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# Wave properties of wind actions on structures

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**SUMMARY**

*In this paper the wave properties of wind actions on structures analyzed on a single degree of freedom (SDOF) system are shown. The wave character of the action was examined by windows-wavelet analysis on several different measured data of wind actions. The constant and then the fluctuating action extracted from the wind force were analyzed by the wavelet transform. Fluctuating component of the wind action is represented by a harmonic function. The wavelet transform of wind action shows that the intensity of harmonic component decreases linearly with the number of cycles which implies that the resonance is not innate to wind action. All data of the wind action are obtained by measurements on SDOF oscillator. Time resolution varied from 0.01 to 0.00001 seconds. Significantly, the characteristics of wind action observed from the data on intensity of the averaging wind force are strongly connected with the length of the averaging wind force. The overall data of dynamic action show that the influence of wind is significantly higher for shorter periods of averaging. In this paper the term resonant spectra is used and the dynamic factor is based on them. The empirical relations for the value of the constant and the first harmonious component of the action are suggested. All results are compared with the classical spectra of the elastic force of SDOF system.*

***Key words***: *wavelet transformation, wind, resonant spectra, dynamic factor.*

## 1. INTRODUCTION

When the $I(t)$ rectangular impulse type of function is chosen for the wavelet transform, the transformation of the given wind force $F(t)$ in the period $0\text{-}t_0$ has the meaning of the average value of wind action, i.e. the average force with the average interval being equal to the duration of the impulse.

If the $I(t)$ harmonious type of function is chosen for the wavelet transform, for example a sinusoidal form, the wavelet transform of the wind force extracts the intensity of the harmonious action that at the same time shows the possible resonant effect on the SDOF system with the period being equal to the period of the chosen sinusoidal function. The special attention was given to the intensity analysis of the sinusoidal function of the same period with different number of cycles.

The experimental measurements of wind actions were done on SDOF system made of smooth steel bar of *60 cm* in length and *4.2 mm* in diameter, clamped in a fixed metal base, with a steel sphere *50 mm* in diameter placed on top. The equivalent mass of SDOF system was 0*.80 kg* and the flexural stiffness was $k=44$ *N/m*. The value of measured equivalent viscous damping was $\xi=0.55\%$. An accelerometer with a resolution of $10^{-5}$ *seconds* was installed in a leeside of the steel sphere. The measurements were carried out in such a way that the horizontal acceleration of the metal sphere centre was measured in the direction of the wind acceleration. This paper is based on the data related to the type of wind named Bora, measured several times at the end of the year 2005. Thirty measurements were registered altogether. The results of the acceleration measurements of the SDOF system have been translated into the wind force using standard dynamics methods.

## 2. FLUCTUATING AND CONSTANT COMPONENT OF WIND ACTION ON SDOF SYSTEM

Wind action on SDOF system is a dynamic action in the streaming direction. It can be extracted on the constant and the fluctuating component. The wind streaming direction was the same as the direction of the SDOF system movement. The constant component in the limited time has a constant value and it represents the Heaviside type of load in the dynamic sense. The fluctuating component in some limited time has the form of the harmonious function. The intensity of both components is important, but the extraction of the fluctuating component is much more complex.

From the observed wind measurement in situ, the fluctuating component can be extracted by the windows-wavelet Fourier transform [1-3].

Windows-wavelet Fourier transform is given by the relation:

$$a_\alpha\left(\alpha,T\right)\big|_{max}=\int_{-\infty}^{\infty}F_w\left(\tau\right)I\left(\tau-t\right)d\tau,\quad \alpha>0 \qquad (1)$$

where:

$$I\left(\tau-t\right)=N\left(\alpha T,\tau-t\right)sin\frac{2\pi}{T}\left(\tau-t\right) \qquad (2)$$

is a sinus impulse with the duration of $t=\alpha T$, while $N$ is a function of impulse normalization shown by the expression:

$$N\left(\alpha T,t\right)=\begin{cases}1/\alpha T & 0<t<\alpha T\\ 0 & other\end{cases} \qquad (3)$$

Figure 1 shows the record of the measured wind force with the time resolution of *0.002 sec*, while it's windows-wavelet transform called the resonant spectra of the action is shown in Figure 2.

Window-wavelet Fourier transform performed on a large number of measuring experiments in situ shows that the relations in the resonant spectra perfectly correlate with the expression:

$$F_\alpha\left(\alpha,T\right)=F_{\alpha=1}\frac{1}{\alpha} \qquad (4)$$

Thus, if the spectrum for one number of waves is known, the spectra for an arbitrary number of waves in wind action are also known.

The constant component of wind action can be extracted from the action by windows-wavelet transform of a rectangular type in the form:

$$F_0\left(T\right)\big|_{max}=\int_{-\infty}^{\infty}F_w\left(\tau\right)I\left(\tau-t\right)d\tau,\quad \alpha>0 \qquad (5)$$

where:

$$I\left(\tau-t\right)=N\left(T,\tau-t\right) \qquad (6)$$

and $T$ represents the averaging period.

Figure 3 shows the result of averaging wind force taken from the data shown on Figure 1.

## 3. RESPONSE OF THE SDOF SYSTEM TO THE FIRST COMPONENT OF FLUCTUATING WIND ACTION

The response of the SDOF system to the fluctuating components of wind action consists of the response of all harmonious components. The action of the first harmonious component is monitored in the form of the resonant action on the SDOF system of the following form:

$$f\left(t\right)=F_1\left(\alpha,T\right)sin\,\omega t \qquad (7)$$

with homogenous initial conditions.

According to Refs. [4, 5], if we neglect the small members connected to viscous damping, the response of the system can be represented by the expression:

$$\ddot{u}=-F_1\left(\alpha,T\right)\left[\left(1-e^{-\xi\omega t}\right)\frac{cos\omega t}{2\xi}-sin\omega t\right] \qquad (8)$$

The highest acceleration amplitude is obtained for these values:

$$\omega t=\frac{2\pi}{T}\alpha T=2\pi\alpha=i\pi\,,\quad \alpha=\frac{i}{2},\quad i=1,2,3... \qquad (9)$$

which gives:

$$\ddot{u}_{max}=F_1\left(\alpha,T\right)\left(1-e^{-\xi 2\pi\alpha}\right)\frac{1}{2\xi} \qquad (10)$$

The maximum value of the response obtained from Eq. (10) is:

$$R_e=F_1\left(\alpha,T\right)\left(1-e^{-\xi 2\pi\alpha}\right)/\,2\xi\big|_{max} \qquad (11)$$

and it represents a value of resonant force spectrum ordinate ($R_e$) of the elastic force for the observed SDOF system.

If the Eq. (4) is included in the Eq. (11), the maximum force response of the resonant spectrum can be shown as:

$$R_e=F_1\left[\frac{1}{\alpha}\left(1-e^{-\xi 2\pi\alpha}\right)/\,2\xi\right]\Big|_{max}=F_1\,A\left(\alpha,\xi\right)\big|_{max} \qquad (12)$$

The expression in square brackets in Eq. (12) is a dynamic factor for the given harmonious component. It is necessary to monitor its extreme value in dependence on the impulse duration, for a given period T. The Figure 4 shows the extremes of the dynamic factor for different values of damping coefficient $\xi$. It is important to notice that the obtained maximum value of the dynamic factor is equal to number $\pi$, in case where $\alpha$=0 is independent of the damping value $\xi$. The values of the dynamic factor for the following semi periods and periods of impulse fall as it is shown in the Figure 4.

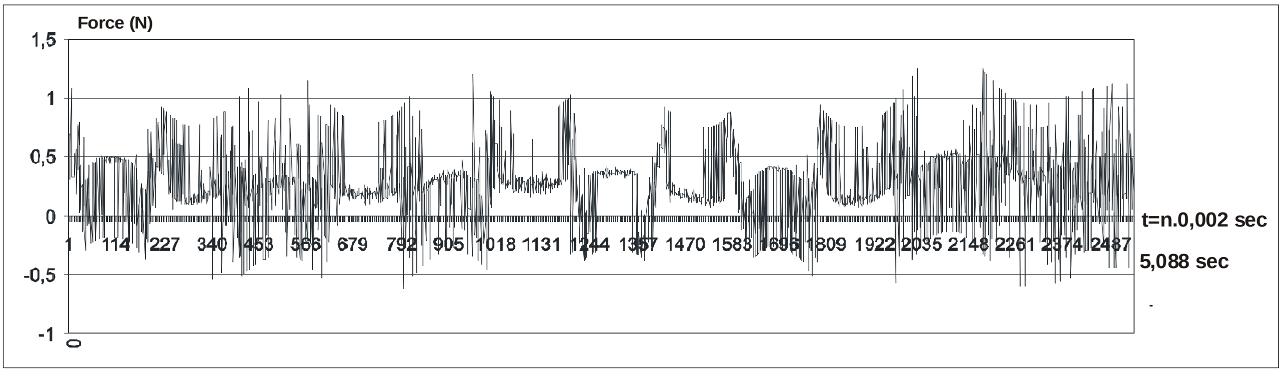


*Fig. 1 Measured wind force record with time resolution 0.002 sec*

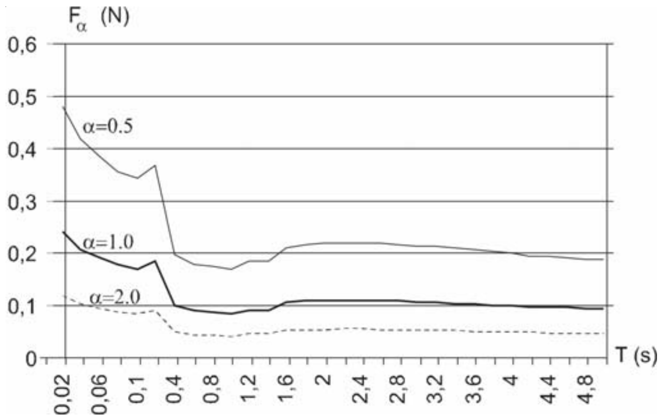


*Fig. 2 Resonant spectra of action force*

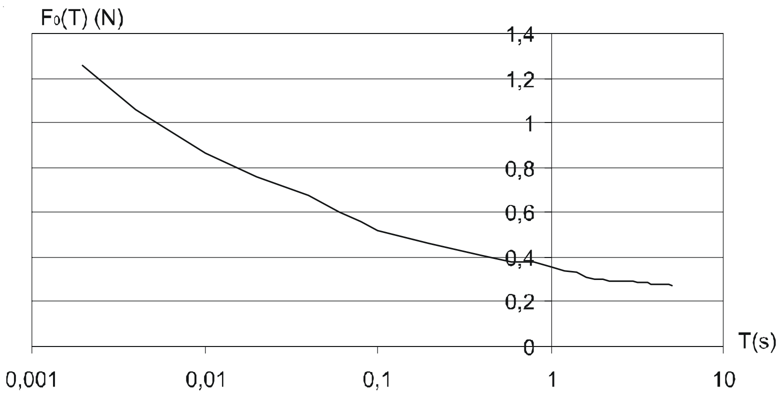


*Fig. 3 Result of averaging wind force*

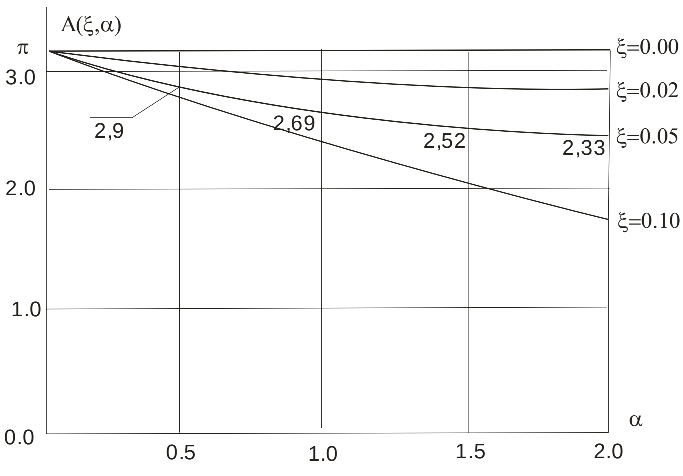


*Fig. 4 Extremes of the waves function A(ξ,α)*

## 4. RESPONSE OF THE SDOF SYSTEM TO THE REMAINING FLUCTUATING COMPONENTS OF WIND ACTION

The remaining harmonious components of wind action on the SDOF system can be written in the form:

$$f\left(t\right)=\sum_{i=2}^{\infty}F_i\left(i,T\right)sin\,i\frac{2\pi}{T}t \qquad (13)$$

According to Refs. [4,5 ] the response of the SDOF system to such action is:

$$R\left(t\right)=\sum_{i=2}^{\infty}\frac{F_1\left(T/i\right)}{k}\frac{1}{1-i^2}sin\,i\frac{2\pi}{T}t \qquad (14)$$

For the periods T of *0.4–5 seconds*, the maximum intensity of every higher member in relation to the first fluctuating component is $1/(1-i^2)$. The sum of absolute intensities of all higher members is approximately *2/3* of the intensity of the first harmonious member. Of course, the extremes do not happen at the same moment, and they can have different signs.

## 5. RESPONSE OF THE SDOF SYSTEM TO THE CONSTANT COMPONENT OF WIND ACTION

Dynamical response of the SDOF system to the constant component with homogenous initial conditions and viscous damping has the following form:

$$u=\frac{F_0\left(T\right)}{k}\left[1-e^{-\xi\omega t}\left(cos\omega t+\xi sin\omega t\right)\right]=\frac{F_0\left(T\right)}{k}A\left(\xi,t\right) \qquad (15)$$

where *A* is a dynamic factor which maximum value is *2*.

## 6. TOTAL WIND FORCE

The total wind action consists of constant and all fluctuating components. All components do not have extremes at the same point in time. If we suppose that it happens at the same point of time, the extreme influence from the upper side is obtained for the value of the SDOF system elastic force. The performed analyses showed that the influence of the constant and the first harmonious component made the main part of the SDOF system elastic force.

The performed analysis also shows that the extreme of the influence of the constant component is followed by the very high percentage of the influence of the first harmonious component.

### 6.1 Wind force measured at the resolution of *t=0.002* seconds

For the wind force shown in Figure 1 the maximum intensity of the elastic force that emerged from the constant component $R_0$ of the wind force, the first harmonious component $R_1$ of wind force and their linear superposition called resonant spectrum $R_{Se}$ are shown in Figure 5.

Accompanying dynamic factor, calculated as a ratio between the resonant elastic force spectrum $R_{Se}$ and constant component $R_0$ of the wind force, is shown in Figure 6.

The comparison between the resonant elastic force spectrum and classical elastic force spectrum $S_e$ is shown in Figure 7.

### 6.2 Wind force measured at the resolution of *t=0.01* seconds

The measured wind force for wind measured at the resolution of 0.01 seconds and an approximate wind speed (*v=95 km/h*), is shown in Figure 8. Analogous analysis gave results shown in Figures 9, 10 and 11.

### 6.3 Wind force measured at the resolution of *t=0.00001* seconds

For wind force measured at the resolution of *0.00001 seconds* Figure 12 graphically shows a part of the force, at the time from *0* to *0.05 seconds* and an approximate wind speed (*v=70 km/h*). In this measurement in situ we note the registration of frequencies from *500* to *18000 Hz* which a human ear can hear. Side effect for the people who made the measurement was the sound of wind on SDOF system. The constant component remains constant below the frequency of *18000 Hz*. One part of the analyzed results, the resonant spectra of action, are shown in Figure 13.

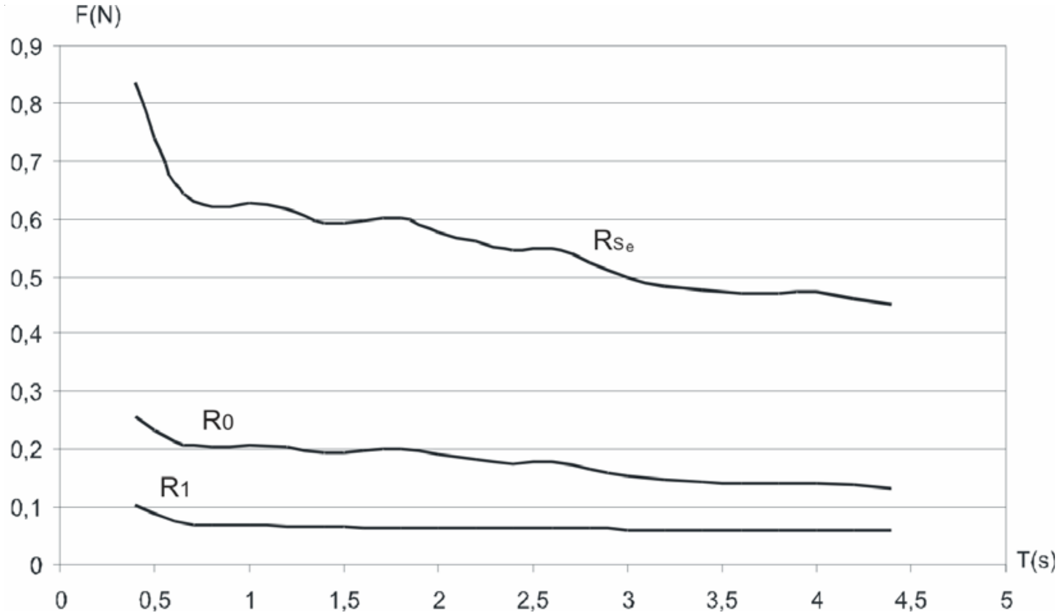


*Fig. 5 Resonant elastic force spectra*

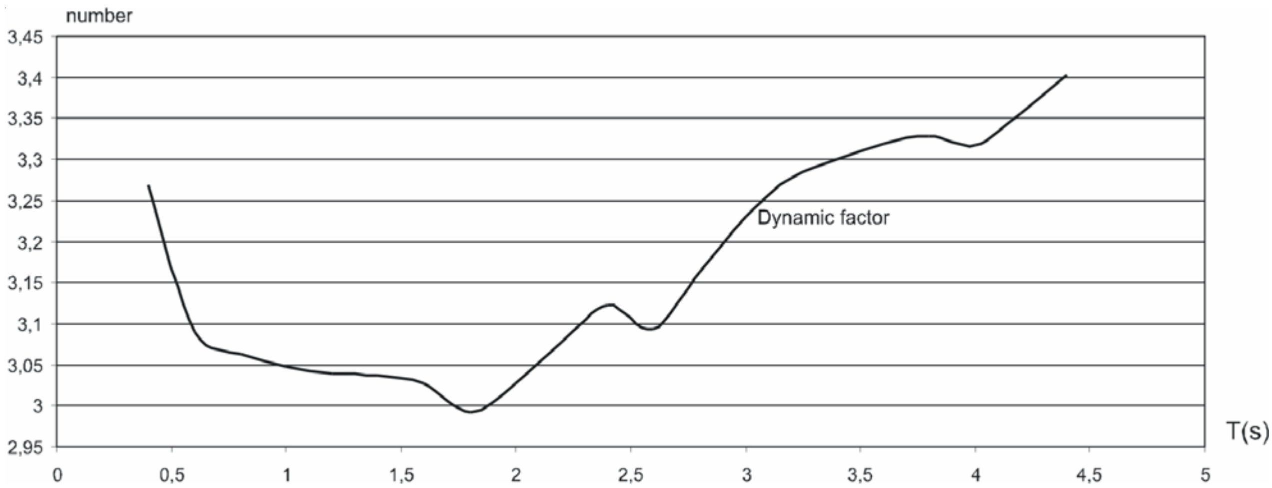


*Fig. 6 Dynamic factor*

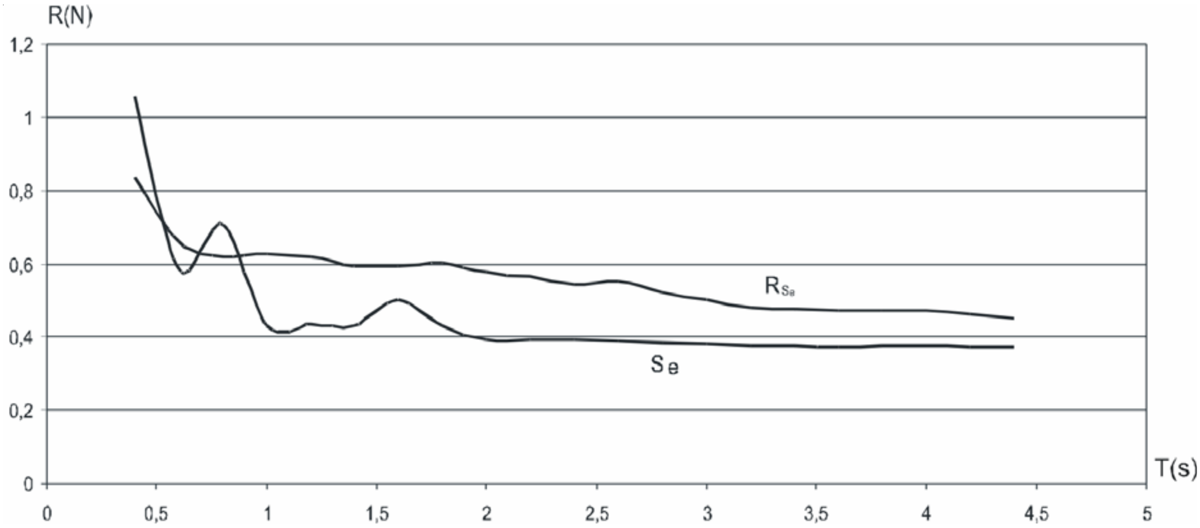


*Fig. 7 Resonant elastic force spectrum ($R_{Se}$) and classical elastic force spectrum ($S_e$)*

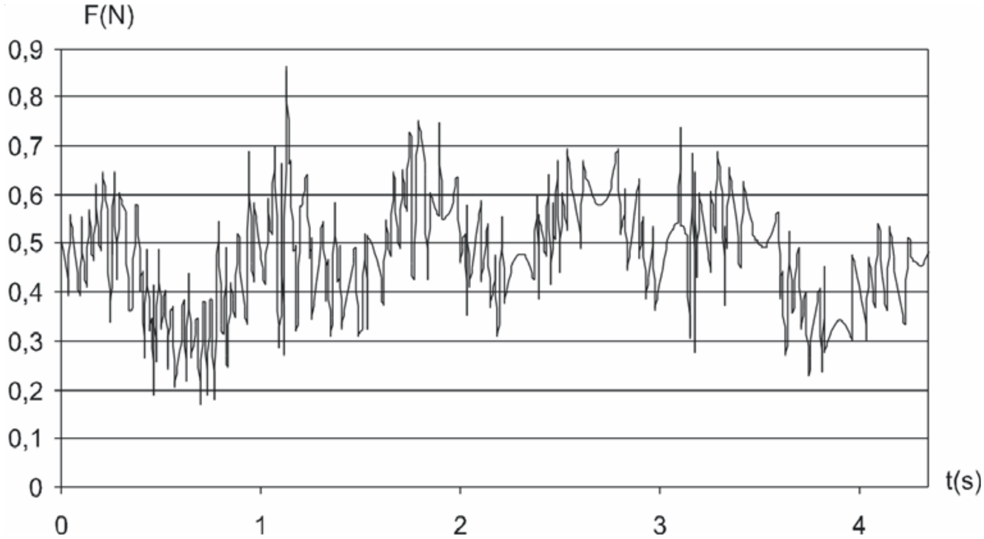


*Fig. 8 Measured wind force at the resolution of 0.01 seconds*

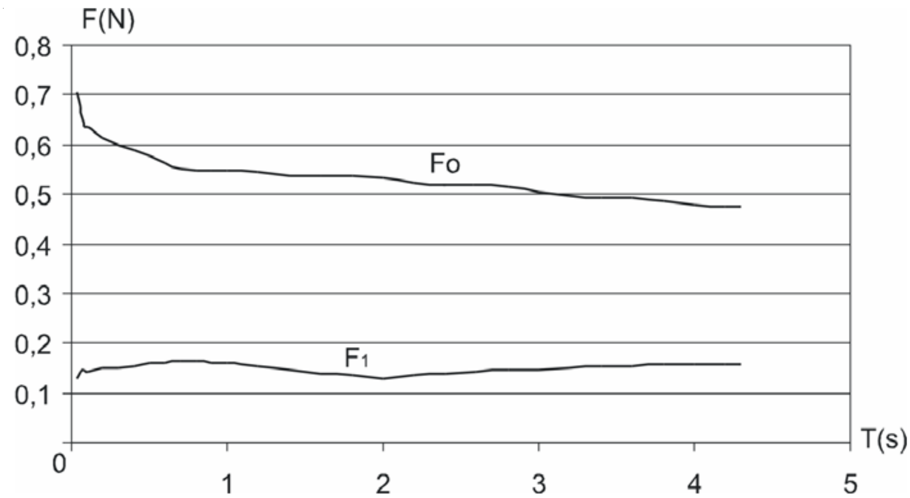


*Fig. 9 Resonant spectra of action force*

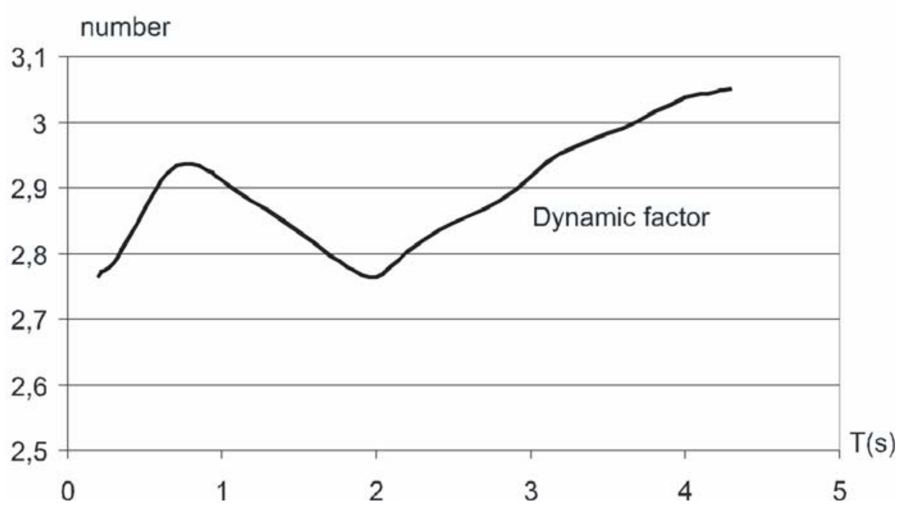


*Fig. 10 Dynamic factor*

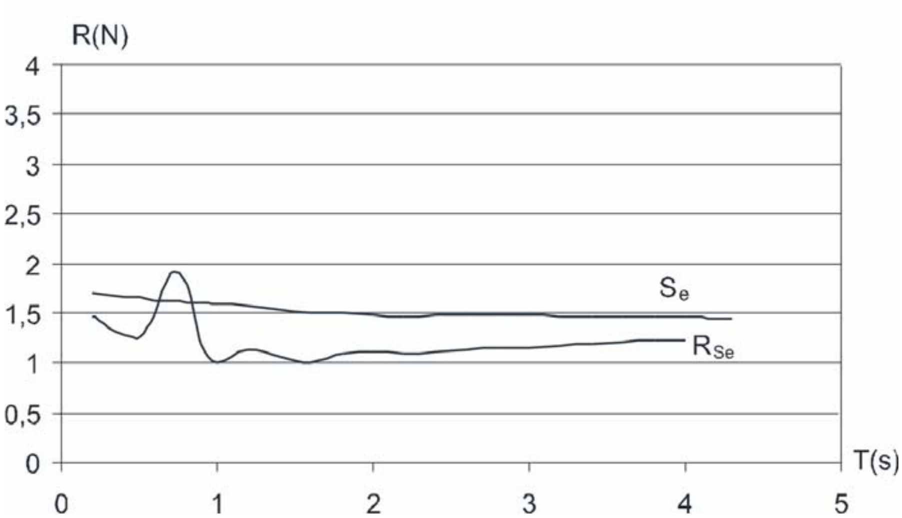


*Fig. 11 Resonant elastic force spectrum ($R_{Se}$) and classical elastic force spectrum ($S_e$)*

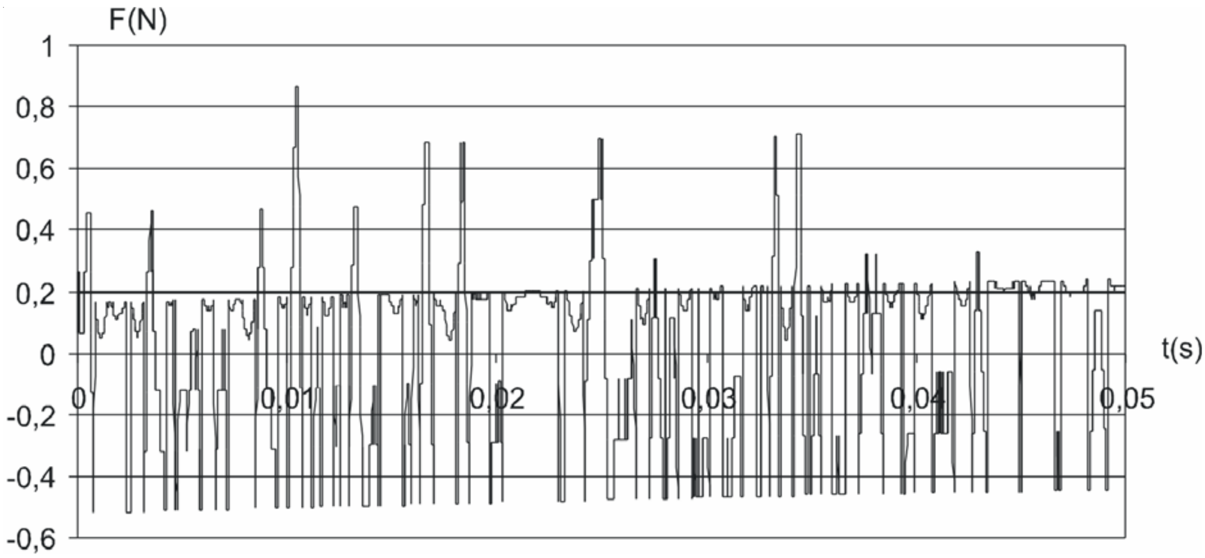


*Fig. 12 Measured wind force at the resolution of 0.00001 seconds*

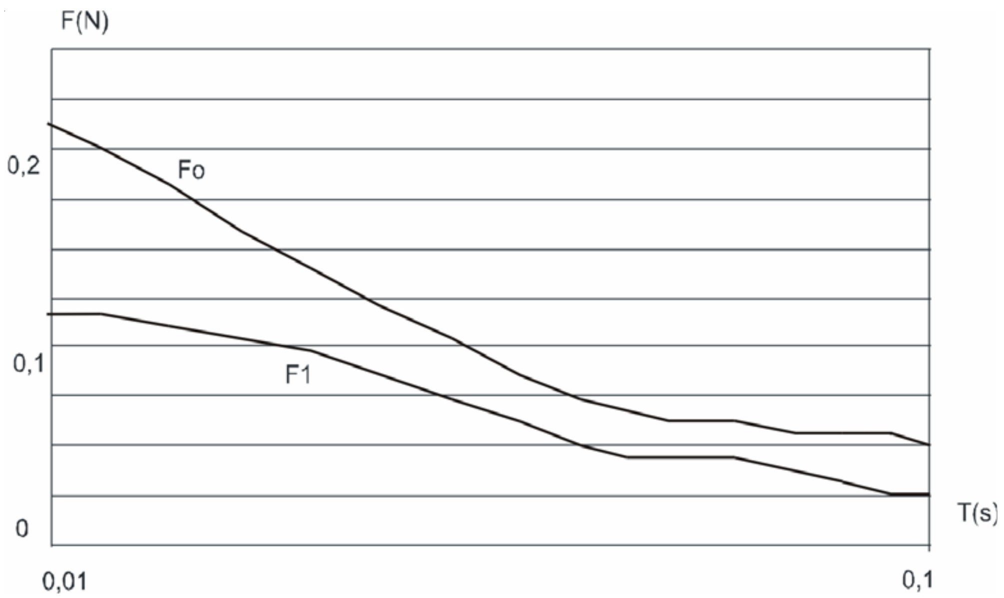


*Fig. 13 Resonant spectra of action*

## 7. NORMALIZED WIND ACTION ON SDOF SYSTEM

A normalized constant wind action component for the range of *0.01-1000 seconds* was constructed from the analysis of the whole range of measurements in situ. The normalization was made on force at the period *T=1.0 second*. For the range of *6-600 seconds* the results from the Ref. [6] were taken. The results were obtained in such a way that an envelope was taken from all data processing. The graphic diagram of the obtained results is given in Figure 14.

The approximate analytical relation for the description of the given curve is:

(16)

Analogue to the constant component, the first harmonious component of the wind action was normalized too. The result of the constant component for the period *T=1.0 second* has been chosen for the normalization. The results have been shown in Figure 14. The approximate relation for the given curve is:

$$F_1(T) = 0.43 F_0(T) \tag{17}$$

Figure 14 shows the accompanying dynamic factor in relation to the constant component. Its size is approximately constant. Upper side limitation of the dynamic factor is *3.35*.

## 8. CONCLUSION

Averaging interval is extremely important for the representation of wind action on a dynamic system. The usage of the averaging interval equal to the duration of the period of the designed system is imposed as a logical approach.

By decreasing the period of the actual system we can observe a significant increase of the intensity of constant component. The portion of the fluctuating component shown in the form of sinus function of one wave has been quantified. Its intensity is on an average somewhat smaller than the half of the constant component intensity.

Special importance in the conclusion is attached to the fact of the intensity of action with several harmonious waves. It is shown that the intensity of waves with several wave lengths decreases inversly to the number of waves, i.e. duration period. This fact shows that the resonant effect is not innate to the analyzed phenomenon.

Total action presented by the extreme elastic force is quantified by the dynamic factor of the elastic force in relation to the force of the constant component. The value of so defined dynamic factor is from 2.74 to 3.35. The data in this paper are obtained from the example of the analysis of three wind action measurements in situ with different time resolutions. In the end the proposal for the normalization of the force of constant and fluctuating component has been given by empirical expression.

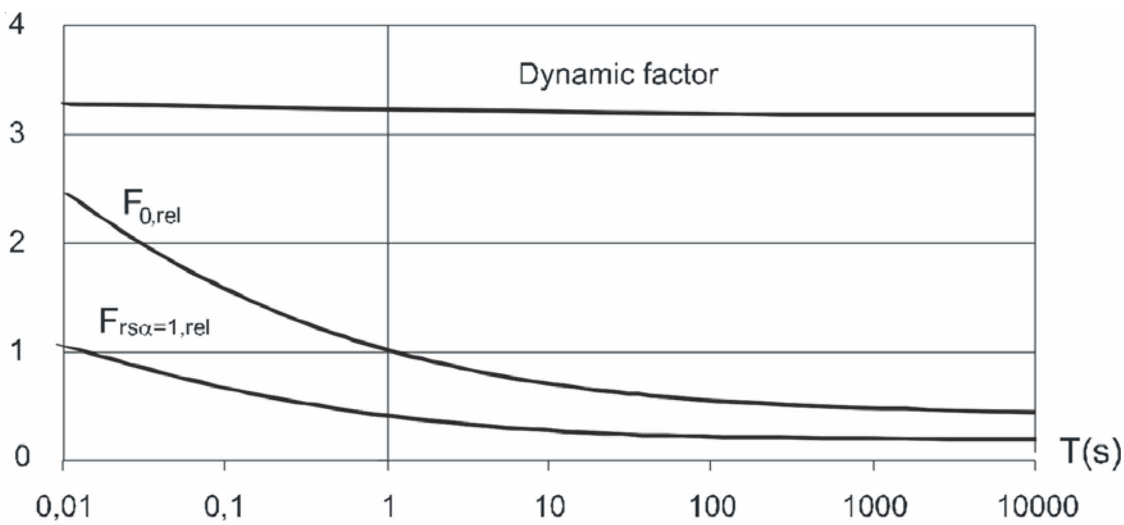


*Fig. 14 Normalized constant and fluctuating component of wind force*

## 9. REFERENCES


[1] A. Mihanović and G. Gusić, Resonant response spectra of structures, Proc. of Society of Croatian Structural Engineers Symposium on Contemporary Construction Processes, Brijuni Islands, Ed. J. Radić, SCSE, Zagreb, pp. 447-454, 1995. (In Croatian)

[2] A. Mihanović and G. Gusić, Resonant spectra obtained by windowed-wavelet analysis, Proc. of Int. Conf. COST TC Technology and Culture as a Basis for Urban Development, Ed. R. Žarnić, University of Ljubljana, pp. 1-6, 1995.

[3] K. Gurley and A. Kareem, Applications of wavelet transforms in earthquake, wind and ocean engineering, *Engineering Structures*, No. 21, pp. 149-167, 1999.

[4] J.L. Humar, *Dynamics of Structures*, Prentice Hall, New Jersey, 1990.

[5] A. Mihanović, *Dynamics of Structures*, Faculty of Civil Engineering, University of Split, Split 1996. (In Croatian)

[6] B. Peroš, I. Boko and T. Šimunović, Analysis of wind action at the Dubrovnik Bridge, *Građevinar*, Vol. 57, No. 2, pp. 87-94, 2005. (In Croatian)


## VALNA SVOJSTVA DJELOVANJA VJETRA NA KONSTRUKCIJE


### SAŽETAK

*U ovom radu su prikazana valna svojstva djelovanja vjetra na konstrukcije analizirana na jednostupnjevnom dinamičkom sustavu. Valni karakter djelovanja istražen je pomoću kratkotrajno-kratkovalne analize na nekoliko različitih pobuda vjetra. Kratkotrajnom transformacijom analizirana je ustaljena, a potom i fluktuirajuća komponenta izlučena iz sile pobude. Fluktuirajuća komponenta sile vjetra predstavljena je harmonijskom funkcijom. Valna transformacija pobude vjetrom pokazala je da intenzitet harmonijske komponente opada linearno s brojem ciklusa. Odatle slijedi da rezonantnost pobude nije prirođena djelovanju vjetra. Svi podaci o veličini sile vjetra dobiveni su pokusnim mjerenjima na JS oscilatoru. Gustoća zapisa varirala je od 0,01 do 0,00001 sekunde. Na podacima o intenzitetu osrednjavanja sile vjetra uočeno je karakteristično svojstvo djelovanja vjetra da je intenzitet osrednjavanja sile izravno povezan s duljinom osrednjavanja brzine vjetra. Ukupni podaci o dinamičkom djelovanju pokazuju da je utjecaj vjetra bitno veći za kraće periode osrednjavanja. U radu je rabljen pojam rezonantnih spektara i na njima utemeljen dinamički faktor. Predloženi su empirijski izrazi za veličinu srednje i prve harmonijske komponente sile pobude. Svi rezultati su uspoređeni s klasičnim spektrom odgovora elastične sile jednostupnjevnog sustava.*


***Ključne riječi***: *valna transformacija, vjetar, rezonantni spektar, dinamički faktor.*