII files.

The optimization approach presented here is based on the assumption that the corrugated core will be produced from simple aluminum plane plates by the process of continuous rolling. The shaped rolls are furthermore assumed to be produced by extrusion, while the extrusion matrix is defined by the simulation (additional program module) of the geer teeth production process based on the shape of the corrugated core obtained by optimization. The elastic springback is currently disregarded (estimated from experience), but the improved models should account for the geometric impact of elastic springback by including the necessary corrections in the geometry.

The idea is one of the optimal product development process. For given constraint conditions, optimal values of the dimensions and shape of the metal sandwich are determined based on the nonlinear programming approach, whereby significant savings in material (both aluminum and glue) are expected to be achieved in mass production of the respective plates. For the optimized sandwich plates, the corresponding geometry of the profiled rolls (Figure 1) is numerically determined, closing the 'product development cycle'.

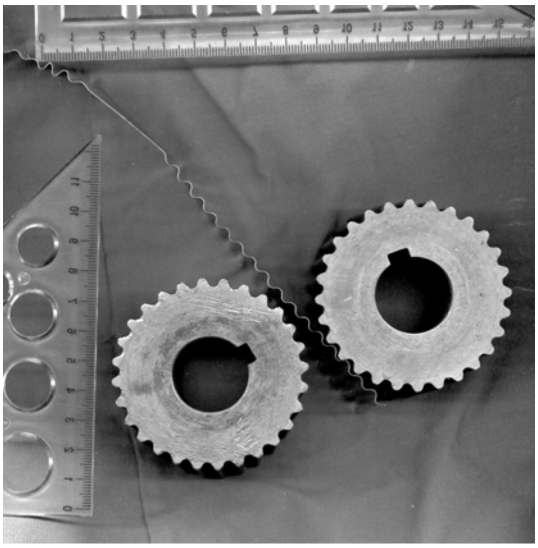

*Fig. 1 Production of test - samples of aluminum sandwich plates - corrugated core*

## 2. BASIC DEFINITIONS

The sandwich plate is simplified and defined in Figure 2. The essential design variables are the respective thicknesses of the outer aluminum plate and of the corrugated core and the 'shape' of the corrugated core. The outer thickness of the sandwich *2H*, loading *F*, and boundary conditions for the sample specimen with width *b* and length *l* are given.

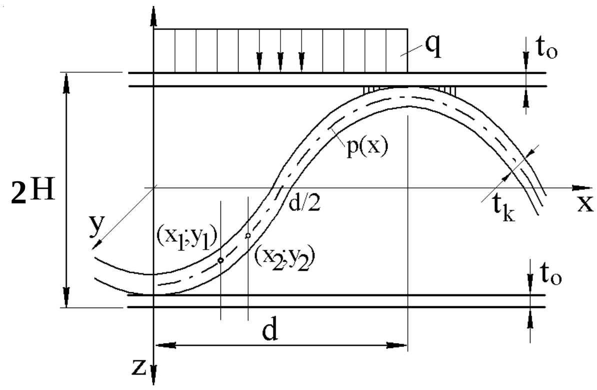


*Fig. 2 Geometry of the metal sandwich optimization problem*

### 2.1 Optimization variables

With the chosen optimization model, the optimization variables include the thicknesses of the outside plates $t_o$, the thickness of the corrugated core $t_k$, the 'half- wavelength' *d*, and the shape of the corrugated core.

The continuous corrugated core shape needs to be described by a finite set of discrete variables, the discretization is in this paper accomplished by means of numeric interpolation. The coordinates of a number of discrete interpolation points serve as optimization variables, while the 'shape' of the corrugated core is obtained by polinomial fitting with $C_1$ continuity through these points (or prescribed local slopes or even prescribed local curvatures). The method of piece-wise polinomial interpolation with $1^{st}$ and $2^{nd}$ order continuity is selected here. A varying number of interpolation points with corresponding piece-wise ranges and degrees of the polinomials were applied with various simulation cases.

The selected design (optimization) variables are:

$t_o$, $t_k$,

$y_i$ in $(x_i, y_i)$, $i=1, mt$,

where *mt* is the number of interpolation points with given coordinate values, respectively.

### 2.2 Shape of the corrugated core as design variable

The problem of having the shape of the corrugated core as one of the key design variables with major impact both on the objective function and the constraint conditions leeds to the problem that the ‘shape of the corrugated core’ needs to be represented by several discrete variables, in order to be able to apply the regular nonlinear programming approach. The solutions therefore could for example include piecewise polinomial interpolation techniques that preserve $C_1$ (and eventually $C_2$ and higher) continuity (Figure 3), such as cubic splines and other interpolation methods, whereby the internal parameters of the interpolating curves become the design (optimization) variables representing the shape of the corrugated core. The degree of the interpolating curves and the degree of the continuity between the segments is naturally chosen to provide a sufficient degree of freedom for the curve in order to be able to model the various geometric and technological constraints that are given (eg. minimum radius of curvature).

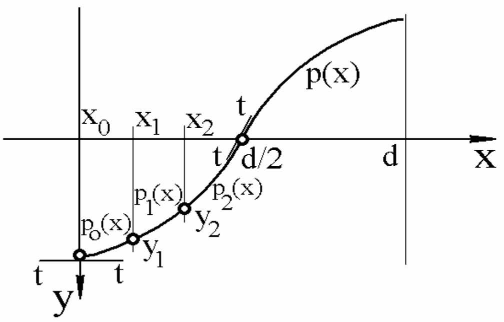


*Fig. 3 Piece-wise polinomial interpolation of the shape of corrugated core (basic 3-segment version shown)*

The simplest method used here is interpolation with piecewise polinomials with continuity of slope at points of contact. This preserves simplicity and still provides the possibility of increasing interpolation quality. Increasing the number of segments, choosing higher degrees of the interpolation polinomials and imposing higher-order continuity at the points of contact results in an increased number of the respective degrees of freedom of the overall interpolating curves that can be deployed with more demanding constraint conditions.

The simplest version (basic model) used is in setting three segments with the following conditions:

$$p_i\left(x_i\right) = y_i \qquad (1)$$

$$p_i\left(x_{i+1}\right) = y_{i+1} \qquad (2)$$

$$p_i{}'\left(x_{i+1}\right) = p_{i+1}{}'\left(x_{i+1}\right) \qquad (3)$$

for all segments, with zero-slope at the amplitudes of the corrugated core (and optionally zero curvature at $x=d/2$). Several other sets of conditions were also applied. Another simple case is piecewise interpolation with $C_2$ continuity, where the additional continuity requirement:

$$p_i{}''\left(x_{i+1}\right) = p_{i+1}{}''\left(x_{i+1}\right) \qquad (4)$$

is imposed at points of contact.

Defined in this way, the coordinate values of the interpolation points become the design - optimization variables that define the shape of the corrugated core. The current values of these variables (steered and ‘refreshed’ directly by the optimization algorithm) are successively used in each iteration of the optimization process to numerically evaluate the corresponding ‘current’ values of the length of the corrugated core, respective approximate stresses, length of the glued bonds, etc.

The piecewise interpolation approach should for real ‘industrial’ cases provide sufficient generality and a capacity to model the corrugated core shapes up to the proximity of the extreme cases of triangular and trapezoid shape. Piecewise interpolation with higher-order polinomials, higher-order continuity and an increased number of segments can provide higher flexibility to deliver this capacity of the overall curve.

## 2.3 Objective functions

For reasons of model simplicity only the cost of aluminum is currently considered in the objective function:

$$f = 2 \cdot t_o + t_k \cdot \int dl = 2 \cdot t_o + t_k \cdot \int \sqrt{1+(y'(x))^2}\, dx \qquad (5)$$

The integrals as well as the slopes and curvatures, are evaluated numerically for current values of the piece-wise interpolation polinomials parameters.

The objective function may also include minimization of the amount of the glue material needed in the sandwich, based on the optimized shape of the corrugated core and the constraint given later by Eq. (12).

Other elements could also be included in the objective function, such as tool depreciation (as a function of complexity of the resulting profiled roll) and tool wear-off, production cost as a function of design, maintenance costs, cost of glued bonds, etc, which would lead towards defining the objective function more realistically in terms of the respective total cost of operation (TCO).

## 2.4 Constraint functions

Considering only the simplified structural model of the sandwich plate which is used at this point, the following are the main corresponding constraints in the optimization model:

- tensile strength of the outer aluminum plates [14]:

$$\sigma_o \leq \sigma_M \qquad (6)$$

- local buckling resistance of the outer aluminum plates [15]:

$$p_o \leq p_{cr} \qquad (7)$$

- strength under compression + bending of the corrugated core [14]:

$$\sigma_i \leq \sigma_M \qquad (8)$$

- local buckling resistance of the corrugated core [16]:

$$p_i \le p_{cr} \tag{9}$$

- shear strength of the glued bonds:

$$\sigma_b \le \sigma_{M,b} \tag{10}$$

- minimum technologically feasible radius of curvature of the corrugated core shape $p(x)$ depending on the condition properties of the material used [17]:

$$R\{p''(x)\} \ge K_1 \tag{11}$$

- sufficient length of glued bonds between the corrugated core and the outer plates (given for example by corrugated core shape curvature or interpolated curve shape at contact points):

$$L\{p(x_b)\} \ge K_2 \tag{12}$$

- technical feasibility of the production of the inner corrugated plate shape using the process of shaped rolling, i.e. manufactureability of the corresponding profiled rolls that generate the corrugated core shape:

$$constraints\{p(x)\} \tag{13}$$

If the geometry and load is approximated as shown in Figure 4, then simplified expressions can be used in expanding the above constraints to the form adequate for nonlinear optimization purposes.

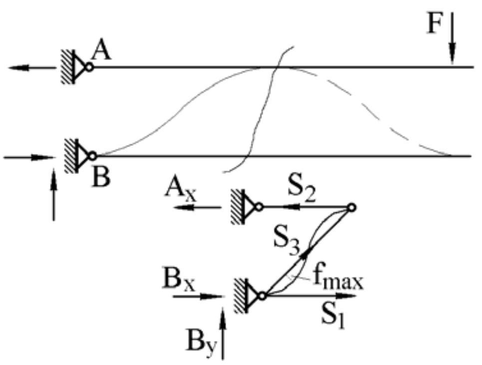


*Fig. 4 Simplified model of sandwich plate segment*

According to Ref. [16], for the case of even compressive load of a hinged plate, the following approximate expressions can be used for the critical buckling load related to buckling constraint, Eq. (7):

$$p_{kr} = K\frac{\pi^2 D}{b^2} \tag{14}$$

$$K = f(b,d) \tag{15}$$

$$D = \frac{E \cdot t_o^3}{12(1-\upsilon^2)} \tag{16}$$

whereby *2d* is the length of the outer plate of width *b* in local buckling (one wavelength of the corrugated core, between two glued amplitudes of the core), and D is the plate stiffness.

The constraint given by Eq. (8) is implemented as stress under combined loading under compression and bending. The bending load is calculated based on the maximum offset $f_{max}$ (Figure 4) which is numerically evaluated (one-dimensional search for maximum along the current curve) based on the interpolation of the current shape of the corrugated core.

As the final (optimal) shape of the corrugated core tends to a straight segment, the constraint given by Eq. (9) is implemented similar to the expressions in Eqs. (14) to (16), but for the close-to-straight segment of the corrugated core.

The constraint given by Eq. (10) is based on the data available for shear strength of glued bonds available in the manufacturer's specifications for epoxy and polyethylene glue. However, this is an area where the respective material data strongly depend on climatic conditions, preparation of surfaces, gluing process parameters and environmental conditions, etc.

The length of the glued bond is again evaluated numerically based on the interpolation of the corrugated core shape, Eqs. (1) to (3). The length of the glued joint (Figure 4) is a function of the maximum thickness of the glued bond and the shape of the corrugated core, which is numerically implemented as a one-dimensional search along the plate segment.

The constraint given by Eq. (11) is implemented based on the manufacturer's data for aluminum alloys used for the construction of the corrugated core, in this paper we used data from Ref. [17].

For the material used for the production of the corrugated core (*AlMg2.5*, hard condition, condition number *0.30*) the minimum feasible curvature as a function of thickness is given as:

| | | | | | |
|---|---|---|---|---|---|
| $t_o$ /mm/ | -0.8 | 0.8-1 | 1-1.5 | 1.5-2 | 2-3 |
| r /mm/ | 3.2 | 4.0 | 6.0 | 8.0 | 12.5 |

which is implemented as a corresponding polinomial approximation in the numerical model.

The actual curvature of the 'current' shape of the corrugated core is once again evaluated numerically based on the interpolated curve as:

$$R_{min} = \frac{\left(1+y'^2\right)^{3/2}}{y''} \tag{17}$$

where the differentials are derived from the piecewise polinomial of the current interpolated curve.

The actual definition of the constraints in Eq. (13) depends on the method of how the corrugated core is actually produced. In this approach, it is based on (continuous) rolling with profiled rolls, in which case, the constraints (13) will be numerically implemented by coupling the optimization functions package (Figure 4) with a function which simulates the process of production of gear teeth, which will be used to prevent shapes that would cause locking and other geometric inadequacies during the production of the corrugated core shape.

## 3. OPTIMIZATION APPROACH

The approach to optimizing the design of the aluminum sandwich plates is a classical one of multivariable nonlinear programming with constraints, [5-9, 18]. The full-scale design optimization would have to include optimization procedures numerically coupled with structural analysis to evaluate the constraint functions. At present, however, optimization and structural analysis are decoupled in a simplified model, where structural analysis is replaced by simplified 'equivalent models' with explicit expressions. More specifically, constraints are based on corresponding approximate formulas for buckling and bending from the Refs. [15, 16].

Plans for follow-up research include optimization fully coupled with structural analysis, as well as neural network based approximation of constraint functions which in the post-training phase may provide for the decoupling of structural analysis and optimization.

The simple test-case for optimization of the sandwich plate is shown in Figure 5.

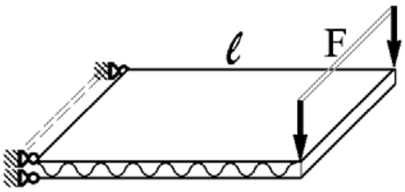


*Fig. 5 Simple test case for the metal sandwich optimization problem (width b)*

In this paper, the penalty-function approach is chosen in a combination with the unconstrained multivariable nonlinear programming methods. With this choice, slower convergence and ill-conditioned design points can appear. Care must be taken in the modified objective function to balance the individual penalty functions terms mutually and with respect to the original objective function.

In this paper, the exterior penalty function approach with inequality constraints was applied with:

$$F(\boldsymbol{x}) = f(\boldsymbol{x}) + r_i \cdot P(\boldsymbol{x}), \qquad r_{i+1} > r_i \tag{18}$$

$$P(\boldsymbol{x}) = \Sigma_k \left(g_k(\boldsymbol{x}) + abs\, g_k(\boldsymbol{x})\right) / 2 \tag{19}$$

where $f(\boldsymbol{x})$ being the original objective function defined by Eq. (5), $g_k(\boldsymbol{x})$ the *k-th* inequality constraint function, $P(\boldsymbol{x})$ the penalty function, $r_i$ the penalty constant in the *i-th* iteration, and $F(\boldsymbol{x})$ the modified 'unconstrained' composite objective function.

The nonlinear programming method deployed here belongs to the quasi-Newton family of methods, in particular the BFGS method, Refs. [5, 13]. These gradient methods make use of both the gradient information as well as the Hessian matrix, as the principle is based on substituting the original objective function by its second-order Taylor series expansion:

$$f\left(\boldsymbol{x}+\Delta\boldsymbol{x}\right) = f\left(\boldsymbol{x}\right) + \boldsymbol{c}^T \Delta\boldsymbol{x} + 0.5 \cdot \Delta\boldsymbol{x}^T \boldsymbol{H} \Delta\boldsymbol{x} \tag{20}$$

on which the optimality condition is imposed, which in turn provides the direction for the 1D line search steps as $\boldsymbol{H} \cdot \Delta\boldsymbol{x} = -\boldsymbol{c}$.

It is therefore generally expected that the optimization process is generically not guaranteed to converge, especially with remote initial solutions and highly nonlinear objective functions, where the quadratic approximation of the original function may not be adequate, therefore the descent condition $\nabla \boldsymbol{f} \cdot \boldsymbol{d} < 0$ needs to be monitored.

Moreover, in this paper the BFGS quasi-Newton method was applied, Ref. [13, 19], where the Hessian matrix is not directly evaluated during the iteration process, but the BFGS approximate of the Hessian is used and successively updated instead.

Similar problems with a higher degree of the complexity of the model, representing more realistic structural models would also be solved using direct optimization methods such as SQP, and have been also solved in this paper by evolutionary methods such as GAs.

## 4. OPTIMIZATION FLOWCHART

The optimization process is based on 'vertical' optimization routines and 'horizontal' problem-specific functionality module (numerical analysis) as shown in Figure 6.

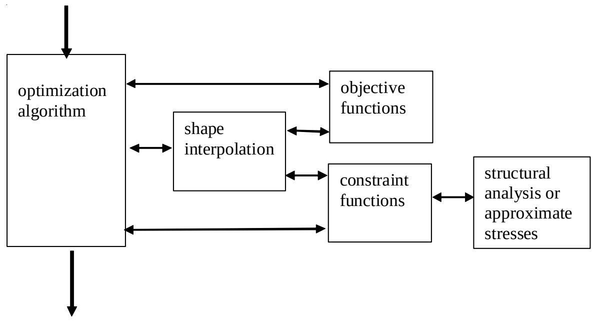


*Fig. 6 Optimization of metal sandwich plates - functional blocks*

During the course of the optimization, a number of fine-tunings of the control parameters were necessary. Before the penalty-function based version of the optimization was launched, the penalty terms for individual constraints and the actual objective function were mutually balanced and the parameter values for convergence determined experimentally.

In the actual implementation of the procedures in Figure 6, several specific numerical algorithms were used for particular blocks [12, 13, 20, 21].

## 5. NUMERICAL RESULTS

A number of numeric tests were performed with different optimization scenarios. Some of the variations included different values of control parameters of optimization methods, different relative weights in balancing the individual constraints and objective functions, variations in respective ranges and individual degrees of complexity of piece-wise polinomial fitting for corrugated core shapes definition, etc.

All optimization cases have been run for hardened aluminum with data and epoxy and polyethylene glued bonds. The assumed test values of dimensions, loads applied, and material properties are:

$H = 3\ mm$,
$d = 12\ mm$,
$b = 20\ mm$,
$F = 50\ N$,
$\sigma_{dop} = 100\ MPa$,
$\nu = 0.3$

Three segment piecewise polinomials with $C_1$ and $C_2$ continuity were used. The maximum thickness of the glued bonds was set to *0.2 mm*, and the minimum permissible length of the glued joint was chosen as *1.2 mm*.

The minimum permissible radius of curvature of the corrugated core shape was set to *3 mm* and depends on the core thickness.

The initial values of some design variables used as the starting point vector are:

$t_0 = 0.5\ mm$,
$t_k = 0.3\ mm$,
*point 1* $= (d/6, 0.7 \cdot H)$,
*point 2* $= (d/3, 0.4 \cdot H)$

In the respective graphs, typically one $d/2$ segment of the corrugated core shape is shown, with the initial shape designated as ‘poc’ and the final optimized shape denoted with ‘kon’.

Case 1

The initial test-case is the case of unconstrained optimization where the objective function is set to be equal to the length of the corrugated core with no constraints imposed, and first interpolation point set to $(0, H)$ instead of $(0, h)$, where $h=H-t_o-t_k/2$. The reason for this is that with $(0, h)$ the optimization process with the length of the corrrugated core as objective function pushes the optimum (minimum length) value of $(t_o+t_k/2)$ towards being equal to $H$, whereby the shape of the corrugated core degrades to the $x$-axis with the length of to $d/2$. Equivalent test-cases were run with the first point set to $(0, h)$ but with constraints defined on the plate thicknesses, in which case the plate thicknesses are pushed to their respective upper limits to minimize core length. These test-cases have, of course, no practical relevance.

In this case, the ‘optimal’ interpolation shape obtained as a result of length minimisation is shown in Figure 7 (one quarter of wave is plotted - segment $0$-$d/2$).

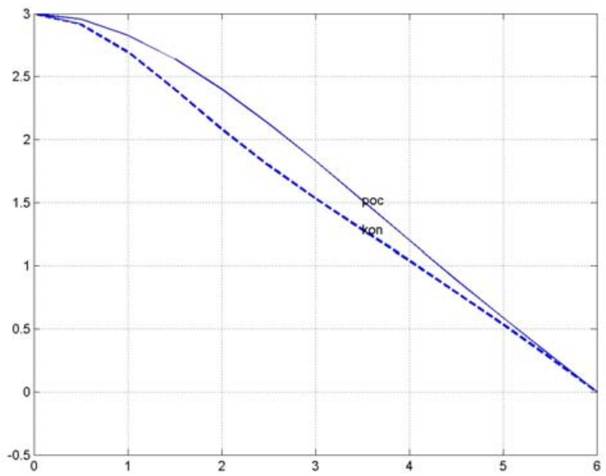


*Fig. 7 Unconstrained optimization of corrugated core case, minimum length of core*

In this case (quasi-unconstrained, minimum length), the length of the segment of the corrugated core is *6.74 mm*, which is here compared with other possible shapes of the corrugated core:

- triangle (through points $(0, H)$ and $(d/2, 0)$):
  * length $l = 6.708\ mm$ (relative factor $f = 1$)
- trapezoid (within $d/2$ segment: base $d/6$, side projection $d/6$, base $d/6$):
  * length $l = 7.606\ mm$ (factor $f = 1.134$)
- cosine wave (stretched through points $(0, H)$ and $(d/2, 0)$):
  * length $l = 6.839\ mm$ (factor $f = 1.02$)
- quadratic parabola (through points $(0, H)$ and $(d/2, 0)$):
  * length $l = 6.887\ mm$ (factor $f = 1.027$)
- interpolation shape by unconstrained optimization as in Figure 7, three-segments third-order piecewise polinomials with $C_2$ continuity:
  * length $l = 6.74\ mm$ (factor $f = 1.005$)
- interpolation shape by unconstrained optimization as in Figure 7, three-segments second-order piecewise polinomials with $C_1$ continuity:
  * length $l = 6.775\ mm$ (factor $f = 1.01$)

Case 2

The following test-case (Figure 8) is similar to the above unconstrained optimization case, but with the following assumptions (one half-wave is plotted for optimized shape):

- first interpolation point is now $(0, h)$ where $h=H-t_o-t_k/2$,
- limits on the thicknesses set to $(0.2 \le t_o \le 0.8)$ for the outer plates and $(0.1 \le t_K \le 0.5)$ for the corrugated core, respectively,
- objective function is now the sectional area of the sandwich plate rather than the length of corrugated core.

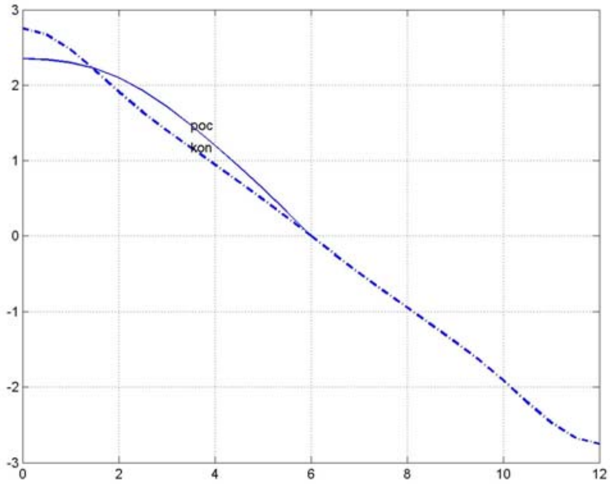


*Fig. 8 Minimization of sandwich mass with constraints on plate thicknesses*

In this case where the objective function is a set equal to the total mass of aluminum in the sandwich (both corrugated core and outer plates), the following (statically unfeasible, purely geometric test) solution is obtained: length of the corrugated core in the optimum is *6.62 mm*, longitudinal cross-section area is *3.06* $mm^2$, and both thickness-design variables are driven down (by the optimization process) to their respective limit values.

Case 3

Including only the constraints in Eqs. (6) and (7) related to the strength of the outer plates (tensile/ compressive stress, local buckling), the following optimal core shape is obtained (Figure 9) where in some 30 iterations an optimum with a length of the corrugated core of *6.45 mm* and cross-sectional area of aluminum (both core and outer plates) of *6.82* $mm^2$ is obtained, with corresponding thicknesses of $t_o=0.514$ *mm* and $t_k=0.1$ mm respectively.

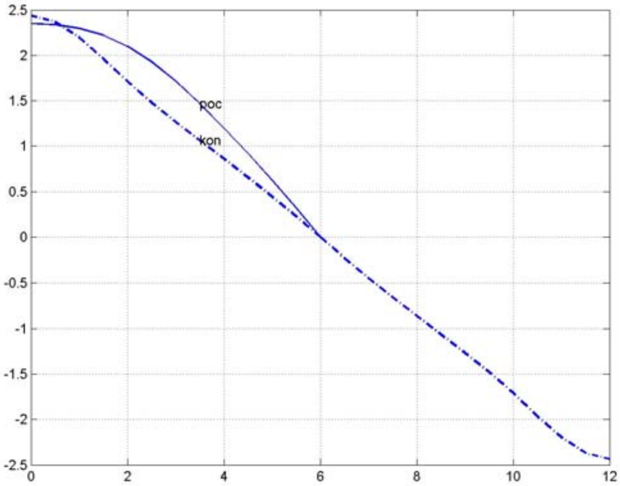


*Fig. 9 Optimization of the sandwich plate towards minimum mass of material with constraints on tensile/compressive strength and local buckling of outer plates*

Case 4

Adding the constraints in Eqs. (8) to (9) related to the strength of both the outer plates and the corrugated core, the following optimal core shape is obtained (Figure 10) where in some 50 iterations an optimum with a length of *6.476 mm* and total longitudinal cross sectional area of aluminum equal to *7.77* $mm^2$ is obtained, with corresponding thicknesses of $t_o=0.517$ *mm* and $t_k=0.243$ *mm* respectively, for the version with the total mass of aluminum in the sandwich as the objective function.

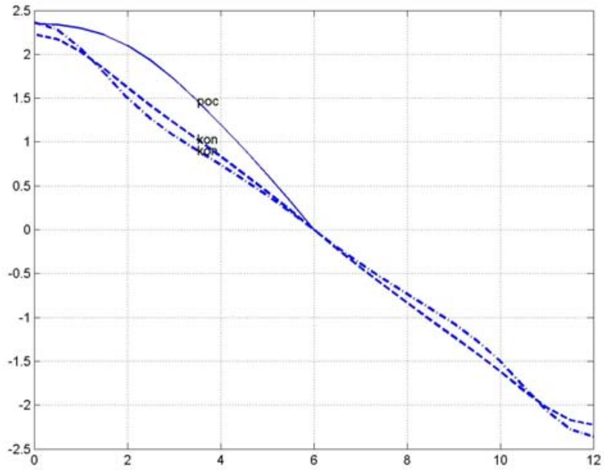


*Fig. 10 Optimization of sandwich plate, minimum mass of material (-.-) and minimum length of core (—), constraints on stresses and buckling of both outer plates and corrugated core*

Case 5

The next test case (Figure 11) also includes the constraints in Eqs. (10) to (12) on glued bond minimum length and shear strength, and additionally the constraint on minimum permissible radius of curvature of the corrugated core, i.e. all constraints listed except the technological one.

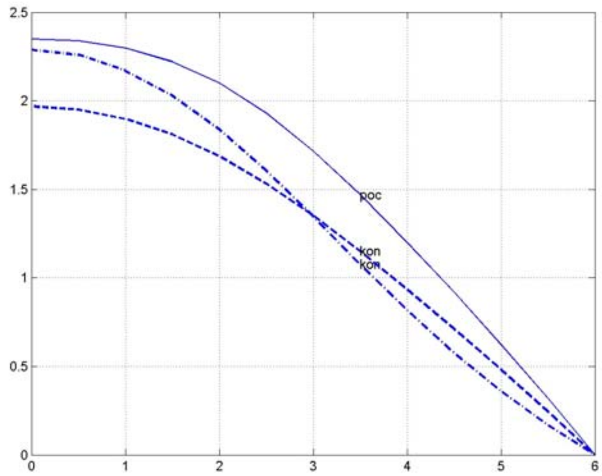


*Fig. 11 Optimization of sandwich plate, minimum mass of material (-.-) and minimum length of core (—), all constraints (6)-(12) except technological manufactureability*

In approximately 50 iterations, an optimum with a core length of *6.486 mm* and aluminum (outer plates and corrugated core) cross-sectional area of *8.785 mm*$^2$ is obtained, with the corresponding thicknesses of $t_o=0.517$ and $t_k=0.396$ *mm* respectively. The constraints are all satisfied within a given tolerance.

Case 6

Amongst many different numerical simulations for various optimization constraints, a few are shown here. The minimum required length of the glued joint was varied as a parameter of the respective constraint equation, Eq. (12), subject to the given precondition that the maximum possible thickness of the glued joint is *0.2 mm*. Since the 'gap' between the outer plate and the corrugated core is determined by the shape of the corrugated core, which is in turn a function of the optimization variables (interpolation of piecewise polinomials obtained by fitting through values of design/optimization variables), this constraint will obviously have an impact on the shape of the corrugated core in the optimum that satisfies the design constraints. Therefore, changes in the prescribed minimum length of the glued joints effectively change the constrained-optimum-shape of the corrugated core (Figure 12).

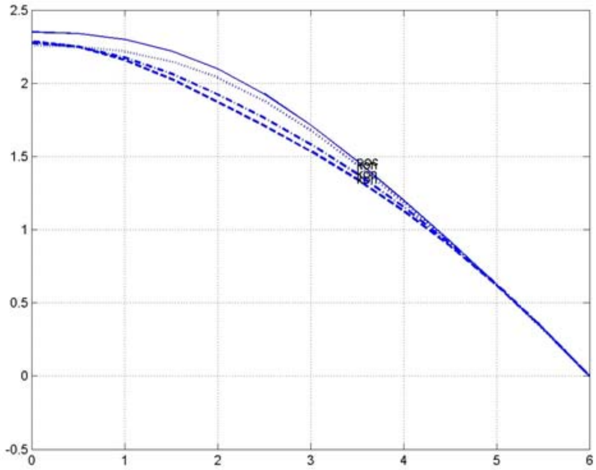

*Fig. 12 Change of the constrained optimum shape of the corrugated core for prescribed minimum values of glued bonds of 1.2 mm (—), 1.5 mm (-.-), and 1.8 mm (..) respectively, the objective function is a total cross-sectional area (mass) of aluminum plates*

Case 7

Another variation is the minimum permissible curvature of the corrugated core, numerically evaluated as 1-dimensional optimum of the respective expression on curvature in Eq. (11) along the interpolated curve. The results are as shown in Figure 13.

The obtained values of the plate thicknesses do not change significantly with the variations in Figures 12 and 13. Of course, higher-degree polinomials need to be used if more specific requirements on the geometry of the corrugated core need to be represented or imposed.

Test-cases with all constraints similar to the one in Figure 11 were also run with the objective function set equal to the length of the corrugated core, and minimum length of glued joint equal to *1.0 mm* and minimum radius of curvature at the amplitude of the corrugated core (point of glued bond between corrugated core and outer plate) set to *2 mm*. The parameters of variation were the interpolation curves deployed, such as the three-segment piecewise interpolation curve with $C_1$ continuity and the interpolation curve with $C_2$ continuity and qubic polinomials.

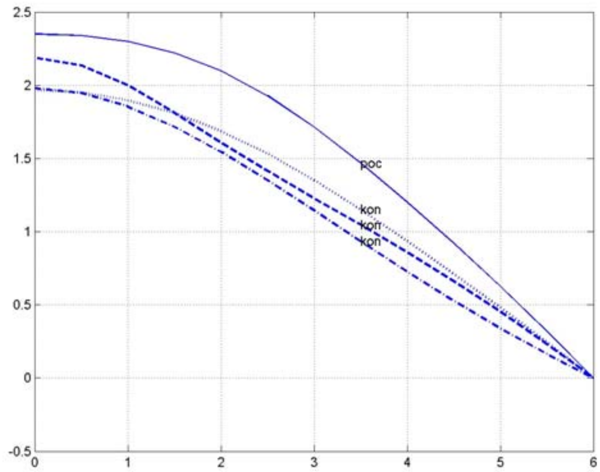


*Fig. 13 Change in the constrained optimum shape of the corrugated core for prescribed minimum values of radius of curvature of 3 mm (—), 3.5 mm (-.-) and 4 mm (..) respectively, objective function is the length of corrugated core*

Case 8

For the sake of mutual comparison, the case with all constraints as shown in Figure 11, in particular the case of the minimum length of core, was also optimized using the genetic algorithm (GA) method [10, 11, 13, 22, 23]. The results using the evolutionary approach fully coincide with the classical nonlinear programming approach from Figure 11, as the matter of fact, using the same program script function for the objective function and constraints, the following results were obtained (Figure 14) with $t_0=0.5194$ *mm*, $t_k=0.3957$ *mm*, core segment length equal to *6.4906 mm* and sandwich segment longitudinal cross-sectional area equal to *8.8397 mm*$^2$. A constant population of 20 members was used, in the respective value ranges given by:

$$(t_0\ t_k\ y_1\ y_2) \in (0.3\ 0.2\ 1.5\ 0\ ;\ 0.8\ 0.6\ 3\ 1.5)$$

whereby a member of the randomly generated initial population is shown in Figure 15. The convergence process of the population in terms of the average distance between the members of the population is shown in Figure 15.

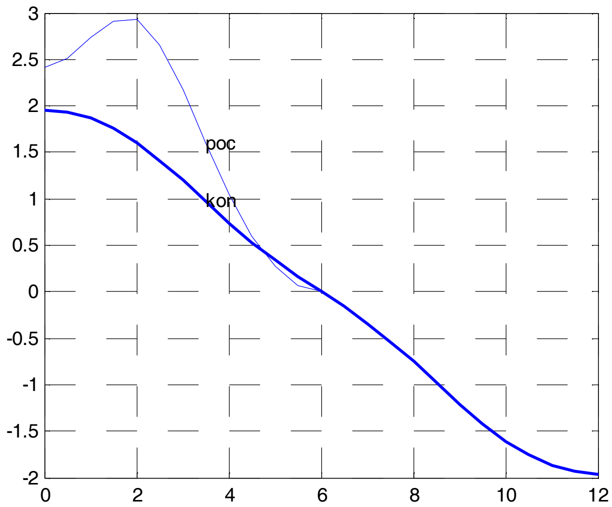


*Fig. 14 Genetic algorithm - based optimization of sandwich plate, minimum length of core (—), all constraints given by Eqs. (6)-(12) except technological manufactureability*

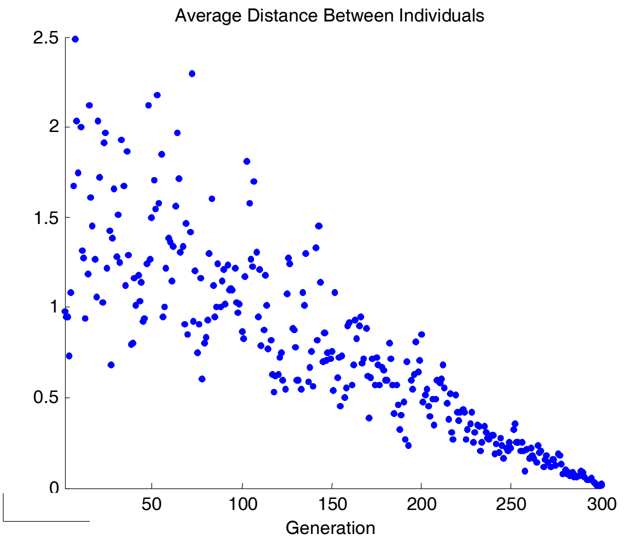


*Fig. 15 Convergence of population with the genetic algorithm - based optimization of sandwich plate shown in Figure 14*

As this is a paper with a conceptual focus, relatively simple interpolations of the corrugated core (such as 9 and 11 parameters ones) were used. In real-life situations, higher-degree polinomials with higher-order continuity and a larger number of segments will be used to represent more complex shapes, because they provide more capacity (internal degrees of freedom) to be geometrically more flexible and adaptible in satisfying constraints while reducing the objective function value. Other simulations have also been performed with different sets of control parameters.

## 6. CONCLUSIONS

Optimization of the designs of metal sandwich plates using nonlinear programming with several objective function and constraint function definitions is developed and presented. The constraint functions applied in the model are based on simplified structural models which provide for overall simplicity of the optimization process. Several test cases were used to verify the developed optimization model, numerical optimizations were performed, and results shown.

The developed model for the optimal design of metal sandwich plates for particular applications with specifically defined loading data and design requirements seems feasible, leading to optimized product development. The research group has so far developed and built several sandwich plate small-scale prototypes for testing purposes.

Follow-up research will include improved approximate structural models which will be introduced and implemented with direct and indirect (via constraint function approximators) coupling of the numerical optimization and the numerical structural analysis. Direct nonlinear programming methods and advanced evolutionary optimization will also be applied and combined with models that include a higher degree shape optimization.

In future research, coupling of evolutionary methods with structural analysis for constraints evaluation will also be based on neural network based approximators [11, 23, 24].

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## OPTIMIZACIJA METALNIH SENDVIČ PLOČA S NABORANOM JEZGROM

### SAŽETAK

*Ovaj rad postavlja mogući pristup optimizaciji aluminijskih sendviča ('metal-metal') s naboranom jezgrom na temelju niza pretpostavki i za odabrane slučajeve. Funkcije cilja i ograničenja izvedene su na temelju zahtjeva za minimalnom masom te zadovoljavanjem statičkih, strukturnih i projektno-konstrukcijskih ograničenja.*

*Cilj ovdje razvijenog i prezentiranog pristupa te pripadnog optimizacijskog modela je mogućnost definiranja 'optimalne geometrije' sendvič-ploče za zadane uvjete. S dodatnim modulom koji je razvijen može se projektirati i alat (profilirani valjci) koji će valjanjem generirati dobivenu optimalnu geometriju naborane jezgre sendviča. U tom smislu, ovaj rad, zapravo uvodi odgovarajući razvoj proizvoda na temelju optimizacije. U konkretnom smislu, polazeći od zadanih uvjeta opterećenja i definicije optimalnosti sendvič-ploče (specifičnih za pojedine slučajeve), numeričkim putem se određuju optimalni oblik i dimenzije sendvič-ploče te nakon toga pripadna geometrija alata kojim se proizvode, čime je kompletiran ciklus 'razvoja optimalnog proizvoda'.*

***Ključne riječi***: *metalne sendvič-ploče, optimizacija, funkcije cilja, ograničenja.*