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On a retailer's EOQ in a supply chain with two-level trade credit

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SUMMARY

Recently, Teng and Goyal [Journal of the Operational Research Society, Vol. 58, pp. 1252-1255, 2007.] extended and modified Huang's model [Journal of the Operational Research Society, Vol. 54, pp. 1011-1015, 2003.] to develop their model and established the proper theoretical results to obtain the optimal solution. Their inventory model is correct and interesting. However, they give the optimal solutions showing that Theorems 1 and 2 in Teng and Goyal are not complete. The main purpose of this paper is to overcome Teng and Goyal's shortcomings and to present complete proofs of their Theorems 1 and 2.

Key words: *inventory, EOQ, two-level trade credit, permissible delay in payments.*

1. INTRODUCTION

The economic order quantity (EOQ) model is widely used by practitioners as a decision-making tool for the control of inventory. However, the assumptions of the EOQ model are rarely met. This has led many researchers to study the EOQ extensively under realistic situations.

A common unrealistic assumption of the EOQ was that the retailer must pay for the items immediately after the items are received. However, in practice, the supplier may provide the retailer many different trade credits, such as a permissible delay in payments to attract new customers and to increase sales. Goyal [1] established a single-item inventory model under trade credit. Khouja and Mehrez [2] investigated the effect of four different supplier credit policies on the optimal order quantity within the EOQ framework. Chung [3] developed an efficient decision procedure to determine the economic order quantity under condition of permissible delay in payments. Teng [4] assumed that the selling price was not equal to the purchasing price to modify Goyal's model [1]. Chung and Huang [5] investigated this issue within EPQ (economic production quantity) framework and developed an efficient solving procedure to determine the optimal replenishment cycle for the retailer. Huang and Chung [6] investigated the inventory policy under cash discount and trade credit. Huang [7] adopted alternative payment rules and assumed finite replenishment rate to investigate the buyer's inventory problem. Huang [8] developed the retailer's inventory policy under two-level trade credit policy. Huang [9] extended Huang [8] to investigate retailer's ordering policy under limited retailer's storage space. Recently, Huang et al. [10] developed retailer's replenishment policy under partially trade credit policy and limited retailer's storage space. Huang [11] incorporated Chung and Huang [5] and Huang [8] to investigate retailer's ordering policy. Recently, Huang et al. [12] developed the retailer's ordering decision-making under two-level trade credit policy when the retailer had a powerful position. There were several interesting and relevant papers related to the permissible delay in payments such as Arcelus and

Srinivasan [13], Chang et al. [14], Ouyang et al. [15], Huang et al. [16], Tsao [17], and others.

Huang [8] first established an economic order quantity (EOQ) model under two-level trade credit policy in which the supplier offers the retailer a permissible delay period M, and the retailer also provides its customer a permissible delay period N (with $N<M$). Then he developed the theoretical results. Recently, Teng and Goyal [18] extended and modified Huang's model to develop their model and established the proper theoretical results to obtain the optimal solution. However, Teng and Goyal [18] at least have the following shortcoming as follows:

(1) Equation (11) in Teng and Goyal [18] always reveals that $TRC_1(T)$ is convex. However, if:

$$2A+D(M-N)^2(cI_c-pI_e)<0$$

in Eq. (11) of Teng and Goyal [18], then:

(i) $TRC_1(T)$ is not convex, and

(ii) T_1^* (Eq. (21) in Teng and Goyal [18]) does not exist such that Theorem 1 (A) in Teng and Goyal [18] is invalid.

(2) Even, if $2A+D(M-N)^2(cI_c-pI_e)>0$, processes to prove Theorem 1 in Teng and Goyal [18] are not complete so that the accuracy of Theorem 1 in Teng and Goyal [18] is questionable.

(3) If $2A+D(M-N)^2(cI_c-pI_e)\leq 0$, then T_1^* does not exist so that the accuracy of Theorem 2 in Teng and Goyal [18] is questionable.

Therefore, the main purpose of this paper is to overcome the above shortcomings and to present an easy-reading proof to complete the proofs for Theorems 1 and 2 in Teng and Goyal [18].

2. MATHEMATICAL FORMULATION

For simplicity, we use the similar notation and assumptions as was used by Teng and Goyal in Ref. [18] as follows:

- D - the annual demand rate,
- A - the ordering cost per order,
- c - the purchasing cost per unit,
- p - the selling price per unit, with $p>c$,
- h - the unit holding cost per year excluding interest charge,
- I_e - the interest earned per dollar per year,
- I_c - the interest charged per dollar per year,
- M - the retailer's trade credit period in years offered by the supplier,
- N - the customer's trade credit period in years offered by the retailer,
- T - the replenishment cycle time in years,
- Q - the order quantity,
- $TRC(T)$ - the annual total relevant cost, which is a function of T,
- T^* - the optimal replenishment cycle time of $TRC(T)$,
- Q^* - the optimal order quantity = DT^*.

The annual total relevant cost can be expressed as follows:

$$TRC(T) = TRC_1(T) \quad \text{if} \quad T \geq M-N \tag{1a}$$

$$TRC(T) = TRC_2(T) \quad \text{if} \quad T < M-N \tag{1b}$$

where:

$$TRC_1(T) = \\ = \frac{A}{T} + \frac{hDT}{2} + \frac{cI_cD}{2T}[T+N-M]^2 - \frac{pI_eD(M-N)^2}{2T} \tag{2}$$

and:

$$TRC_2(T) = \frac{A}{T} + \frac{hDT}{2} - pI_eD(M-N) + \frac{pI_eDT}{2} \tag{3}$$

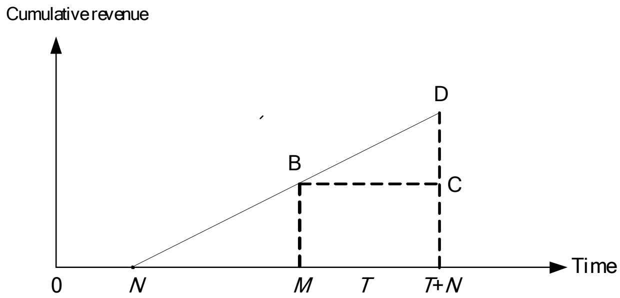

Fig. 1 $N \leq M$ and $M \leq T+N$

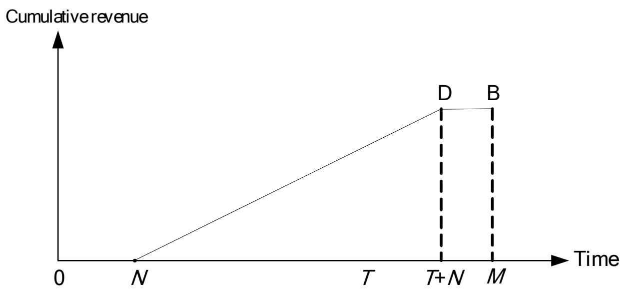

Fig. 2 $N \leq M$ and $T+N \leq M$

From Eqs. (2) and (3), we obtain:

$$TRC_1(M-N) = TRC_2(M-N). \tag{4}$$

Furthermore, $TRC(T)$ is continuous on $T > 0$. Notice that $TRC_1(T)$ is not equal to the right-hand-side terms of Eq. (2) if $T < M-N$. Likewise, $TRC_2(T)$ is not equal to the right-hand-side terms of Eq. (3) if $T > M-N$. In fact, only one of the following two mutually exclusive events can occur: (1) $T > M-N$, and (2) $T < M-N$.

3. ANALYSIS AND EXPLANATION

To minimize the annual total relevant cost, taking the first-order and the second-order derivatives of $TRC_1(T)$ and $TRC_2(T)$ with respect to T, we obtain:

$$\frac{\partial TRC_1(T)}{\partial T}= =\frac{-1}{T^2}\left[A+\frac{D(M-N)^2(cI_c-pI_e)}{2}\right]+\frac{D(h+cI_c)}{2} \tag{5}$$

$$\frac{\partial^2 TRC_1(T)}{\partial T^2}=\frac{1}{T^3}[2A+D(M-N)^2(cI_c-pI_e)], \tag{6}$$

$$\frac{\partial TRC_2(T)}{\partial T}=\frac{-A}{T^2}+\frac{D(h+pI_e)}{2} \tag{7}$$

and:

$$\frac{\partial^2 TRC_2(T)}{\partial T^2}=\frac{2A}{T^3}>0\cdot \tag{8}$$

Equation (8) implies $TRC_2(T)$ as a strictly convex function on $T > 0$. Let $\frac{\partial TRC_2(T)}{\partial T}=0$, we obtain the corresponding unique optimal cycle time T_2^* as:

$$T_2^*=\sqrt{2A/[D(h+pI_e)]}\,. \tag{9}$$

To ensure $T_2^* \le M-N$, we substitute Eq. (9) into $T \le M-N$, then we can obtain that:

if and only if: $\Delta_1=2A-(h+pI_e)D(M-N)^2\le 0$.

Equation (6) implies that $TRC_1(T)$ is strictly convex on $T > 0$ when $2A+D(M-N)^2(cI_c-pI_e)>0$. Likewise, let $\frac{\partial TRC_1(T)}{\partial T}=0$, we can easily obtain the unique optimal cycle time T_1^* as:

$$T_1^*=\sqrt{[2A+D(M-N)^2(cI_c-pI_e)]/[D(h+cI_c)]}\,. \tag{10}$$

It is obvious from Eq. (10) that if $2A+D(M-N)^2(cI_c-pI_e)>0$, then T_1^* exists. Otherwise, T_1^* does not exist. To ensure $T_1^* \ge M-N$, we substitute Eq. (10) into $T \ge M-N$, then we can obtain that:

$$\text{if and only if: } \Delta_1 \ge 0, \text{ then } T_1^* \text{ is as shown in Eq. (10).} \tag{11}$$

From the above arguments, we can obtain following results.

Theorem 1: Let $\Delta_1=2A-(h+pI_e)D(M-N)^2$

(A) If $\Delta_1 > 0$, then $T^* = T_1^*$.
(B) If $\Delta_1 < 0$, then $T^* = T_2^*$.
(C) If , $\Delta_1 = 0$, then $T^* = T_1^* = T_2^* = M-N$.

Proof :

(A) If $\Delta_1 > 0$,

$$\frac{\partial TRC_2(T)}{\partial T}=\frac{-A}{T^2}+\frac{D(h+pI_e)}{2}< <\left[1-\frac{(M-N)^2}{T^2}\right]\left[\frac{D(h+pI_e)}{2}\right]<0, \quad \text{if: } T<M-N. \tag{12}$$

Consequently, if $\Delta_1 > 0$, then $TRC_2(T)$ is a strictly decreasing function for all $T < M-N$. We know that if $\Delta_1 > 0$, then T_1^* is the optimal solution of $TRC_1(T)$. Therefore, we have:

$TRC_1(T_1^*) < TRC_1(T)$, for all $T > T_1^*$;
Likewise, we obtain:
$TRC_1(T_1^*) < TRC_1(M-N) =$
$= TRC_2(M-N) < TRC_2(T)$, for all $T < T_1^*$.
This proves that if $\Delta_1 > 0$, then $T^* = T_1^*$.

(B) If $\Delta_1 < 0$,

$$\frac{\partial TRC_1(T)}{\partial T}= =\frac{-1}{T^2}\left[A+\frac{D(M-N)^2(cI_c-pI_e)}{2}\right]+\frac{D(h+cI_c)}{2}> >\left[1-\frac{(M-N)^2}{T^2}\right]\left[\frac{D(h+cI_c)}{2}\right]>0, \quad \text{if: } T>M-N. \tag{13}$$

Consequently, if $\Delta_1 < 0$, then $TRC_1(T)$ is a strictly increasing function for all $T > M-N$. We know that if $\Delta_1 < 0$, then T_2^* is the optimal solution of $TRC_2(T)$. Therefore, we have:

$TRC_2(T_2^*) < TRC_2(T)$, for all $T < T_2^*$;
Likewise, we obtain:
$TRC_2(T_2^*) < TRC_2(M-N) =$
$= TRC_1(M-N) < TRC_1(T)$, for all $T > T_2^*$.
This proves that if $\Delta_1 < 0$, then $T^* = T_2^*$.

(C) If $\Delta_1 = 0$,
we can easily obtain that $T^* = T_1^* = T_2^* = M-N$ from Eqs. (9) and (10).

Incorporating the above arguments, we have completed the proof of Theorem 1.

Adopting the same notation as Teng and Goyal [18] we have:

$$Q_1^*=DT_1^*= =\sqrt{D[2A+D(M-N)^2(cI_c-pI_e)]/(h+cI_c)} \quad \text{if: } 2A+D(M-N)^2(cI_c-pI_e)>0. \tag{14}$$

and:

$$Q_2^*=T_2^*D=\sqrt{2AD/(h+pI_e)}\,. \tag{15}$$

In the classical EOQ model, both the retailer and the customer are assumed to pay the products as soon as they receive them. Hence, it is a special case with $M = N = 0$. Therefore, the classical optimal EOQ is:

$$Q_4^* = \sqrt{2AD/(h + cI_c)}\,. \tag{16}$$

Equation (10) shows that if $2A + D(M - N)^2(cI_c - pI_e) \le 0$, then T_1^* does not exist. Therefore, Eq. (14) is not defined. Consequently, Theorem 2 in Teng and Goyal [18] should be modified as follows.

Theorem 2:

(A) Suppose that $2A + D(M - N)^2(cI_c - pI_e) \le 0$, then:

(a1) if $pI_e < cI_c$, then Q_2^* is larger than Q_4^*.
(a2) if $pI_e > cI_c$, then Q_2^* is smaller than Q_4^*.
(a3) if $pI_e = cI_c$, then $Q_2^* = Q_4^*$.

(B) Suppose that $2A + D(M - N)^2(cI_c - pI_e) > 0$, then:

(b1) if $pI_e < cI_c$, then both Q_1^* and Q_2^* are larger than Q_4^*.
(b2) if $pI_e > cI_c$, then both Q_1^* and Q_2^* are smaller than Q_4^*.
(b3) if $pI_e = cI_c$, then $Q_1^* = Q_2^* = Q_4^*$.

4. NUMERICAL EXAMPLE

In order to illustrate the results obtained in this paper, let us apply the theoretical results to solve the following real-world example. This example is the same as in Teng and Goyal [18] except the value of the specific parameters.

A store buys nail cutters from a supplier at $c=\$0.50$ a piece and sells at $p=\$2.5$ a piece. The supplier offers a permissible delay if the payment is made within 120 days (i.e., $M=4/12=1/3$). This credit term in finance management is usually denoted as "net 120" (e.g., see Brigham [19]). However, if the payment is not paid in full by the end of 120 days, then 8% interest (i.e., $I_c=0.08$) is charged on the outstanding amount. We assume that the customer purchases by a credit card (i.e., $N=1/12$), $D=5{,}000$ units, $h=\$0.5$/unit/year, $A=\$10$ per order, and $I_e=5\%$ if the store deposits its revenue into a money-market account; or $I_e=10\%$ if it invests its revenue into a mutual fund account.

Since $2A + D(M - N)^2(cI_c - pI_e) = 20 + (5000)(3/12)^2(0.04 - 2.5I_e) < 0$ and $\Delta_1 = 2A - (h + pI_e)D(M - N)^2 = 20 - (0.5 + 2.5I_e)(5000)(3/12)^2 < 0$ for $I_e = 5\%$ or 10%, we know from Theorem 1(B) that the optimal replenishment interval is $T^* = T_2^*$. Substituting the numerical values into Eq. (15), we obtain the optimal economic order quantity Q_2^* as follows:

$$Q_2^* = \begin{cases} 400.00 & \text{if}: I_e = 5\%, \\ 365.15 & \text{if}: I_e = 10\%. \end{cases} \tag{17}$$

From Eq. (16), we have the classical optimal economic order quantity (i.e., $M=N=0$):

$$Q_4^* = \sqrt{2AD/(h + cI_c)} = 430.33\,. \tag{18}$$

The results in Eqs. (17) and (18) reveal that if $pI_e > cI_c$, we know from Theorem 2(A)-(a2) that the store should order less quantity than the classical EOQ, and to take the benefits of the permissible delay more frequently, and vice versa.

5. CONCLUSIONS

The inventory problem consists of two parts: (1) the modelling and (2) the solution procedure. Teng and Goyal's [18] inventory models are correct and interesting. The modelling can provide researchers and practitioners some concepts of analyzing the inventory problem. However, their solution procedure ignored the explorations of the functional behaviors to help to find the optimal solutions which will result in the proofs of their solution procedure that is not perfect from the viewpoint of logic. Therefore, Theorems 1 and 2 in this paper are used to overcome shortcomings of Theorems 1 and 2 given by Teng and Goyal [18].

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O EOQ TRGOVACA NA MALO I O NAČINU SNABDIJEVANJA POMOĆU TRGOVINSKOG KREDITA NA DVIJE RAZINE

SAŽETAK

Nedavno su Teng i Goyal [Journal of the Operational Research Society, 58 (2007) 1252-1255] proširili i preinačili Huang-ov model [Journal of the Operational Research Society, 54 (2003) 1011-1015] kako bi razvili taj model i utemeljili prave teorijske rezultate i dobili optimalno rješenje. Model njihovog inventara je točan i zanimljiv. Međutim, oni daju optimalna rješenja pokazujući da Teoremi 1 i 2 Tenga i Goyal-a nisu potpuni. Osnovni cilj ovog rada je svladavanje Teng-ovih i Goyal-ovih nedostataka radi iznošenja potpunih dokaza za objašnjenje njihovih Teorema 1 i 2.

Ključne riječi: *inventar, EOQ, trgovinski kredit na dvije razine, dozvoljena odgoda plaćanja.*