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Time varying Hartmann flow with heat transfer of a power-law fluid with uniform suction and injection under exponential decaying pressure gradient

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SUMMARY

The time varying Hartmann flow of an electrically conducting viscous incompressible non-Newtonian power-law fluid between two parallel horizontal non-conducting porous plates is studied with heat transfer under exponential decaying pressure gradient. An external uniform magnetic field that is perpendicular to the plates and uniform suction and injection through the surface of the plates are applied. The two plates are kept at different but constant temperatures while the Joule and viscous dissipations are taken into consideration. Numerical solutions for the governing nonlinear momentum and energy equations are obtained using finite difference approximations. The effect of the magnetic field, the parameter describing the non-Newtonian behavior, and the velocity of suction and injection on both the velocity and temperature distributions as well as the dissipation terms are examined.

***Key words**: MHD flow, heat transfer, non-Newtonian fluids, numerical analysis.*

1. INTRODUCTION

The study of the rectangular channel flow of an electrically conducting viscous fluid under the action of a transversely applied magnetic field, known as Hartmann flow, has immediate applications in many devices such as magnetohydrodynamic (MHD) power generators, MHD pumps, accelarators, aerodynamics heating, electrostatic precipitation, polymer-technology, petroleum-industry, purification of crude oil and fluid droplets-sprays. Channel flows of a Newtonian fluid with heat transfer have been studied, with or without Hall currents, by many authors [1-9]. These results are important for the design of the duct wall and the cooling arrangements.

A number of industrially important fluids such as multon plastics, polymers, pulps and foods exhibits non-Newtonian fluid behavior [10]. Due to the growing use of these non-Newtonian materials, in various manufacturing and processing industries, considerable efforts have been directed towards understanding their flow and heat transfer characteristics. Many of the inelastic non-Newtonian fluids, encountered in chemical engineering processes, are known to follow the so-called "power-law model" in which the shear stress varies according to a power function of the strain rate [11]. The power-law fluid flows, within parallel plate ducts and rectangular ducts, have been considered by many authors [12-15].

In the present study, the unsteady Hartmann flow of a conducting non-Newtonian power-law fully developed fluid between two infinite non-conducting horizontal parallel and porous plates is studied. The flow starts from rest through the application of a uniform and exponential decaying pressure gradient with a uniform suction from above and a uniform injection from below. The flow is subjected to a uniform magnetic field perpendicular to the plates. The induced magnetic field is neglected by assuming a very small magnetic Reynolds number [5]. The two plates are kept at two different but constant temperatures. The Joule and viscous dissipations are taken into consideration in the energy equation. The governing nonlinear momentum and energy equations are solved numerically using the finite difference approximations. The inclusion of the magnetic field, the suction and injection, and the non-Newtonian fluid characteristics leads to some interesting effects, on both the velocity and temperature fields.

2. FORMULATION OF THE PROBLEM

The fluid is assumed to be laminar viscous incompressible and obeying the power-law model and flows between two infinite horizontal parallel non-conducting plates located at the $y=\pm h$ planes and extend from $x=-\infty$ to ∞ and from $z=-\infty$ to ∞ (Figure 1). The upper and lower plates are kept at two constant temperatures T_2 and T_1 respectively, with $T_2>T_1$. The flow is driven by a uniform and constant pressure gradient dp/dx in the x-direction, and a uniform suction from the above and injection from below which are applied at $t=0$. A uniform magnetic field with magnetic flux density vector $\boldsymbol{B}_0$ is applied in the positive y-direction. The uniform suction implies that the y-component of the velocity is constant and is taken equal to v_0. Thus, the velocity vector of the fluid is given by:

$$\boldsymbol{v}(y,t) = u(y,t)\boldsymbol{i} + v_0\boldsymbol{j}$$

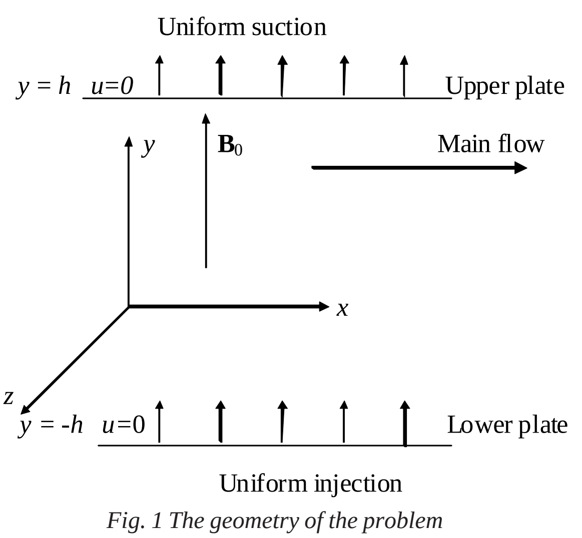


Fig. 1 The geometry of the problem

The fluid motion starts from rest at $t=0$, and the no-slip condition at the plates implies that the fluid velocity has neither z nor an x-component at $y=\pm h$. The initial temperature of the fluid is assumed to be equal to T_1.

The flow of the fluid is governed by the Navier-Stokes equation [16, 17]:

$$\rho\frac{D\boldsymbol{v}}{Dt} = \nabla\cdot\left(\mu\nabla\boldsymbol{v}\right) - \nabla p + \boldsymbol{J}\times\boldsymbol{B}_o \tag{1}$$

where ρ is the density of the fluid and μ is the apparent viscosity of the model and is given by:

$$\mu = K\left(\frac{\partial u}{\partial y}\right)^{n-1} \tag{2}$$

where K is the consistency index, n is the flow behavior index which corresponds to the type of the fluid (n less than, equal to, and greater than 1 gives pseudoplastic, Newtonian and dilatant fluids respectively), $\boldsymbol{B}_0$ is the magnetic field, which is assumed to be also the total magnetic field, as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number [5]. Using Ohm's law [5] the Navier-Stokes Eq. (1) read:

$$\rho\frac{\partial u}{\partial t} + \rho v_0\frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \sigma B_o^2 u \tag{3}$$

The energy equation with viscous and Joule dissipations is given by:

$$\rho c_p\frac{\partial T}{\partial t} + \rho c_p v_o\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_o^2 u^2 \tag{4}$$

where σ, c_p and k are, respectively, the electrical conductivity, specific heat capacity at constant volume and the thermal conductivity of the fluid. The second and the third terms on the right-hand side represent the viscous and Joule dissipations respectively. The viscous dissipation term may often be neglected for Newtonian fluids, however, depending on the duct geometry and relative volumetric flow rate, viscous dissipation may have a dramatic effect on the thermal flow field in non-Newtonian fluids [18].

The initial and boundary conditions of the problem are given by:

$$u=0 \text{ at } t\le 0 \text{ and } u=0 \text{ at } y=\pm h \text{ for } t>0 \tag{5}$$

$$T=T_1 \text{ at } t\le 0,\ T=T_1 \text{ at } y=-h,\ T=T_2 \text{ at } y=-h \text{ for } t>0 \tag{6}$$

It is expedient to write the above equations in the non-dimensional form. To do this, we introduce the following non-dimensional quantities:

$$\bar{x}=\frac{x}{h},\quad \bar{y}=\frac{y}{h},\quad \bar{z}=\frac{z}{h},\quad \bar{t}=\frac{tu_o}{h},\quad \bar{u}=\frac{u}{u_o},$$

$$\bar{p}=\frac{p}{\rho u_o^2},\quad \bar{T}=\frac{T-T_1}{T_2-T_1},\quad \bar{\mu}=\frac{\mu}{\mu_r}$$

$Re = \rho u_o h / \mu_r$ is the Reynolds number,

$S = \rho v_o h / \mu_r$ is the suction parameter,

$Pr = \rho c_p u_o h / k$ is the Prandtl number,

$Ec = u_o \mu_r / (\rho c_p h (T_2 - T_1))$ is the Eckert number,

$Ha^2 = \sigma B_o^2 h^2 / \mu_r$ is the Hartmann number squared,

$\mu_r = K u_o^{1-n} / h^{1-n}$ is the generalized reference viscosity,

where $d\bar{p} / d\bar{x} = Ce^{\alpha t}$. The generalized reference viscosity is chosen so that when $n=1$ (Newtonian fluid), the viscosity becomes constant [9, 17]. Here u_0 is the characteristic velocity which is arbitrarily chosen such that $Re=1$. Also, in terms of the above non-dimensional variables and parameters Eqs. (3) and (4) are written as (where the bars are dropped for convenience):

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) - Ha^2 u \quad (7)$$

$$\frac{\partial T}{\partial t} + S\frac{\partial T}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} + Ec\mu\left(\frac{\partial u}{\partial y}\right)^2 + Ha^2 Ecu^2 \quad (8)$$

where:

$$\mu = \left(\frac{\partial u}{\partial y}\right)^{(n-1)} \quad (9)$$

The initial and boundary conditions for the velocity and temperature in the dimensionless form are written as:

$u=0$ at $t \le 0$ and $u=0$ at $y=\pm 1$ for $t>0$ (10)

$T=0$ at $t \le 0$, $T=0$ at $y=-1$, $T=1$ at $y=1$ for $t>0$ (11)

3. NUMERICAL SOLUTION

Equations (7) and (9) represent a coupled system of non-linear partial differential equations which can not be solved analytically. Therefore, they are integrated numerically under the initial and boundary conditions, Eq. (10), using central differences for the derivatives and Thomas algorithm for the solution of the set of discretized equations. A linearization technique is first applied to replace the nonlinear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. Then the Crank-Nicolson implicit method [19] is used at two successive time levels. An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step and the iterations are continued till convergence, within a prescribed accuracy. Finally, the resulting block tridiagonal system is solved using the generalized Thomas-algorithm [19]. The energy Eq. (8) is a linear inhomogeneous second-order ordinary differential equation whose right-hand side is known from the solutions of the flow Eqs. (7), (9) and (10). The values of the velocity u and its gradient are substituted in the right-hand side of Eq. (8) which is solved numerically with the initial and boundary conditions, Eq. (11), using central differences for the derivatives and Thomas-algorithm for the solution of the set of discritized equations. Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximations in the y-direction. The diffusion terms are replaced by the average of the central differences at two successive time-levels. The computational domain is divided into meshes each of dimension Δt and Δy in time and space, respectively. We define the variables $v = \partial u / \partial y$ and $H = \partial T / \partial y$ to reduce the second order differential Eqs. (7) and (8) to the first order differential equations. The finite difference representations for the resulting first order differential take the form:

$$\left(\frac{u_{i+1,j+1} - u_{i,j+1} + u_{i+1,j} - u_{i,j}}{2\Delta t}\right) + S\left(\frac{v_{i+1,j+1} + v_{i,j+1} + v_{i+1,j} + v_{i,j}}{4}\right) =$$
$$= -\frac{dp}{dx} + \left(\frac{\bar{\mu}_{i,j+1} + \bar{\mu}_{i,j}}{2}\right)\left[\frac{(v_{i+1,j+1} + v_{i,j+1}) - (v_{i+1,j} + v_{i,j})}{2\Delta y}\right] + \left(\frac{\bar{\mu}'_{i,j+1} + \bar{\mu}'_{i,j}}{2}\right)\left[\frac{v_{i+1,j+1} + v_{i,j+1} + v_{i+1,j} + v_{i,j}}{4}\right] -$$
$$- Ha^2\left[\frac{u_{i+1,j+1} + u_{i,j+1} + u_{i+1,j} + u_{i,j}}{4}\right] \quad (12)$$

The variables with bars are given initial guesses from the previous time steps and an iterative scheme is used at every time to solve the linearized system of difference equations. Then the finite difference form for the first-order form of the energy Eq. (8) can be written as:

$$\left(\frac{T_{i+1,j+1}-T_{i,j+1}+T_{i+1,j}-T_{i,j}}{2\Delta t}\right)+S\left(\frac{H_{i+1,j+1}+H_{i,j+1}+H_{i+1,j}+H_{i,j}}{4}\right)= \\ =\frac{1}{Pr}\left[\frac{(H_{i+1,j+1}+H_{i,j+1})-(H_{i+1,j}+H_{i,j})}{2\Delta y}\right]+DISP \qquad (13)$$

where DISP represents the Joule and viscous dissipation terms which are known from the solution of the momentum equations and can be evaluated at the mid point (i,j) of the computational cell. Computations have been made for $C=-5$, $\alpha=1$, $Pr=1$, and $Ec=0.2$. Grid-independence studies show that the computational domain $0<t<\infty$ and $-1<y<1$ can be divided into intervals with step sizes $\Delta t=0.0001$ and $\Delta y=0.005$ for time and space, respectively as shown in Figure 2. Smaller step sizes do not show any significant change in the results. Convergence of the scheme is assumed when every one of u, v, T and H for the last two approximations differ from unity by less than 10^{-6} for all values of y in $-1<y<1$ at every time step. Less than 7 approximations are required to satisfy this convergence criteria for all ranges of the parameters studied here. In order to examine the accuracy and correctness of the solutions, the results obtained here are tested and compared with the results for the Newtonian case reported by Attia in Ref. [9] ($n=1$). The comparison shows a complete agreement between the results of both solutions. While comparisons with previously published theoretical work on this problem were performed, no comparisons with experimental data were done because, as far as the author is aware, such data are lacking at the present time.

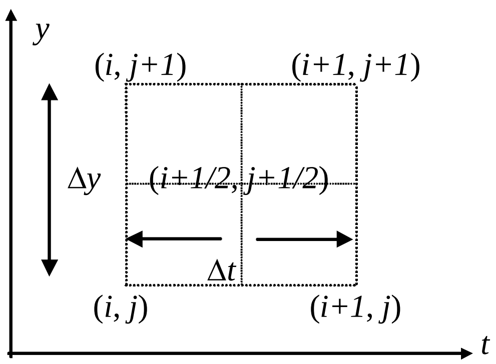


Fig. 2 Mesh network

4. RESULTS AND DISCUSSION

Figures 3 and 4 show the time development of the profile of the velocity u and the temperature T, respectively, for various values of time t and for $n=0.5$, 1, and 1.5. The figures are evaluated for $Ha=3$ and $S=1$. As shown in Figure 3, the profiles are asymmetric about the $y=0$ plane because of the suction. Figure 3 shows that the velocity u decreases with time and reaches its steady state monotonically. It is clear from Figure 3 that the effect of the flow index n on u depends on t and y. For small t, increasing n decreases u for all y apart from the central region due to the decrease in viscosity resulting from the large velocity gradient in this area. However, near the center u increases with increasing n since the velocity gradient is small and therefore the viscosity decreases with increasing n. As time develops, increasing n increases u for all y due to the overall decrease in velocity and its gradient which decreases viscosity. Figure 3a indicates that small values of n affect the parabolic shape of the velocity profile and lead to the suppression of the peaks.

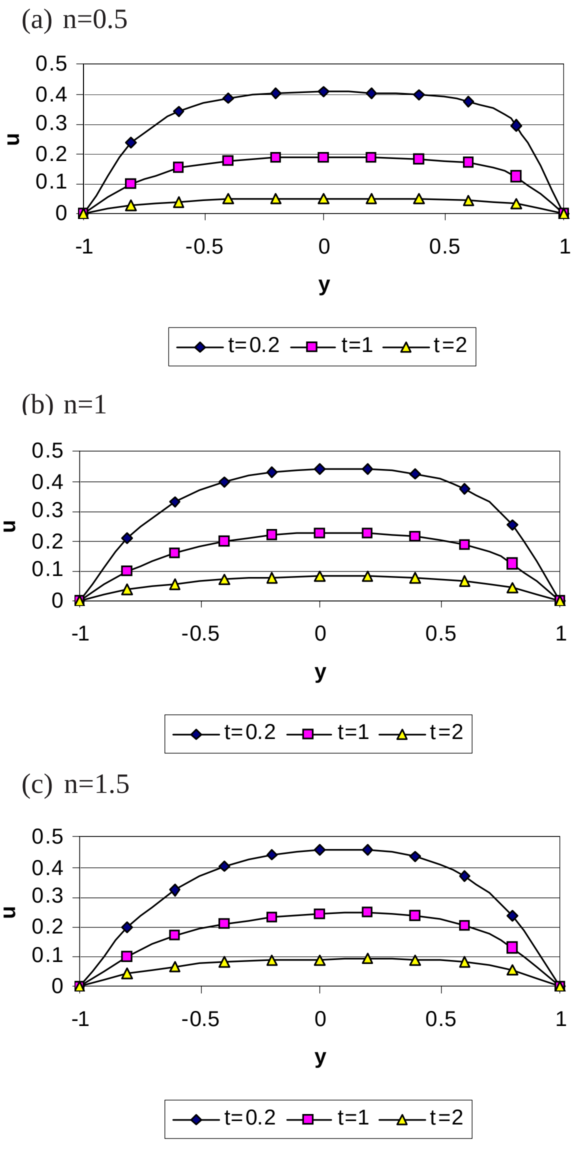
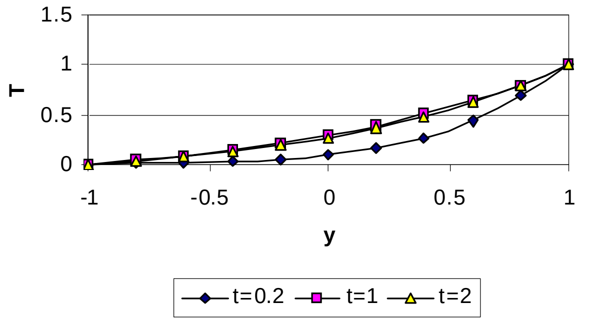


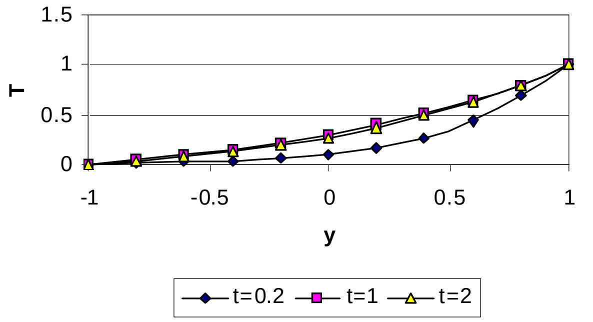
Fig. 3 Time variation of the profile of u for various values of n (Ha=3, S=1)

Figure 4 shows that the temperature profile does not reach its steady state monotonically. The temperature increases with time up to a maximum value and then decreases up to the steady state. Figure 4 shows also, that the effect of *n* on the temperature depends on *t*. For small *t*, increasing *n* decreases *T*, but as time develops increasing *n* increases *T*. This is due to the effect of *n* in increasing or decreasing *u* which affects the dissipations.

(a) n=0.5



(b) n=1



(c) n=1.5

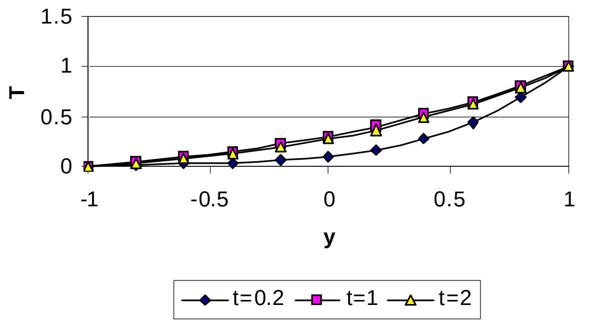
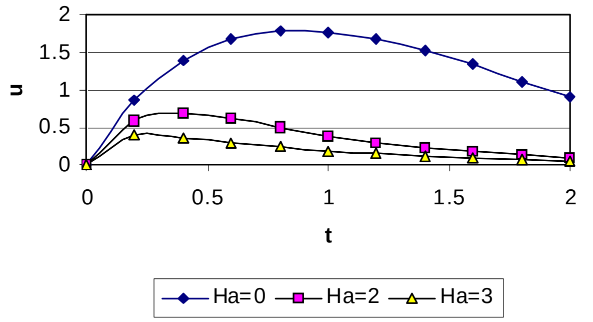


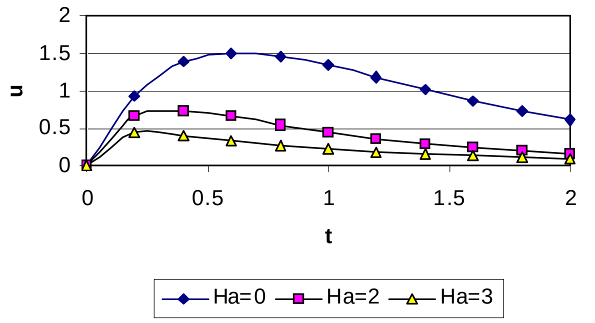
Fig. 4 Time variation of the profile of T for various values of n (Ha=3, S=1)

Figures 5 and 6 show the effect of the Hartmann number *Ha* on the time development of *u* and *T* at *y=0* with time, respectively, for various values of Hartmann number *Ha* and for *n=0.5*, *1*, and *1.5*. In these figures *S=0*. Figure 5 shows that increasing *Ha* decreases *u* as it increases the damping force on *u*. It is also clear from Figure 5 that the effect of *n* on *u* depends on *Ha* and *t*. For small values of *Ha*, increasing *n* decreases *u* for small and moderate time, but increases *u* for large time. For large values of *Ha*, increasing *n* increases *u* for all *t*. This is due to the decrease in *u* and its gradient with increasing *Ha* or with time progression and both results in decreasing the viscosity with increasing *n*.

(a) n=0.5



(b) n=1



(c) n=1.5

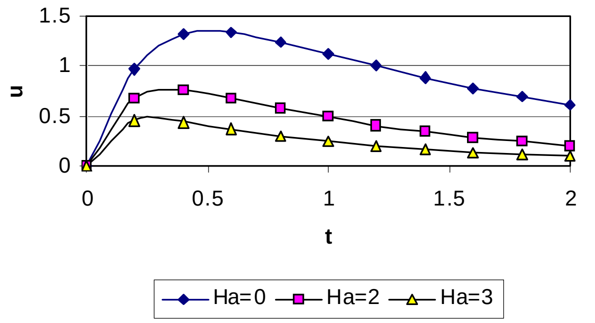
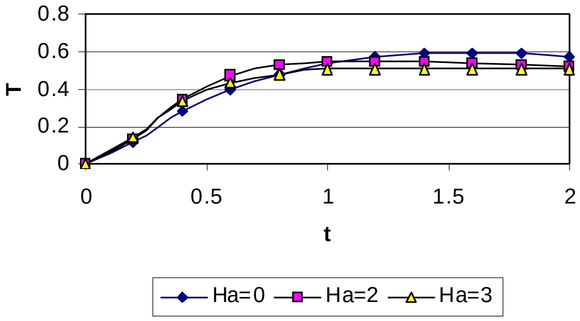


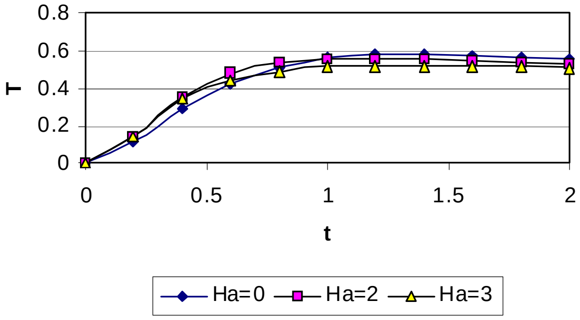
Fig. 5 Effect of Ha on u at y=0 for various values of n (S=0)

Figure 6 shows that the effect of *Ha* on the temperature *T* depends on *t*. For small values of *t*, increasing *Ha* increases *T* since the velocity *u* is small and increasing *Ha*, although it decreases *u* and its gradient, increases the Joule dissipation and then increases *T*. However, for large values of *t* increasing *Ha* decreases *T* due to the corresponding reduction in the Joule and viscous dissipations. It is also observed from Figure 6 that the effect of *n* on *T* depends on *Ha* and *t*. For small values of *Ha*, increasing *n* increases *T* for small and moderate time, but decreases *T* for large time. Increasing *n* more decreases *T* for small and moderate time, but increases it for large time. For large values of *Ha*, increasing *n* always increases *T* due to the increase in Joule dissipations.

(a) n=0.5



(b) n=1



(c) n=1.5

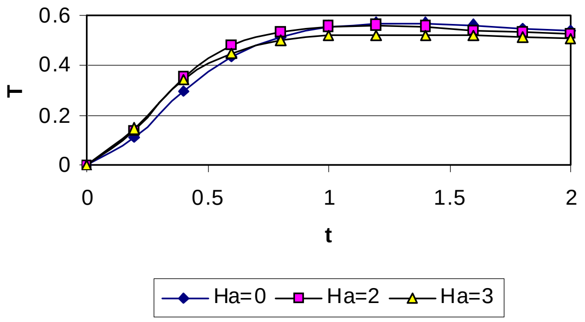
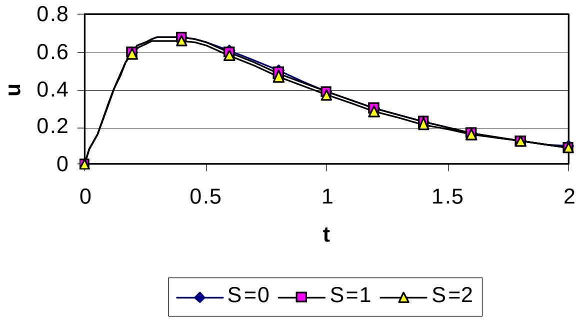


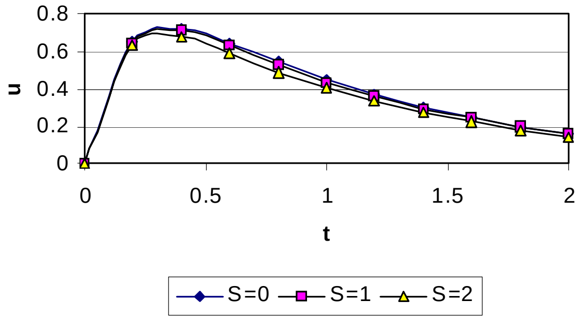
Fig. 6 Effect of Ha on T at y=0 for various values of n (S=0)

Figures 7 and 8 show the effect of the suction parameter *S* on the time development of *u* and *T* at *y=0* with time respectively for various values of the suction parameter *S* and for *n=0.5*, *1*, and *1.5*. In these figures *Ha=2*. Figure 7 shows that *u* at the centre decreases with increasing *S* for all values of *n* due to the convection of the fluid from regions in the lower half to the centre, which has a higher fluid speed. It is clear from Figure 7 that the influence of *S* on *u* is more pronounced for the case of large *n*.

(a) n=0.5



(b) n=1



(c) n=1.5

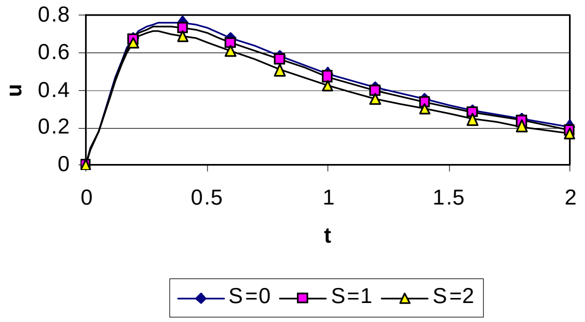


Fig. 7 Effect of S on u at y=0 for various values of n (Ha=2)

Figure 8 indicates that increasing *S* decreases the temperature at the centre of the channel for all values of *n*. This is due to the influence of the convection in pumping the fluid from the cold lower half towards the centre of the channel. The parameter *S* has a marked effect of the temperature for all values of *n*.

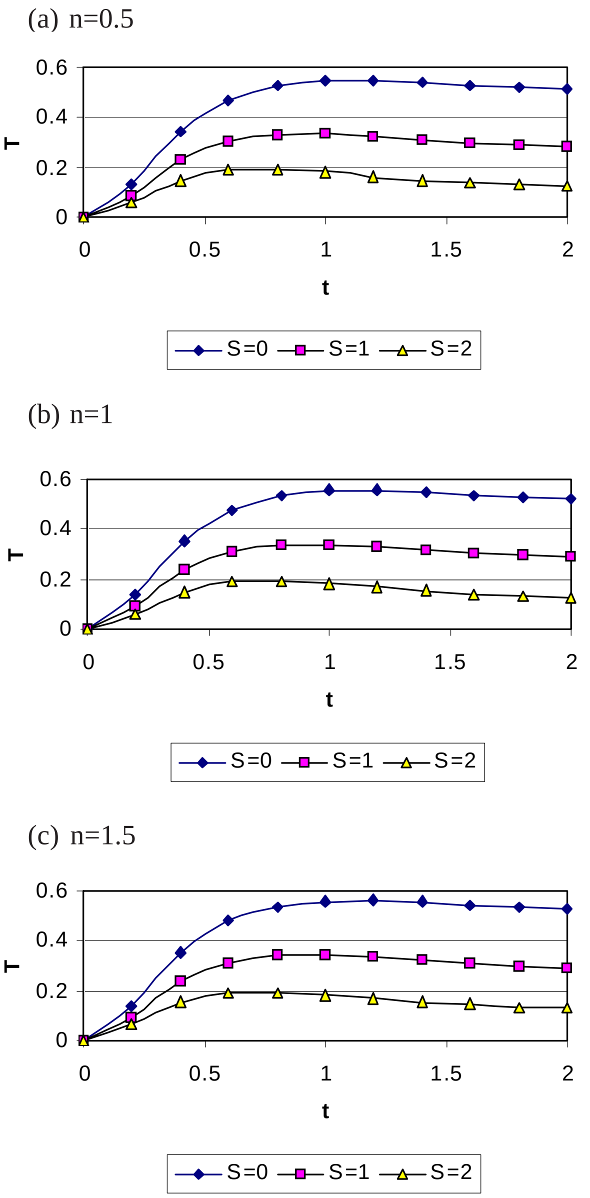


Fig. 8 Effect of S on T at y=0 for various values of n (Ha=2)

5. CONCLUSIONS

The transient Hartmann flow of a power-law non-Newtonian fluid under the influence of an applied uniform magnetic field is studied with heat transfer. The effects of the non-Newtonian fluid behavior (flow index n), the magnetic field (Hartmann number Ha), and the suction or injection velocity (suction parameter S) are studied. It was found that the effect of the flow index on the velocity depends on the magnetic field, time and the coordinate y. Also, the effect of the flow index on the temperature T depends on the magnetic field and time. The effect of the suction velocity on u is more pronounced for large values of the flow index, while it has a marked effect on the temperature for all values of the flow index.

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VREMENSKI OVISAN HARTMANNOV TOK S EKSPONENCIJALNIM PRIJENOSOM TOPLINE FLUIDA PRI JEDNOLIKOM USISAVANJU I UBRIZGAVANJU POD EKSPONENCIJALNO OPADAJUĆIM GRADIJENTOM PRITISKA

SAŽETAK

U ovom se radu pruočava vremenski ovisan Hartmannov tok električno sprovodljive viskozne nestlačive ne-Newtonove power-law tekućine između dvije paralelne horizontalne neprovodljive porozne ploče s prijenosom topline pod gradijentom eksponencijalno opadajućeg pritiska. Primijenjeni rubni uvjeti su vanjsko jednoliko magnetsko polje okomito na ploče, jednoliko usisavanje i ubrizgavanje kroz površinu ploča. Uzimajući u obzir Youleove i viskozne disipacije, na ovim dvjema pločama održavala se različita, ali konstantna temperatura. Koristeći aproksimacije konačne razlike dobila su se numerička rješenja za glavni nelinearni moment sile kao i jednadžbe energije. Ispitali su se djelovanje magnetskog polja, parametar koji opisuje to ne-Newtonsko ponašanje, brzina usisavanja i ubrizgavanja na raspodjelu brzine i temperature te uvjeti disipacije.

Ključne riječi: *MHD-tok, prijenos topline, ne-Newtonove tekućine, numerička analiza.*