# Effect of pressure on free convection heat transfer from a horizontal cylinder at constant wall temperature 

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#### Abstract

SUMMARY Free convection heat transfer from a horizontal solid cylinder is examined by experimental methods when the pressure of the surrounding air is varied. The surface temperature of the cylinder is maintained at a uniform temperature using the electric current. The cylindrical element is suspended horizontally in a steel pressure vessel. The vessel is charged with air at a wide range of pressures. The objective of this paper is to show that as the pressure approaches zero, the free convective heat transfer approaches a limiting non-zero value. The surface temperature of the element is measured by a thermocouple at the mid point. The element is sufficiently remote from the walls of the vessel to give a substantially free convection. The temperature of the atmosphere in which the element is suspended is taken as equal to that of the vessel and is measured by a thermocouple in the vessel wall. Experimental measurements are carried out at a number of pressures when the voltage and current (electric power) are held constant. It is observed that the data of temperature difference between the element and the vessel as well as the data of mean temperature with respect to the fourth root of pressure fall approximately on a straight line. This, supports the idea of extrapolating the temperature difference to near zero pressure. The method of extrapolation to near zero pressure is repeated for a variety of electric power inputs and the data of temperature difference at near zero pressure are obtained. It is also observed that at a given power input, the temperature difference between the element and the vessel decreases as the pressure is increased. The method of extrapolation is extended to high pressures to find the pressure at which the temperature difference between the element and the vessel approaches zero at a given power input. The data are then compared with standard convective heat transfer correlations in terms of dimensionless groups.


Key words: heat transfer, heated cylinder, radiation, Nusselt number, pressure.

## 1. INTRODUCTION

Free convection heat transfer from horizontal cylinders has been of interest to several investigators since the middle of the last century. Most of the research has been carried out at a standard atmospheric pressure. The experimental data at non-atmospheric pressures are scarce. Fishenden and Saunders [1] recommended the following relation for the calculation of heat transfer coefficient in laminar and turbulent free convection from horizontal heated cylinder with a uniform temperature at the wall:

$$
\begin{equation*}
N u_{m, T}=0.47 R a^{1 / 4} \text { for } 10^{4}<R a<10^{9} \tag{1}
\end{equation*}
$$

and:

$$
\begin{equation*}
N u_{m, T}=0.10 R a^{1 / 3} \quad \text { for } \quad R a>10^{9} \tag{2}
\end{equation*}
$$

McAdams [2] gives the correlation for isothermal cylinders as:

$$
\begin{equation*}
N u_{m, T}=0.53 R a^{1 / 4} \text { for } 10^{4}<R a<10^{9} \tag{3}
\end{equation*}
$$

and:

$$
\begin{equation*}
N u_{m, T}=0.13 R a^{1 / 3} \text { for } 10^{9}<R a<10^{12} \tag{4}
\end{equation*}
$$

The Rayleigh number is based on the cylinder diameter $d$. For smaller values of $R a$, graphs are presented by McAdams [2].

A general expression of the form:

$$
\begin{equation*}
N u_{m, T}=c R a^{n} \tag{5}
\end{equation*}
$$

with the following tabulation of $c$ and $n$, is given by Morgan [3]:

| $R a$ | $c$ | $n$ |
| :---: | :---: | :---: |
| $10^{-10}-10^{-2}$ | 0.675 | 0.058 |
| $10^{-2}-10^{2}$ | 1.02 | 0.148 |
| $10^{2}-10^{4}$ | 0.85 | 0.188 |
| $10^{4}-10^{7}$ | 0.48 | 0.25 |
| $10^{7}-10^{12}$ | 0.125 | 0.333 |

Churchill and Chu [4] have given a correlation covering a very wide range of $R a$ for isothermal cylinders:

$$
\begin{gather*}
N u_{m, T}=\left[0.60+0.387\left\{\frac{R a}{\left[1+(0.559 / P r)^{9 / 16}\right]^{16 / 9}}\right\}^{1 / 6}\right]^{2} \\
\text { for } 10^{-5} \leq R a \leq 10^{12} \tag{6}
\end{gather*}
$$

Again, $N u_{m, T}$ and $R a$ are based on the cylinder diameter $d$.

At low pressures the mechanism of convective heat transfer changes as a consequence of the increase in the mean free path of the gaseous molecules with a falling pressure. Once the length of the mean free path becomes comparable with the dimensions of the body and the thickness of the boundary layer, intermolecular collisions become less frequent and molecules arriving at the solid surface are unable to come into equilibrium with the surface. As a result, significant velocity and temperature discontinuities may develop at the gassolid boundary. The rates of transfer are no longer governed by intermolecular collisions within the boundary layers, but depend also upon the effectiveness of property exchange between the gas molecules and the wall. The mechanism of interaction between molecules and wall is extremely complicated, and empirical parameters called reflection coefficients [5] in the case of momentum transfer and called accommodation coefficients [5] in the case of energy transfer may be used to account for their effect. If the air is highly rarefied so that $\lambda \gg L$, the frequency of intermolecular collisions may be totally negligible. At the solid surface, the impinging and re-emitted streams of molecules do not interact to any significant degree; the boundary layers to all practical purposes disappear, with molecules adjacent to the surface maintaining their free stream identity. The physical model becomes simple enough to permit calculation of transfer rates by the kinetic theory of gases. It is important to note that the mean free path is not the distance between molecules so that even in a highly rarefied gas, there is still a sufficient number of molecules in a unit element
of the gas to give meaning to gas properties such as density, viscosity and temperature. The mean free path for air at sea level conditions is almost $6 \times 10^{-6} \mathrm{~cm}$. It varies with altitude according to a Tabulation given by Schaaf and Chambre [6].

Empirical equations such as those mentioned above are no longer applicable and more elaborate expressions such as the following [7] must be used:
$\frac{2}{N u}=\ln \left\{1+\frac{6.82}{R a^{1 / 3}}\right\}+K n \frac{8 \gamma}{0.96(\gamma+1)}-\ln (1+2 K n)(7)$
The Nusselt number, which is a measure of the rate of heat transfer, becomes a function of the Knudsen number, the ratio between the mean free path and a characteristic dimension of the body. The influence of the Knudsen number begins to be significant, $K n>0.001$, at absolute pressures of less than about 1200 Pa .

When the vessel is charged with compressed air and pressures above the atmospheric one are being employed, the following assumptions can be made for pressures up to 1000 bars and temperatures up to 500 K [8]:

1. The ideal gas equation holds,
2. The specific heats remain constant,
3. The Prandtl number remains constant, and
4. The viscosity and thermal conductivity depend only on temperature.
As the pressure is increased, the air temperature will rise proportionally. It is then expected the air temperature will reach the element temperature, and accordingly the rate of heat transfer from the element to the vessel will approach zero.

The objective of this paper is to study the influence of pressure on free convection heat transfer from a horizontal heated cylinder at uniform wall temperature. The element and vessel temperatures are measured under a variety of pressure conditions. The experimental results show a linear relationship between the temperature difference and the fourth root of pressure. This interesting result helps to extrapolate the measured data to zero pressure as well as to the pressure at which the temperature difference vanishes. Experimental data indicate that the empirical method of extrapolation proposed in this paper, yields effectively the same results as a more elaborate analysis based on Eq. (5).

## 2. EXPERIMENTAL SETUP

The apparatus consists of a cylindrical element suspended horizontally in a steel pressure vessel, as shown in Figure 1. The vessel is charged with air at a wide range of pressures. The element is made of copper and heated internally by means of a glass insulated electrical heater. The element surface temperature is measured by a thermocouple at the mid point. The top cover plate, from which the element is suspended, is bolted on. The element is sufficiently remote from the walls of the vessel to give substantially free convection.


Fig. 1 Schematic diagram of experimental rig
The heat input to the element may range up to about 10 Watts, and the maximum working temperature is $200^{\circ} \mathrm{C}$. With this very small heat input, heating of the pressure vessel is negligible and the temperature of the atmosphere in which the element is suspended may be taken as equal to that of the vessel and is measured by a thermocouple in the vessel wall.

The pressure vessel is connected by a copper pipe of a large bore and an isolating valve to an electrically driven vacuum pump to be used when pressures below the atmospheric one are being employed. An external connection for a compressed air supply is provided to be used when pressures above the atmospheric one are being used.

The instruments and controls used for experimental measurement of power input to element, pressure in vessel, temperature of element and vessel are as follows:

1. Voltmeter and ammeter for indicating power supply to element,
2. On/off switch for element power supply,
3. Rheostat for regulating element power supply,
4. Digital vacuum gauge for measuring low pressures in a vessel,
5. Digital pressure gauge for measuring high pressures in a vessel,
6. Thermocouple indicator for temperature of element and a vessel,
7. Valves to put vessel in communication either with atmosphere, or with the compressed air supply or with the vacuum pump,
8. Pressure regulator for controlling compressed air supply,
9. Change-over switch to connect the element thermocouple or the vessel thermocouple to the indicator, and
10. On/off switch for vacuum pump.

## 3. EXPERIMENTAL PROCEDURE

The overall convective heat transfer characteristics of the element are investigated experimentally. The steps taken are as follows:

1. The power supply is switched on and the rheostat is adjusted to give a desired power input, say 5 W .
2. The valves leading to atmosphere and to the vacuum pump are closed. The pressure regulator is turned anti-clockwise to minimize the air supply pressure.
3. The valve between the air supply and the vessel is opened carefully. The pressure regulator is turned clockwise.
4. When the pressure in the vessel reaches a desired value, say 250 kPa , the isolating valve on the compressed air supply is closed.
5. The temperature of the element on the thermocouple indicator is observed. This temperature takes some minutes to stabilize and is recorded when no further change is taking place.
6. The change-over switch is turned to connect the vessel thermometer to the indicator. The temperature of the vessel is observed. This temperature is quickly stabilized and then recorded.
7. The voltmeter, ammeter, and pressure gauge are observed. The voltage, current and vessel pressure are recorded.
8. The isolating valve is opened to reduce the pressure in the vessel to another desired value, say 200 kPa and then closed again. Steps 5 to 7 are repeated. This step is repeated for as many pressures above the atmospheric one as required.
9. The isolating valve between the vessel and the air supply is closed. The valve leading to atmosphere is opened. Steps 5-7 are repeated for atmospheric pressure.
10. Once step 9 is carried out, the atmospheric pressure in the laboratory is recorded. Step 9 is repeated with the isolating valve open and the atmospheric valve closed.
11.The isolating valve and the atmospheric valve are closed. The vacuum pump is switched on. The valve between the vessel and the vacuum pump is opened. The pump is run until the pressure in the vessel has been reduced to a desired value, say $70 k P a$. Steps 5-7 are repeated. This step is repeated for as many pressures below atmospheric as required.
12.The whole procedure including steps 1-11 can be repeated for a different power supply.

## 4. EXPERIMENTAL RESULTS

The experimental procedure for each run is to set a pre-defined power input (voltage and current) and a high pressure in the vessel, then to take a series of readings at progressively lower pressures. Finally leaving the vacuum pump running for as long as necessary to reach the ultimate vacuum for which the apparatus is capable, this is nominally $4 P a$; however, under favorable conditions even lower pressures may be obtained.

Table 1 shows a set of observations made in accordance with the experimental procedure explained above, with the vessel charged with air. The electrical
input to the element was maintained constant (6.56 W) while the air pressure varied from 421866 Pa to 3.2 Pa .

Table 1 Typical experimental pressure and temperature data for air

| $V$ <br> [Volt] | $I$ <br> [Amper] | $Q$ <br> [Watt] | $P$ <br> [Pascal] | $T_{E}$ <br> $\left[{ }^{\circ} C\right]$ | $T_{V}$ <br> $\left[{ }^{\circ} C\right]$ | $T_{E^{-}} T_{V}$ <br> $\left[{ }^{\circ} C\right]$ | $P^{1 / 4}$ <br> $\left[\mathrm{~Pa}^{1 / 4}\right]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8.2 | 0.8 | 6.56 | 421866 | 83.8 | 24.3 | 59.5 | 25.48 |
| 8.2 | 0.8 | 6.56 | 338800 | 84.6 | 28.2 | 64.7 | 24.12 |
| 8.2 | 0.8 | 6.56 | 252933 | 95 | 23.8 | 71.2 | 22.42 |
| 8.2 | 0.8 | 6.56 | 225600 | 97.4 | 23.6 | 73.8 | 21.79 |
| 8.2 | 0.8 | 6.56 | 204933 | 97.6 | 21.4 | 76.2 | 21.27 |
| 8.2 | 0.8 | 6.56 | 161733 | 100.5 | 20.6 | 79.9 | 20.05 |
| 8.2 | 0.8 | 6.56 | 132933 | 105.2 | 22.2 | 83 | 19.09 |
| 8.2 | 0.8 | 6.56 | 101333 | 110.5 | 23.4 | 87.1 | 17.84 |
| 8.2 | 0.8 | 6.56 | 45333 | 124.2 | 22.4 | 101.8 | 14.59 |
| 8.2 | 0.8 | 6.56 | 9866 | 138.6 | 21.7 | 116.9 | 9.96 |
| 8.2 | 0.8 | 6.56 | 1600 | 153.6 | 21.5 | 132.1 | 6.32 |
| 8.2 | 0.8 | 6.56 | 306 | 161.4 | 23.4 | 138 | 4.18 |
| 8.2 | 0.8 | 6.56 | 24 | 165.5 | 22.6 | 142.9 | 2.21 |
| 8.2 | 0.8 | 6.56 | 3.2 | 172 | 20.2 | 151.8 | 1.33 |

Heat transfer from the element to its surroundings takes place by two processes, namely by free or natural convection and radiation. While the process of convection is a function of air pressure, the heat loss by radiation is effectively independent of this pressure.

The heat loss due to radiation from a body at temperature $T_{E}$ located in a space of dimensions substantially larger than that of the body and at temperature $T_{V}$ is given by the Stefan-Boltzman equation [9]. The emissivity of the element surface (copper finished with a matt black surface), is specified by the manufacturer, 0.98 .

If the electrical resistance of the leads that supply power to the element and support it and the heat losses by conduction along the current carrying the thermocouple leads are sufficiently small, one can assume that when the steady state condition prevails, i.e. when the element temperature is stabilized, the heat input to the element is equal to the sum of heat losses due to radiation and natural convection.

Having the data of the element and vessel temperatures, heat transfer surface and emmisivity, the heat loss due to radiation can be worked out using Eq. (8) and then the heat loss due to convection is calculated using Eq. (9). The heat transfer coefficient is then deduced from the following relationship:

$$
\begin{equation*}
Q_{C}=A h\left(T_{E}-T_{V}\right) \tag{8}
\end{equation*}
$$

Table 2 shows the results of the above sequence leading to the convective heat transfer coefficient corresponding to the data reported in Table 1. Table 2 also includes the data of mean temperature, thermal conductivity and Nusselt number.
$T_{M}$ in Table 2 is the arithmetic mean of the element and vessel temperatures, namely:

$$
\begin{equation*}
T_{M}=\frac{T_{E}+T_{V}}{2} \tag{9}
\end{equation*}
$$

where $k$ is the thermal conductivity at air mean temperature. It is assumed that in the range of low
pressures, the thermal conductivity of gases increases with temperature [10]. For monatomic and diatomic gases, the rise in conductivity is almost proportional to the absolute temperature:

$$
\begin{equation*}
\frac{k_{2}}{k_{1}}=\frac{T_{2}}{T_{1}} \tag{10}
\end{equation*}
$$

The conductivity of air at reference temperature of $300 K$ is $0.02624 \mathrm{~W} / \mathrm{mK}$ [9]. The conductivity of air at other temperatures $\left(T_{M}\right)$ is calculated using this reference value and Eq. (11).

Table 2 Typical experimental heat transfer coefficient ( $h$ ) and Nusselt number ( Nu ) data for air

| $Q_{R}[$ Watt $]$ | $Q_{C}[$ Watt $]$ | $h\left[W / m^{2} \mathrm{~K}\right]$ | $T_{M}\left[{ }^{\circ} \mathrm{C}\right]$ | $\mathrm{k}[\mathrm{W} / \mathrm{mK}]$ | Nu |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.527 | 5.033 | 26.32 | 54.05 | 0.028619 | 5.77 |
| 1.478 | 5.082 | 28.04 | 56.4 | 0.028825 | 6.1 |
| 1.925 | 4.635 | 20.26 | 59.4 | 0.029087 | 4.37 |
| 2.017 | 4.543 | 19.16 | 60.5 | 0.029183 | 4.12 |
| 2.065 | 4.495 | 18.36 | 59.5 | 0.029096 | 3.96 |
| 2.189 | 4.371 | 17.02 | 60.55 | 0.029188 | 3.66 |
| 2.341 | 4.219 | 15.82 | 63.7 | 0.029463 | 3.37 |
| 2.531 | 4.029 | 14.39 | 66.95 | 0.029748 | 3.03 |
| 3.143 | 3.417 | 10.44 | 73.3 | 0.030303 | 2.16 |
| 3.849 | 2.711 | 7.22 | 80.15 | 0.030902 | 1.46 |
| 4.657 | 1.903 | 4.48 | 87.55 | 0.031549 | 0.89 |
| 5.074 | 1.486 | 3.35 | 92.4 | 0.031973 | 0.66 |
| 5.337 | 1.223 | 2.66 | 94.05 | 0.032118 | 0.52 |
| 5.789 | 0.771 | 1.58 | 96.1 | 0.032297 | 0.31 |

## 5. COMPARISON

The Nusselt numbers deduced from experimental data in Table 2 can be compared with those predicted by standard correlations mentioned in the introduction section. To carry out such a comparison, the thermal properties of air at mean temperature should be known. The density of air is calculated from the perfect gas equation:

$$
\begin{equation*}
\rho=\frac{P}{R T} \tag{11}
\end{equation*}
$$

where $R$ is the gas constant, being $0.287 \mathrm{~kJ} / \mathrm{kgK}$ for air.

The specific heat of gases is temperature dependent, the following equation is recommended [11] for air:

$$
\begin{equation*}
C_{p}=0.917+2.58 \times 10^{-4} T-3.98 \times 10^{-8} T^{2} \tag{12}
\end{equation*}
$$

where $T$ is in Kelvin and $C p$ in $\mathrm{kJ} / \mathrm{kgK}$.
Viscosity is also strongly temperature dependant; Sutherland's law [11] may be used for air as:

$$
\begin{equation*}
\mu=\frac{1.46 \times 10^{-6} T^{3 / 2}}{110+T} \tag{13}
\end{equation*}
$$

where $T$ is in Kelvin and $\mu$ in $k g / m s$. The range of validity of Eq. (14) is $280-1500 \mathrm{~K}$.

The coefficient of volume expansion for perfect gases is the reciprocal of absolute temperature [12]:

$$
\begin{equation*}
\beta=\frac{l}{T} \tag{14}
\end{equation*}
$$

The dimensionless Grashof number, Prandtl number and Nusselt number are defined as follows:

$$
\begin{gather*}
G r=\frac{g \beta\left(T_{E}-T_{V}\right) d^{3} \rho^{2}}{\mu^{2}}  \tag{15}\\
\operatorname{Pr}=\frac{\mu C_{p}}{k}  \tag{16}\\
N u=\frac{h d}{k} \tag{17}
\end{gather*}
$$

Table 3 shows the thermal properties of air and dimensionless groups, leading to calculation of Nusselt number from standard correlations.

Table 3 Thermal properties of air and dimensionless groups at mean temperature

| $\rho$ <br> $\left[\mathrm{kg} / \mathrm{m}^{3}\right]$ | $C_{p}$ <br> $[J / \mathrm{kgK}]$ | $\mu$ <br> $[\mathrm{kg} / \mathrm{ms}]$ | $\beta$ <br> $\left[K^{-1}\right]$ | $G r$ | $P r$ | $R a$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4.492416 | 997.1566 | $1.97648 E-05$ | 0.003056 | 22716.94 | 0.688655 | 15644.13 |
| 3.58212 | 997.7015 | $1.98713 E-05$ | 0.003034 | 15427.01 | 0.687793 | 10610.59 |
| 2.650127 | 998.3964 | $2.00067 E-05$ | 0.003007 | 9083.978 | 0.686721 | 6238.154 |
| 2.35595 | 998.6511 | $2.00562 E-05$ | 0.002997 | 7380.247 | 0.68633 | 5065.284 |
| 2.146557 | 998.4196 | $2.00112 E-05$ | 0.003006 | 6373.481 | 0.686679 | 4376.532 |
| 1.688731 | 998.6626 | $2.00585 E-05$ | 0.002997 | 4103.815 | 0.686297 | 2816.437 |
| 1.375037 | 999.3913 | $2.01997 E-05$ | 0.002969 | 2760.896 | 0.68518 | 1891.71 |
| 1.038155 | 1000.142 | $2.03448 E-05$ | 0.00294 | 1612.502 | 0.684026 | 1102.993 |
| 0.455924 | 1001.607 | $2.06263 E-05$ | 0.002886 | 347.1537 | 0.681761 | 236.6759 |
| 0.097301 | 1003.184 | $2.0927 E-05$ | 0.00283 | 17.29664 | 0.67936 | 11.75064 |
| 0.015456 | 1004.882 | $2.12485 E-05$ | 0.002772 | 0.468551 | 0.676796 | 0.317113 |
| 0.002917 | 1005.994 | $2.14574 E-05$ | 0.002736 | 0.016867 | 0.675132 | 0.011387 |
| 0.000228 | 1006.371 | $2.15281 E-05$ | 0.002723 | 0.000105 | 0.674553 | $7.1 E-05$ |
| $3.02 E-05$ | 1006.84 | $2.16158 E-05$ | 0.002708 | $1.94 E-06$ | 0.67386 | $1.31 E-06$ |

Among the standard correlations mentioned in the introduction section, Eq. (5) with the accompanying tabulation of $c$ and $n$, covers the full range of Rayleigh numbers in Table 3. The values obtained for Nusselt number corresponding to the Rayleigh numbers in Table 3 are listed in Table 4. The corresponding experimental values are also included in Table 4.
Table 4 Comparison of experimental Nusselt numbers with those predicted by Eq. (5) [3]

| $R a$ | $c$ | $n$ | $N u(E q .(5))$ | $N u($ Table 2) |
| :---: | :---: | :---: | :---: | :---: |
| 15644.13 | 0.48 | 0.25 | 5.37 | 5.77 |
| 10610.59 | 0.48 | 0.25 | 4.87 | 6.1 |
| 6238.154 | 0.85 | 0.188 | 4.39 | 4.37 |
| 5065.284 | 0.85 | 0.188 | 4.22 | 4.12 |
| 4376.532 | 0.85 | 0.188 | 4.11 | 3.96 |
| 2816.437 | 0.85 | 0.188 | 3.78 | 3.66 |
| 1891.71 | 0.85 | 0.188 | 3.51 | 3.37 |
| 1102.993 | 0.85 | 0.188 | 3.17 | 3.03 |
| 236.6759 | 0.85 | 0.188 | 2.37 | 2.16 |
| 11.75064 | 1.02 | 0.148 | 1.47 | 1.46 |
| 0.317113 | 1.02 | 0.148 | 0.86 | 0.89 |
| 0.011387 | 1.02 | 0.148 | 0.53 | 0.66 |
| $7.1 E-05$ | 0.675 | 0.058 | 0.39 | 0.52 |
| $1.31 E-06$ | 0.675 | 0.058 | 0.31 | 0.31 |

It can be seen that the Nusselt numbers deduced from the experimental method developed in this paper are in good agreement with those predicted by Morgan [3] correlation, Eq. (5). This comparison is more elucidated when two sets of Nusselt numbers are
plotted against the Rayleigh number, this is done in Figure 2.


Fig. 2 Comparison of experimental Nusselt numbers with those predicted by Morgan [3]

## 6. ANALYSIS

Table 2 indicates that as the air pressure is decreased, the contribution of convective heat transfer decreases, while the contribution of radiative heat transfer increases. It is further evident that even at a low pressure as 3.2 Pa, convective heat transfer still constitutes about 12 percent of the total heat load. It is thus not true to assume that by further reducing the air pressure, the convective heat transfer vanishes. It is actually the objective of this paper to show that the convective heat transfer exists even at zero pressure, having in mind that all empirical correlations for $R a<10^{-5}$ (Eqs. (5) and (7)) predict a zero Nusselt number at zero pressure. Consequently, as pressure goes to zero, density goes to zero, Grashof number goes to zero, Rayleigh number goes to zero, and eventually Nusselt number goes to zero.

Careful consideration of experimental data reveals that as low pressures are approached, the successive increases in element temperature become greater and a direct plot of the temperature difference against pressure would be of little use as a means of determining the value of $T_{E}-T_{V}$ corresponding to zero pressure. It is however observed that a plot of $T_{E}-T_{V}$ against the fourth root of pressure gives approximately a straight line and this provides a satisfactory basis for estimating temperature difference at zero pressure. The data of temperature difference and fourth root of pressure are listed in Table 1. These data are plotted in Figure 3.


Fig. 3 Variation of temperature difference and mean temperature of air versus the fourth root of pressure

Furthermore, a plot of air mean temperature versus the fourth root of pressure gives also a straight line and this provides a reasonable basis to estimate the air mean temperature at zero pressure. The data of air mean temperature are listed in Table 2. These data are also plotted in Figure 3.

The line of $\mathrm{T}_{\mathrm{E}}-\mathrm{T}_{\mathrm{V}}$ has a slope of $-13^{\circ} \mathrm{C} / \mathrm{Pa}^{1 / 4}$ and when extrapolated to a near zero pressure, it gives a temperature difference of about $157{ }^{\circ} \mathrm{C}$. The line of $\mathrm{T}_{\mathrm{M}}$ has a slope of $-5.9^{\circ} \mathrm{C} / \mathrm{Pa}^{1 / 4}$ and when extrapolated to a zero pressure, it gives a mean temperature of roughly $98.5^{\circ} \mathrm{C}$. The element and vessel temperatures at a zero pressure are then easily obtained. Having the element and the vessel temperatures at a zero pressure, the radiative heat transfer (Eq. (8)), the convective heat transfer (Eq. (9)), the convective heat transfer coefficient (Eq. (10)) and finally the Nusselt number (Eq. (18)) at zero pressure are obtained. A listing of these information is available in Table 5. Care should be taken that these data correspond to the electric power input to the element ( 6.56 W ). Evidently, at a different power condition these data will change, but still the convective heat transfer approches a non-zero limiting value. To find out how this limiting value varies with respect to the power input the whole experimental procedure is repeated for a different power input ( 4.21 W ). The intermediate steps are not revealed to avoid lengthening the paper. The final results are listed in Table 5.

Table 5 Heat transfer characteristics of a horizontal isothermal cylinder under near zero pressure condition

| $\begin{gathered} Q \\ {[W]} \\ \hline \end{gathered}$ | $\begin{gathered} \hline T_{E} \\ {\left[{ }^{\circ} \mathrm{C}\right]} \\ \hline \end{gathered}$ | $\begin{gathered} \hline T_{V} \\ {\left[{ }^{\circ} \mathrm{C}\right]} \\ \hline \end{gathered}$ | $\begin{gathered} \hline Q_{R} \\ {[W]} \\ \hline \end{gathered}$ | $\begin{gathered} \hline Q_{c} \\ {[W]} \\ \hline \end{gathered}$ | $\begin{gathered} h \\ {\left[W / m^{2} \cdot K\right]} \\ \hline \end{gathered}$ | $N u$ | $\begin{gathered} Q_{C} Q \\ \% \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6.56 | 177 | 20 | 6.119058 | 0.440942 | 0.874011 | 0.17 | 7.2 |
| 4.21 | 145 | 26 | 4.099941 | 0.110059 | 0.287817 | 0.06 | 2.6 |

Table 5 shows that as the pressure approaches zero, the free convection heat transfer approaches a non-zero limiting value depending on the electric power input entering the element.

## 7. ACCURACY OF RESULTS

The accuracy of the results depends on the errors encountered when conducting the experiment. Some of these errors are identified as:

1. The heat input to the element warms up the pressure vessel, causing the temperature of the atmosphere, in which the element is suspended, to be less than that of the vessel. The air temperature has been taken as equal to that of the vessel and is measured by a thermocouple in the vessel wall.
2. The electrical resistance of the leads that supply power to the element and support it, cause a
power loss that has been ignored when calculating power input to the element. This affects the radiative as well as the convective heat loss from the element.
3. Heat is lost by conduction through the thermocouple leads (conductors) and carried radially to the surface of the insulating sleeves covering the leads where it is dissipated by radiation and convection. This has been ignored when calculating the power input to the element.
4. The heat transfer area is taken as the total surface area of the element and then the convective heat transfer coefficient is calculated using Eq. (10). In reality, the two ends of the cylinder must be treated as two vertical circular discs having different heat transfer characteristics than the horizontal cylinder.

## 8. CONCLUSION

Heat transfer from a heated cylinder to its quiescent atmosphere takes place by two mechanisms, free convection and radiation. While the mechanism of radiation is effectively independent of pressure, the convective heat transfer in gases is a function of the gas pressure. As the gas pressure is decreased, the contribution of convective heat transfer decreases, but never vanishes. At the same time, the contribution of radiative heat transfer increases, but never equals the total heat input to the element. It is worth pointing out that even at a very low pressure of 3.2 Pa , the convective heat transfer accounts for nearly $12 \%$ of the total heat loss from the cylinder. This is when the heat load to the element is 6.56 W and it decreases by further reducing the pressure. It accounts for almost $7.2 \%$ of the total heat loss in the limiting case when the pressure approaches zero.

The data of temperature difference between the element and the free atmosphere, with respect to the fourth root of pressure fall approximately on a straight line. The method of extrapolation allows to predict the temperature difference at a zero pressure. There is no specific theoretical justification for this method. It is merely a matter of experience that the observations at low pressures fall nearly on a straight line when plotted against the fourth root of pressure.

Although the empirical correlations developed for the laminar free convection from a horizontal cylinder, predict a zero Nusselt number at zero pressure, this paper shows that the Nusselt number approaches to a non-zero limiting value at zero pressure. This limiting value is easily obtained using the method of extrapolation explained in this paper and avoids using the complicated relationships such as Eq. (7) which involve evaluating the mean free path of molecules and gas properties based on kinetic theory of gases.

## 9. REFERENCES

[1] M. Fishenden and O.A. Saunders, An Introduction to Heat Transfer, Oxford University Press, 1950.
[2] W.H. McAdams, Heat Transmission, 3rd ed., McGraw-Hill, New York, 1954.
[3] V.T. Morgan, The Overall convective heat transfer from smooth circular cylinders, Advances in Heat Transfer, Vol. 11, Academic Press, New York, 1975.
[4] S.W. Churchill and H.H.S. Chu, Correlating equations for laminar and turbulent free convection from a horizontal cylinder, Int. J. Heat Mass Transfer, Vol. 18, 1975.
[5] W.M. Rohsenow and H. Choi, Heat, Mass, and Momentum Transfer, Prentice-Hall, Inc., New Jersey, 1961.
[6] S.A. Schaaf and P.L. Chambre, Flow of rarefied gases, High Speed Aerodynamics and Jet Propulsion, Sec. H, Vol. 3, Princeton University Press, 1958.
[7] L. Boswirth and M.A. Plint, Technische Stromungslehre, Schroedel, 1975.
[8] H. Schlichting and K. Gersten, Boundary Layer Theory, $8^{\text {th }}$ ed., Springer, 1999.
[9] J.P. Holman, Heat Transfer, 9th Edition, McGraw-Hill, New York, 2002.
[10] M. Schunck, Temperature dependency of the thermal conductivity of gases, Heat Exchanger Design Handbook, Ed. E.W. Schlunder, Hemisphere Publishing Corporation, 1989.
[11] F.J. Bayley, J.M. Owen and A.B. Turner, Heat Transfer, Nelson, 1972.
[12] H.Y. Wong, Handbook of Essential Formula and Data on Heat Transfer for Engineers, Longman, 1977.

## UTJECAJ PRITISKA NA SLOBODAN PRIJENOS TOPLINE KONVEKCIJOM IZ HORIZONTALNOG CILINDRA PRI STALNOJ TEMPERATURI STIJENKI


#### Abstract

SAŽETAK Slobodni prijenos topline konvekcijom iz horizontalnog čvrstog cilindra ispituje se eksperimentalnim metodama pri promjeni tlaka okolnog zraka. Površinska temperatura cilindra održava se jednolikom koristeći elekričnu struju. Cilindrični element je obješen horizontalno u čeličnoj posudi pod tlakom. Cilj ovog rada je pokazati da se približavanjem tlak nuli, slobodni prijenos topline konvekcijom približava graničnoj ne-nultoj vrijednosti.

Površinska temperatura elementa mjeri se pomoću termoelementa udaljenog od zidova posude tako da se omogućava prilično slobodno strujanje. Pretpostavlja se da je temperatura atmosfere u kojoj je element obješen jednaka temperaturi posude i ona se mjeri pomoću termoelementa u stijenki posude. Eksperimentalna mjerenja se vrše za određeni broj tlakova kada su i napon i struja stalni. Uočava se da se podaci o razlici u temperaturi između elementa i posude kao i podaci o prosječnoj temperaturi u odnosu na četvrti korjen tlaka nalaze otprilike na pravcu. To potvrđuje ideju ekstrapoliranja razlike temperature na tlak blizu nule. Metoda ekstrapolacije do tlaka blizu nule se ponavlja za različite unose električne energije i tako se dobivaju podaci o razlici u temperaturi pri tlaku blizu nule. Također se uočava da, pri određenom unosu energije, dolazi do smanjenja razlike u temperaturi između elementa i posude kako se povećava tlak. Metoda ekstrapolacije se proširuje na visoke tlakove kako bi pronašli tlak pri kojem se razlika u temperaturi između elementa i posude približava nuli pri određenom unosu energije. Podaci se tada uspoređuju sa standardnim korelacijama konvekcijskog prijenosa topline u odnosu na bezdimenzionalne grupe.


Ključne riječi: prijenos topline, zagrijavani cilindar, radijacija, Nusselt-ov broj, tlak.

