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Identification of the boundary condition on the skin surface subjected to external heat source

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SUMMARY

In the paper the inverse problem consisting in the identification of external heat source influencing the skin surface is presented. On the basis of the knowledge of heating curves at selected points from the domain considered the time dependent value of boundary heat flux is identified. In order to solve the problem the sequential function specification method [1, 2] and the whole-domain estimation of heat flux [2] have been used. In the stage of numerical computations the boundary element method has been applied. From the practical point of view the algorithm presented can be applied for the burns prediction.

Key words: *identification of external heat source, skin surface, sequential approach, boundary element method, burns prediction.*

1. INTRODUCTION

The skin is treated as a multi-layer domain. If the 1D model is considered (such assumption is quite acceptable) then the equations describing the heat transfer processes in the successive layers are of the form [3, 4, 5]:

$$x \in \Omega_e :$$
$$c_e \partial_t T_e(x,t) = \lambda_e \partial_{xx} T_e(x,t) + k_e \left[T_B - T_e(x,t) + Q_{me} \right] \tag{1}$$

where $e=1,2,3$ correspond to the epidermis, dermis and sub-cutaneous sub-domain (Figure 1), c_e, λ_e and k_e denote the volumetric specific heat, thermal conductivity and perfusion coefficient for sub-domains Ω_e, T_B is the arterial blood temperature and Q_{me} is the metabolic heat source.

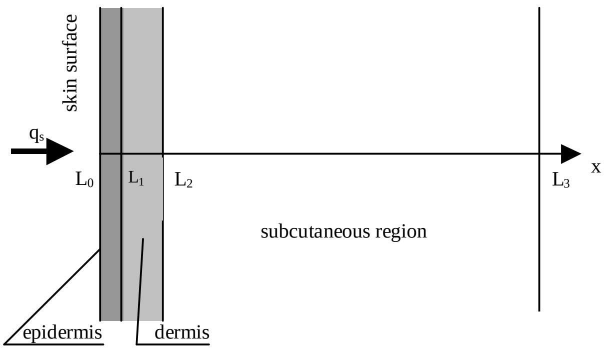

Fig. 1 Skin tissue

For $x \in \Gamma_0$ (skin surface) we introduce the Neumann condition:

$$q_1(0,t) = \lambda_1 \partial_x T_1(0,t) = q_s(t) \tag{2}$$

and course of function $q_s(t)$ is unknown.

On the contact surfaces between sub-domains the continuity conditions are given:

$$x \in \Gamma_{e,e+1} : \begin{cases} -q_e(x,t) = q_{e+1}(x,t) \\ T_e(x,t) = T_{e+1}(x,t) \end{cases}, \quad e = 1,2 \tag{3}$$

For the internal boundary Γ_3 limiting the system the no-flux condition is assumed. The initial temperature distribution is also determined [3].

Additionally, the values T_{di}^f at the selected set of points x_i for times t^f are known. Namely:

$$T_{di}^f = T_d(x_i, t^f) \; ; \; i = 1,2,..., M \; ; \; f = 1,2,..., F \tag{4}$$

In order to solve the inverse problem the two different algorithms have been applied. The first of them is called the sequential function specification method [1, 2] and uses the information about the future temperature measurements. The second one (whole-domain estimation of heat flux [2]) consists of the estimation of coefficients determining the form of $q_s(t)$.

It should be pointed out that comparing the task discussed with the solutions presented in literature we consider essentially more complex problem (the non-homogeneous domains with internal heat sources).

The 'measured' heating curves (Eq. (4)) result from the solution of the direct problem for assumed course of function $q_s(t)$, next they have been disturbed in a random way.

2. SEQUENTIAL FUNCTION SPECIFICATION METHOD

In the sequential function specification method [1, 2] the sensitivity coefficients are used.

In order to calculate them, the governing equations are differentiated with respect to the unknown boundary heat flux. So, for each sub-domain of skin one has (Eq. (1)):

$$x \in \Omega_e : c_e \partial_t Z_e(x,t) = \lambda_e \partial_{xx} Z_e(x,t) - k_e Z_e(x,t) \tag{5}$$

where:

$$Z_e(x,t) = \frac{\partial T_e(x,t)}{\partial q_s} \tag{6}$$

is the sensitivity function.

A differentiation of the boundary and initial conditions with respect to q_s gives:

– continuity conditions:

$$x \in \Gamma_{e,e+1} : \begin{cases} -\lambda_e \partial_x Z_e(x,t) = \lambda_{e+1} \partial_x Z_{e+1}(x,t) \\ Z_e(x,t) = Z_{e+1}(x,t) \end{cases}, \quad e = 1,2 \tag{7}$$

– Neumann condition:

$$x \in \Gamma_0 : -\lambda_1 \partial_x Z_1(x,t) \tag{8}$$

– adiabatic condition for $x \in \Gamma_3$:

$$-\lambda_3 \partial_x Z_3(x,t) = 0 \tag{9}$$

– initial condition:

$$t=0 : Z_e(x,t) = 0, \; e = 1,2,3 \tag{10}$$

The additional problem can be solved directly - it is correctly posed because both the differential equations determining the distribution of Z_e in the domain considered and the boundary initial conditions are known too.

Due to the discrete nature of temperature data, Eq. (4), the unknown function $q_s(t)$ must also be expressed in a discrete form, for example:

$$t \in \left[t^{f-1}, t^f\right] : q^f = q_s\left(t^f\right), \; f = 1,2,...,F \tag{11}$$

It is assumed that the heat flux is known at times t^1, t^2, ...t^{f-1} and we want to determine the heat flux q^f at time t^f. Of course, some measured temperature histories are given at interior locations x_i, namely T_{di}^1, T_{di}^2, ..., T_{di}^{f-1}, $i=1,2,...,M$. This variant of function specification method is called the sequential approach [1, 2]. Additionally, we assume that the temperature histories are known for R future intervals, namely:

$$T_{di}^{f+r-1} = T_d(x_i, t^{f+r-1}) \; ; \; r=1,2,..., R \; ; \; i=1,2,..., M \tag{12}$$

and the heat flux is constant over R future steps:

$$q^f = q^{f+1} = q^{f+R-1} \tag{13}$$

Function $T_i^{f+r-1} = T(x_i, t^{f+r-1})$ is expanded in a Taylor series about arbitrary but known value of heat flux $\hat{q}^f$:

$$T_i^{f+r-1} = \hat{T}_i^{f+r-1} + \left.\frac{\partial T_i^{f+r-1}}{\partial q^f}\right|_{q^f = \hat{q}^f} \left(q^f - \hat{q}^f\right) \tag{14}$$

where $\hat{T}_i^{f+r-1}$ is the temperature at time t^{f+r-1} and location x_i obtained under the assumption that for $t \in \left[t^{f-1}, t^{f+r-1}\right]$ the heat flux equals $q^f = q^{f+1} = q^{f+R-1} = \hat{q}^f$.

We introduce the sensitivity coefficients and then:

$$T_i^{f+r-1} = \hat{T}_i^{f+r-1} + Z_i^{f+r-1}\left(q^f - \hat{q}^f\right) \tag{15}$$

In order to solve the inverse problem, the least squares method is applied [1, 2, 6]:

$$S\left(q^f\right) = \sum_{i=1}^{M} \sum_{r=1}^{R} \left(T_i^{f+r-1} - T_{di}^{f+r-1}\right)^2 \rightarrow MIN \tag{16}$$

Putting Eq. (15) into Eq. (16) one has:

$$S\left(q^f\right) = \sum_{i=1}^{M} \sum_{r=1}^{R} \left(\hat{T}_i^{f+r-1} + Z_i^{f+r-1}\left(q^f - \hat{q}^f\right) - T_{di}^{f+r-1}\right)^2 \rightarrow MIN \tag{17}$$

Differentiating the criterion given by Eq. (12) with respect to the unknown heat flux q^f and using the necessary condition of minimum, one obtains:

$$q^f = \hat{q}^f + \frac{\sum_{i=1}^{M}\sum_{r=1}^{R}\left(T_{di}^{f+r-1} - \hat{T}_i^{f+r-1}\right) Z_i^{f+r-1}}{\sum_{i=1}^{M}\sum_{r=1}^{R}\left(Z_i^{f+r-1}\right)^2} \tag{18}$$

3. WHOLE-DOMAIN ESTIMATION OF HEAT FLUX

We assume the time variation of boundary heat flux in the form:

$$x \in \Gamma_0 : q_s(t) = \beta_1 + \beta_2 t + \beta_3 t^2 \tag{19}$$

where the coefficients β_1, β_2 and β_3 are unknown.

As in the sequential approach of function specification method in the whole-domain estimation of heat flux [2] the sensitivity coefficients are used.

So, we differentiate the governing equations with respect to the unknown parameters β_j, $j = 1, 2, 3$. For each sub-domain of skin one has (Eq. (1)):

$$x \in \Omega_e : \quad c_e \partial_t U_{ej}(x,t) = \lambda_e \partial_{xx} U_{ej}(x,t) - k_e U_{ej}(x,t) \tag{20}$$

where:

$$U_{ej}(x,t) = \frac{\partial T_e(x,t)}{\partial \beta_j} \tag{21}$$

are the sensitivity functions.

A differentiation of the boundary and initial conditions with respect to β_j gives:

– continuity conditions:

$$x \in \Gamma_{e,e+1} : \quad \begin{cases} -\lambda_e \partial_x U_{ej}(x,t) = \lambda_{e+1} \partial_x U_{e+1,j}(x,t) \\ U_{ej} = U_{e+1,j} \end{cases}, \quad e = 1,2 \tag{22}$$

– Neumann condition:

$$x \in \Gamma_0 : \quad -\lambda_1 \partial_x U_{1j} = \begin{cases} 1, & j = 1 \\ t, & j = 2 \\ t^2, & j = 3 \end{cases} \tag{23}$$

– adiabatic condition for $x \in \Gamma_3$:

$$-\lambda_3 \partial_x U_{3j}(x,t) = 0 \tag{24}$$

– initial condition:

$$t = 0 : U_{ej}(x,t) = 0 \tag{25}$$

Summing up, three additional boundary initial problems connected with the sensitivity of temperature field with respect to the unknown parameters β_1, β_2 and β_3 must be solved. It should be pointed out that these additional problems can be solved directly - they are correctly posed because both the differential equations determining the distribution of U_{ej} in the domain considered and also the boundary initial conditions are known.

We assume that the values of temperature T_{di}^f at the internal nodes x_i for time t^f, $f=1,2,\ldots,F$ are known, Eq. (4). In order to solve the inverse problem, the sum of squares criterion is applied [1, 2, 6]:

$$S(\beta_1, \beta_2, \beta_3) = \sum_{f=1}^{F}\sum_{i=1}^{M}\left(T_i^f - T_{di}^f\right)^2 \tag{26}$$

where T^f is the calculated temperature at the points x_i.

Differentiating the criterion Eq. (26) with respect to the unknown parameters β_j, $j=1,2,3$ and using the necessary condition of minimum, one obtains:

$$\frac{\partial S}{\partial \beta_j} = 2\sum_{f=1}^{F}\sum_{i=1}^{M}\left(T_i^f - T_{di}^f\right)^2 \frac{\partial T_i^f}{\partial \beta_j} = 0, \quad j = 1,2,3 \tag{27}$$

or:

$$\sum_{f=1}^{F}\sum_{i=1}^{M}\left(T_i^f - T_{di}^f\right) U_{ij}^f = 0, \quad j = 1,2,3 \tag{28}$$

At first, we solve the basic boundary initial problem for the arbitrary assumed values of parameters β_1, β_2 and β_3. The solution obtained we denote by $\hat{T}_i^f$. The temperature T_i^f is expanded into Taylor's series in the vicinity of point $\hat{T}_i^f$ taking into account the first and second components, this means [1, 2, 6]:

$$T_i^f = \hat{T}_i^f + \sum_{k=1}^{3} \frac{\partial T_i^f}{\partial \beta_k}\left(\beta_k - \hat{\beta}_k\right) \tag{29}$$

or:

$$T_i^f = \hat{T}_i^f + \sum_{k=1}^{3} U_{ik}^f \left(\beta_k - \hat{\beta}_k\right) \tag{30}$$

Putting Eq. (30) into Eq. (28) one has:

$$\sum_{f=1}^{F}\sum_{i=1}^{M}\left[\hat{T}_i^f + \sum_{k=1}^{3} U_{ik}^f\left(\beta_k - \hat{\beta}_k\right) - T_{di}^f\right] U_{ij}^f = 0 \tag{31}$$

$$j = 1,2,3$$

or:

$$\sum_{f=1}^{F}\sum_{i=1}^{M} U_{ij}^f \sum_{k=1}^{3} U_{ik}^f \beta_k = \sum_{f=1}^{F}\sum_{i=1}^{M} U_{ij}^f \sum_{k=1}^{3} U_{ik}^f \hat{\beta}_k + \sum_{f=1}^{F}\sum_{i=1}^{M}\left(T_{di}^f - \hat{T}_i^f\right) U_{ij}^f, \quad j = 1,2,3 \tag{32}$$

The Eq. (32) can be written in the matrix form, namely:

$$\boldsymbol{A\beta} = \boldsymbol{A\hat{\beta}} + \boldsymbol{B} \tag{33}$$

where:

$$A=\begin{bmatrix}\sum_{f=1}^{F}\sum_{i=1}^{M}\left(U_{i1}^{f}\right)^2 & \sum_{f=1}^{F}\sum_{i=1}^{M}U_{i1}^{f}U_{i2}^{f} & \sum_{f=1}^{F}\sum_{i=1}^{M}U_{i1}^{f}U_{i3}^{f}\\ \sum_{f=1}^{F}\sum_{i=1}^{M}U_{i2}^{f}U_{i1}^{f} & \sum_{f=1}^{F}\sum_{i=1}^{M}\left(U_{i2}^{f}\right)^2 & \sum_{f=1}^{F}\sum_{i=1}^{M}U_{i2}^{f}U_{i3}^{f}\\ \sum_{f=1}^{F}\sum_{i=1}^{M}U_{i3}^{f}U_{i1}^{f} & \sum_{f=1}^{F}\sum_{i=1}^{M}U_{i3}^{f}U_{i2}^{f} & \sum_{f=1}^{F}\sum_{i=1}^{M}\left(U_{i3}^{f}\right)^2\end{bmatrix} \quad (34)$$

and:

$$B=\begin{bmatrix}\sum_{f=1}^{F}\sum_{i=1}^{M}\left(T_{di}^{f}-\hat{T}_i^{f}\right)U_{i1}^{f}\\ \sum_{f=1}^{F}\sum_{i=1}^{M}\left(T_{di}^{f}-\hat{T}_i^{f}\right)U_{i2}^{f}\\ \sum_{f=1}^{F}\sum_{i=1}^{M}\left(T_{di}^{f}-\hat{T}_i^{f}\right)U_{i3}^{f}\end{bmatrix} \quad (35)$$

while:

$$\boldsymbol{\beta}=\begin{bmatrix}\beta_1\\ \beta_2\\ \beta_3\end{bmatrix}\quad,\quad \hat{\boldsymbol{\beta}}=\begin{bmatrix}\hat{\beta}_1\\ \hat{\beta}_2\\ \hat{\beta}_3\end{bmatrix} \quad (36)$$

The system of Eq. (33) allows to find the values of β_1, β_2 and β_3.

4. BOUNDARY ELEMENT METHOD

In order to solve the basic boundary initial problem and the additional problems connected with the sensitivity functions, the boundary element method has been used [7, 8].

The following equations for successive layers of skin are considered:

$$c_e\partial_t F_e(x,t)=\lambda_e\partial_{xx}F_e(x,t)+S_e(x,t)\,,\quad e=1,2,3 \quad (37)$$

where $F_e(x,t)$ denotes the temperature $T_e(x,t)$ functions $Z_e(x,t)$ or $U_{ej}(x,t)$ resulting from the sensitivity analysis, $S_e=S_e(x,t)$ are the source functions.

At first, the time grid is introduced:

$$0=t^0<t^1<\ldots<t^{f-1}<t^f<\ldots<\infty,\ \Delta t=t^f-t^{f-1} \quad (38)$$

If the 1st scheme of the BEM is taken into account then the boundary integral equations (for successive layers of skin - $e=1,2,3$) corresponding to transition $t^{f-1}\to t^f$ are of the form [6, 7]:

$$\begin{aligned}F_e\left(\xi,t^f\right)+\left[\frac{1}{c_e}\int_{t^{f-1}}^{t^f}F_e^*\left(\xi,x,t^f,t\right)J_e(x,t)dt\right]_{x=L_{e-1}}^{x=L_e}=\left[\frac{1}{c_e}\int_{t^{f-1}}^{t^f}J_e^*\left(\xi,x,t^f,t\right)F_e(x,t)dt\right]_{x=L_{e-1}}^{x=L_e}+\\+\int_{L_{e-1}}^{L_e}F_e^*\left(\xi,x,t^f,t^{f-1}\right)F_e\left(x,t^{f-1}\right)dx+\frac{1}{c_e}\int_{L_{e-1}}^{L_e}S_e\left(x,t^{f-1}\right)\int_{t^{f-1}}^{t^f}F_e^*\left(\xi,x,t^f,t\right)dtdx\end{aligned} \quad (39)$$

where F_e^* are the fundamental solutions given by formulas:

$$F_e^*\left(\xi,x,t^f,t\right)=\frac{1}{2\sqrt{\pi a_e\left(t^f-t\right)}}\exp\left[-\frac{(x-\xi)^2}{4a_e\left(t^f-t\right)}\right] \quad (40)$$

where ξ is the point in which the concentrated heat source is applied and $a_e=\lambda_e/c_e$.

The heat fluxes resulting from the fundamental solutions are equal to:

$$J_e^*\left(\xi,x,t^f,t\right)=-\lambda_e\frac{\partial F_e^*\left(\xi,x,t^f,t\right)}{\partial x}=\frac{\lambda_e(x-\xi)}{4\sqrt{\pi}\left[a_e\left(t^f-t\right)\right]^{3/2}}\exp\left[-\frac{(x-\xi)^2}{4a_e\left(t^f-t\right)}\right] \quad (41)$$

Assuming that:

$$t\in\left[t^{f-1},t^f\right]:\quad \begin{cases}F_e(x,t)=F_e\left(x,t^f\right)\\ J_e(x,t)=J_e\left(x,t^f\right)\end{cases} \quad (42)$$

one has the following form of Eq. (39):

$$\begin{aligned}&F_e\left(\xi,t^f\right)+g_e(\xi,L_e)J_e\left(L_e,t^f\right)-g_e(\xi,L_{e-1})J_e\left(L_{e-1},t^f\right)=\\&h_e(\xi,L_e)F_e\left(L_e,t^f\right)-h_e(\xi,L_{e-1})F_e\left(L_{e-1},t^f\right)+p_e(\xi)+z_e(\xi)\end{aligned} \quad (43)$$

where:

$$h_e(\xi,x)=\frac{1}{c_e}\int_{t^{f-1}}^{t^f} J_e^*\left(\xi,x,t^f,t\right)dt=\frac{sgn(x-\xi)}{2}erfc\left(\frac{|x-\xi|}{2\sqrt{a_e\Delta t}}\right) \tag{44}$$

and:

$$g_e(\xi,x)=\frac{1}{c_e}\int_{t^{f-1}}^{t^f} F_e^*\left(\xi,x,t^f,t\right)dt=\frac{\sqrt{\Delta t}}{\sqrt{\pi\lambda_e c_e}}exp\left[-\frac{(x-\xi)^2}{4a_e\Delta t}\right]-\frac{|x-\xi|}{2\lambda_e}erfc\left(\frac{|x-\xi|}{2\sqrt{a_e\Delta t}}\right) \tag{45}$$

while:

$$p_e(\xi)=\int_{L_{e-1}}^{L_e} F_e^*\left(\xi,x,t^f,t^{f-1}\right)F_e\left(x,t^{f-1}\right)dx=\frac{1}{2\sqrt{\pi a_e\Delta t}}\int_{L_{e-1}}^{L_e} exp\left[-\frac{(x-\xi)^2}{4a_e\Delta t}\right]F_e\left(x,t^{f-1}\right)dx \tag{46}$$

at the same time:

$$z_e(\xi)=\int_{L_{e-1}}^{L_e} S_e\left(x,t^{f-1}\right)g_e(\xi,x)dx \tag{47}$$

For $\xi\to L_{e-1}^+$ and $\xi\to L_e^-$ for each domain considered one obtains the system of equations (Eq. (43)):

$$\begin{cases} F_e\left(L_{e-1},t^f\right)+g_e\left(L_{e-1},L_e\right)J_e\left(L_e,t^f\right)-g\left(L_{e-1},L_{e-1}\right)J_e\left(L_{e-1},t^f\right)= \\ h_e\left(L_{e-1}^+,L_e\right)F_e\left(L_e,t^f\right)-h\left(L_{e-1}^+,L_{e-1}\right)F_e\left(L_{e-1},t^f\right)+p\left(L_{e-1}\right)+z\left(L_{e-1}\right) \\ F_e\left(L_e,t^f\right)+g\left(L_e,L_e\right)J_e\left(L_e,t^f\right)-g\left(L_e,L_{e-1}\right)J_e\left(L_{e-1},t^f\right)= \\ h_e\left(L_{e-1}^-,L_e\right)F_e\left(L_e,t^f\right)-h_e\left(L_e^-,L_{e-1}\right)F_e\left(L_{e-1},t^f\right)+p_e\left(L_e\right)+z_e\left(L_e\right) \end{cases} \tag{48}$$

This system can be written in the matrix form:

$$\begin{bmatrix} g_{11}^e & g_{12}^e \\ g_{21}^e & g_{22}^e \end{bmatrix}\begin{bmatrix} J_e\left(L_{e-1},t^f\right) \\ J_e\left(L_e,t^f\right) \end{bmatrix}=\begin{bmatrix} h_{11}^e & h_{12}^e \\ h_{21}^e & h_{22}^e \end{bmatrix}\begin{bmatrix} F_e\left(L_{e-1},t^f\right) \\ F_e\left(L_e,t^f\right) \end{bmatrix}+\begin{bmatrix} p_1^e+z_1^e \\ p_2^e+z_2^e \end{bmatrix} \tag{49}$$

Taking into account the continuity conditions for $x=L_1$ and $x=L_2$ and adiabatic condition for $x=L_3$ one has:

$$\begin{bmatrix} -h_{11}^1 & -h_{12}^1 & g_{12}^1 & 0 & 0 & 0 \\ -h_{21}^1 & -h_{22}^1 & g_{22}^1 & 0 & 0 & 0 \\ 0 & -h_{11}^2 & g_{11}^2 & -h_{12}^2 & g_{12}^2 & 0 \\ 0 & -h_{21}^2 & g_{21}^2 & -h_{22}^2 & g_{22}^2 & 0 \\ 0 & 0 & 0 & -h_{11}^3 & g_{11}^3 & -h_{12}^3 \\ 0 & 0 & 0 & -h_{21}^3 & g_{21}^3 & -h_{22}^3 \end{bmatrix}\begin{bmatrix} F_s\left(L_0,t^f\right) \\ F_b\left(t^f\right) \\ J_b\left(t^f\right) \\ F_d\left(t^f\right) \\ J_d\left(t^f\right) \\ F_3\left(L_3,t^f\right) \end{bmatrix}=\begin{bmatrix} -g_{11}^1F_1\left(L_0,t^f\right)+p_1^1+z_1^1 \\ -g_{21}^1F_1\left(L_0,t^f\right)+p_2^1+z_2^1 \\ p_1^2+z_1^2 \\ p_2^2+z_2^2 \\ p_1^3+z_1^3 \\ p_2^3+z_2^3 \end{bmatrix} \tag{50}$$

where $F_1(L_1,t^f) = F_2(L_1,t^f) = F_b(t^f)$, $F_2(L_2,t^f) = F_3(L_2,t^f) = F_d(t^f)$, $J_1(L_1,t^f) = J_2(L_1,t^f)=J_b(t^f)$ and $J_2(L_2,t^f) = J_3(L_2,t^f)=J_d(t^f)$. Next, we introduce the boundary condition for $x=L_0$ associated with the primary problem or additional problems resulting from the sensitivity analysis.

The values of functions $F_e(x,t)$ at the internal points $\xi\in(L_{e-1}, L_e)$ for time t^f can be found using the formula:

$$\begin{aligned} F_e\left(\xi,t^f\right)&=g_e\left(\xi,L_{e-1}\right)J_e\left(L_{e-1},t^f\right)-g_e\left(\xi,L_e\right)J_e\left(L_e,t^f\right)+ \\ &h_e\left(\xi,L_e\right)F_e\left(L_e,t^f\right)-h_e\left(\xi,L_{e-1}\right)F_e\left(L_{e-1},t^f\right)+p_e(\xi)+z_e(\xi) \end{aligned} \tag{51}$$

5. EXAMPLES OF COMPUTATIONS

In numerical computations the following skin parameters have been assumed [1]: $\lambda_1=0.235$ $W/(mK)$, $\lambda_2=0.445$ $W/(mK)$, $\lambda_3=0.185$ $W/(mK)$, $c_1=4.3068\cdot10^6$ $J/(m^3K)$, $c_2=3.96\cdot10^6$ $J/(m^3K)$, $c_3=2.674\cdot10^6$ $J/(m^3K)$, $k_1=0$, $k_2=k_3=4995.25$ $W/(m^3K)$, $T_B=37$ °C, $Q_{m1}=0$, $Q_{m2}=Q_{m3}=245$ W/m^3 [3]. The thicknesses of successive skin layers: 0.1, 2 and 10 *mm*. The layers have been discretized using 10, 40 and 120 internal cells, while the time step equals $\Delta t=0.05$ *s*.

At first, it is assumed that the boundary heat flux $q_s(t)$ (Eq. (2)) is known, namely:

$$q_s(t) = 60 + 6400t - 640t^2 \tag{52}$$

and we solve the direct problem of Eqs. (1), (3) and (52) supplemented by the initial condition and no-flux condition on the internal surface $x=L_3$. The initial condition determines the quadratic temperature distribution between *32.5 °C* at the surface and *37 °C* at the base of the subcutaneous region [3]. In Figure 2 the temperature field in the skin domain for times *2*, *4*, *6*, *8* and *10 s* is shown. Figure 3 illustrates the course of heating curves at the points $x_1=0.0001$ *m*, $x_2=0.00015$ *m* and $x_3=0.00025$ *m*.

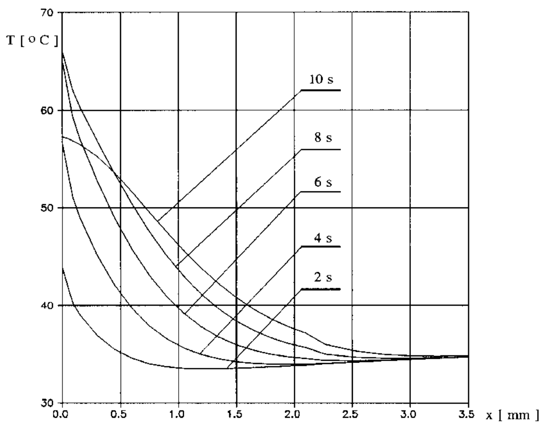

Fig. 2 Temperature distribution in the skin domain

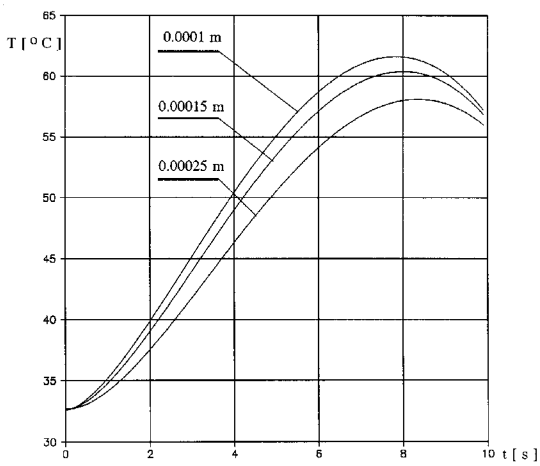

Fig. 3 Heating curves

Next, the inverse problem connected with the boundary heat flux $q_s(t)$ identification using previously presented methods has been solved. The average relative error between the exact $q_{ex}(t)$ and estimated values $q_s(t)$ is defined as:

$$B=\frac{1}{F}\sum_{f=1}^{F}\left|\frac{q_s\left(t^f\right)-q_{ex}\left(t^f\right)}{q_{ex}\left(t^f\right)}\right|100\,\% \tag{53}$$

In the first version of computations it is assumed that the 'measured' temperature at the point x_1 is known and this temperature corresponds to the exact solution of the direct problem - Figure 3. Using the sequential function specification method (SFSM) for $R=3$, the error of identification equals $B=3.99$ *%*, while the whole-domain estimation procedure (WDP) leads to the exact solution of the inverse problem ($B=0$). If the temperature courses are given at three internal nodes (Figure 3) then we obtain the similar results, this means for the first method $B=4.67$ *%*, while for the second one $B=0$.

Simulation of inexact measurement of temperatures can be obtained under the assumption that measurement errors are normally distributed with zero mean and constant standard deviation σ, and then:

$$\hat{T}_{di}^{f}=T_{di}^{f}+\sigma f\left(x_i\right) \tag{54}$$

where $x_i\in(-3,3)$ is a random variable, $f(x)$ is the probability density function of normal distribution. Figure 4 presents the simulated measurement temperatures for $\sigma=1$, while in Figure 5 the results of inverse problem solution are shown.

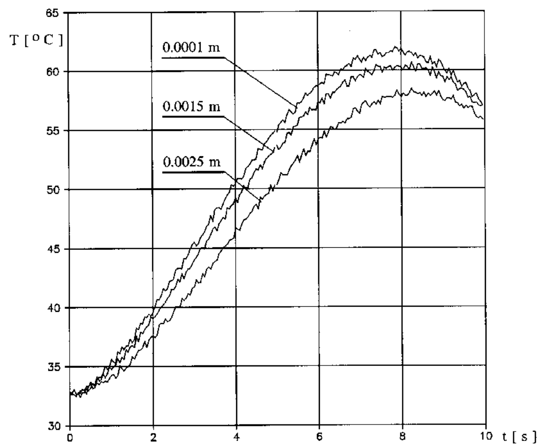

Fig. 4 Disturbed heating curves (σ = 1)

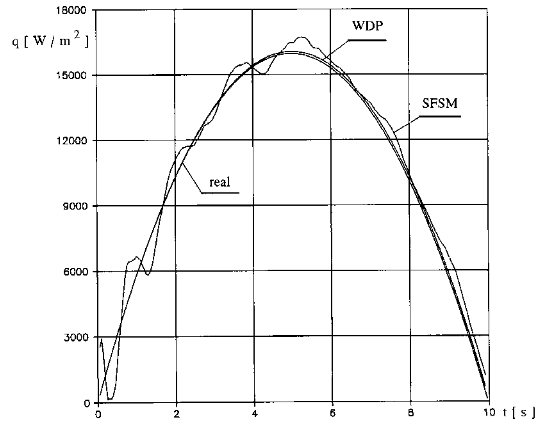

Fig. 5 Solution of inverse problem

6. CONCLUSIONS

The sequential function specification method and the whole-domain procedure allow to identify the boundary heat flux, but in the case of exact measurements of temperatures only the second leads to the exact solution of inverse problem. In the case of the inexact measurements the whole-domain procedure gives the results with the smaller errors than the sequential function specification method. It should be pointed out the whole-domain procedure can be applied only in the case when the searched heat flux is described by the function in which the parameters are unknown.

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IDENTIFICIRANJE RUBNIH UVJETA NA POVRŠINI KOŽE KOJA JE IZLOŽENA VANJSKOM IZVORU TOPLINE

SAŽETAK

Ovaj rad opisuje inverzni problem identificiranja vanjskog izvora topline koji utječe na površinu kože. Poznavajući krivulje zagrijavanja na odabranim točkama promatranog područja, identificira se vrijednost rubnog strujanja topline koja ovisi o vremenu. U svrhu rješavanja problema koristi se specifikacijska metoda pravilne funkcije [1,2] i procjena strujanja topline cijelog područja [2]. Primjenjuje se i metoda rubnih elemenata u fazi numeričkog izračuna. S praktične točke gledišta, korišteni algoritam može se primijeniti u predviđanju opekotina.

Ključne riječi: *identifikacija vanjskog izvora topline, površina kože, uzastupni pristup, metoda rubnih elemenata, predviđanje opekotina.*