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Comparison of two-dimensional and three-dimensional analysis of reinforced and prestressed concrete structures

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SUMMARY

This paper presents the comparison of two-dimensional and three-dimensional analysis of the reinforced and prestressed concrete structures. The curved prestressing tendons and reinforcing bars, which are modelled by one-dimensional finite element, are embedded into adequate two-dimensional and three-dimensional finite elements. The influence of the prestressing tendons on the concrete is modelled by distributed normal and tangential forces along the tendons and two concentrated forces at the anchors. The computation of the post-tensioned prestressed structures is organized in three phases: before, during and after prestressing of the tendons. A few numerical examples are given to compare the results obtained by these two analyses.

***Key words**: Finite element analysis, reinforced structures, prestressed concrete structures, phase prestressing.*

1. INTRODUCTION

The finite element method offers a powerful and general analytical tool for studying the behaviour of reinforced and prestressed concrete structures. During the last two decades different models and modelling techniques, material laws and failure criteria including fracture mechanics have been introduced and exploited but there is no general consensus which one is the most suitable for the numerical modelling of reinforced concrete and prestressed concrete structures.

This paper presents two numerical models for the computation of reinforced and prestressed concrete structures. The first one is a model for the analysis of plane structures, Refs. [1, 2], while the second one is for the three-dimensional analysis, Refs. [3, 4].

Generally, curved prestressing tendons and reinforcing bars are embedded into a two-dimensional 8-node isoparametric element in the case of two-dimensional analysis, while a three-dimensional 20-node element is used for three-dimensional analysis, Ref. [5]. Prestressing tendons and reinforcing bars are modelled by one-dimensional isoparametric three-node elements independently of the concrete element meshes i.e. the performed analysis, Ref. [6]. The influence of the prestressing tendons on the concrete is modelled by distributed normal and tangential forces along the tendons and two forces concentrated at the anchors, Ref. [7]. The developed models make it possible to compute friction losses and losses caused by short-term deformation of concrete.

The computation for the post-tensioned prestressed structures is organized in three phases. The load can be applied incrementally in each phase. In the phase which precedes the prestressing of the tendons the structure is computed taking into account the dead load and one part of the permanent load. Concrete or reinforced concrete structures are analyzed herein. In

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the prestressing phase the tendons are tensioned individually. The prestressing force can be applied at once or incrementally. In the third phase which follows the tensioning of all tendons, the structure is computed taking into account the remaining part of the dead load and all kinds of the live load.

The formulations, necessary for the numerical modelling of these structures, for two-dimensional and three-dimensional analysis will be presented in this paper, Refs. [1-4]. The described models are implemented in the computer programs PRECON, Refs. [1, 2] and PRECON3D, Ref. [3]. In both cases, the advantage of the proposed modelling is complete freedom in prescribing the location and geometry of reinforcing bars and prestressing tendons.

A few numerical examples will be given to compare the results obtained by two-dimensional and three-dimensional analyses. In each case, the obtained results have shown good agreement with the published ones, numerical or experimental, Refs. [1-4, 8].

The full advantage of three-dimensional modelling over two-dimensional modelling is evident when the width of the cross-section over the height is not constant, e.g. when we have **I**, **T**, **Π** or similar cross-sections, and when the prestressing tendon is placed out of the cross-section symmetry plane.

2. DETERMINATION OF TENDON GEOMETRY

The proposed model for the numerical treatment of reinforced and prestressed concrete structure consists of 2D or 3D concrete elements with embedded reinforcing bars and/or prestressing tendons. The two-dimensional 8-node elements and three-dimensional 20-node elements are used for concrete modelling and one-dimensional 3-node elements are used for reinforcing bars and prestressing tendons.

Prestressed tendons can occupy a general position within the concrete element; they can be either straight or parabolic. They can also consist of straight and parabolic parts (Figure 1 and Figure 2). All tendons can be simulated in this way, whether straight or parabolic, if the incontinuity of the first derivation is obtained during the transfer from the region of one curvature to another.

For two-dimensional analysis, geometry of the tendon is described by square parabola. The tendon position is determined by two boundary nodes coordinates defined in global coordinate system.

For 3D analysis, geometry of the tendon is described by the space function of the second order. In this way any position of the tendon can be described, curved into one or more planes. This model offers possibilities for cable description but it requires more input data necessary for defining its position. In this model the tendon position is defined by coordinates of two nodes and the location of the tangent at boundary nodes.

3. THE PRESTRESS FORCE TRANSFER

After defining the tendon position, it is necessary to determine the influence of the prestress force upon the concrete element. The tendon force at any cross-section depends upon the applied force and prestress losses. Generally, it is necessary to determine the prestress influence in the internal points of the tendon if the external forces at the tendon ends are known. The influence of the prestressing tendons on the concrete is modelled by distributed normal and tangential forces along the tendons and two forces concentrated at the anchors. The developed models make it possible to compute friction losses and losses caused by the short-term deformation of concrete.

Figure 3 shows the force acting on an infinitesimal element *ds* of a curved tendon. From the equilibrium equation, the normal and the tangential components of continuously distributed load can be expressed, for both models, as:

$$p_n(s) = k(s)F(s) \qquad p_t(s) = \pm \mu\, p_n(s) \tag{1}$$

Normal load $p_n(s)$ at any cross-section of the tendon depends upon of the curvature of the tendon and the intensity of the prestress force at that section, while tangential load $p_t(s)$ is the frictional force per unit length.

As it can be seen, for both models, it is important to determine tendon curvature $k(s)$. For two-dimensional model the tendon curvature is always a positive value and can be expressed as:

$$k(s) = \frac{\begin{vmatrix} x' & y' \\ x'' & y'' \end{vmatrix}}{\left(x'^2 + y'^2\right)^{3/2}} \tag{2}$$

The values $x' = \dfrac{dx}{d\chi}$ and $y' = \dfrac{dy}{d\chi}$ are given according to:

$$\frac{dx}{d\chi} = \sum_{k=1}^{3} \frac{dN_k}{d\chi} x_k \qquad \frac{dy}{d\chi} = \sum_{k=1}^{3} \frac{dN_k}{d\chi} y_k \tag{3}$$

while $x'' = \dfrac{d^2x}{d\chi^2}$ and $y'' = \dfrac{d^2y}{d\chi^2}$.

The space curvature of the tendon $k(s)$ can be expressed as:

$$k = \sqrt{\left(\frac{d^2x}{d\chi^2}\left(\frac{d\chi}{ds}\right)^2 + \frac{dx}{d\chi}\frac{d^2\chi}{ds^2}\right)^2 + \left(\frac{d^2y}{d\chi^2}\left(\frac{d\chi}{ds}\right)^2 + \frac{dy}{d\chi}\frac{d^2\chi}{ds^2}\right)^2 + \left(\frac{d^2z}{d\chi^2}\left(\frac{d\chi}{ds}\right)^2 + \frac{dz}{d\chi}\frac{d^2\chi}{ds^2}\right)^2} \tag{4}$$

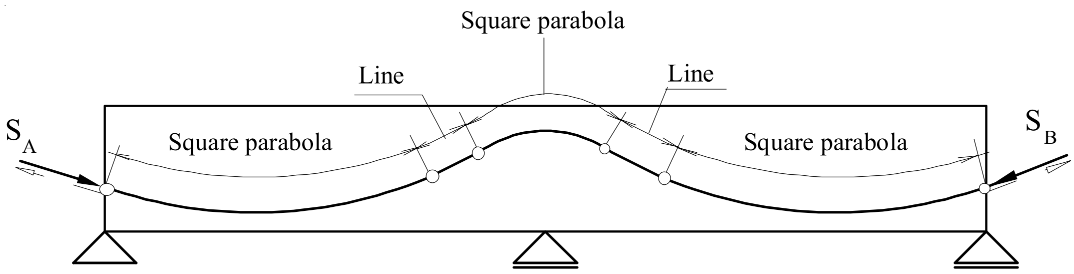


Fig. 1 Possibility of tendon determination for two-dimensional analysis

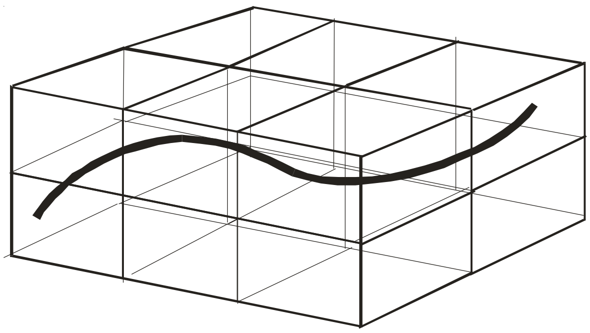
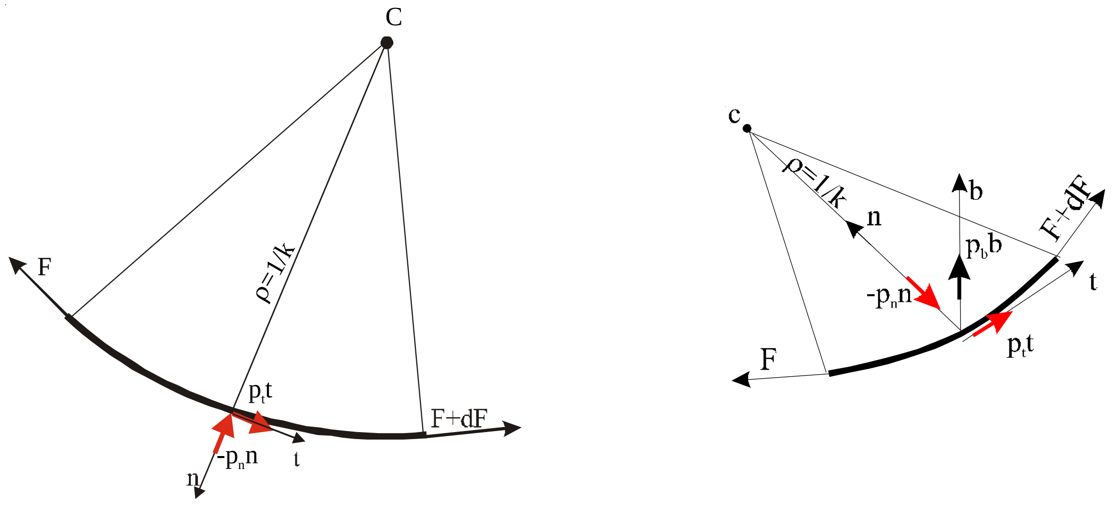


Fig. 2 Possibility of tendon determination for three-dimensional analysis



a) two-dimensional analysis

b) three-dimensional analysis

Fig. 3 The differentially small arc element ds of the tendon and acting forces

As can be seen from Eq. (4), it is necessary to perform double mapping, firstly into the local coordinate system $\xi-\eta-\zeta$, and secondly into the global coordinate system x-y-z of the 3D concrete element. We have to define components d^2x/ds^2, d^2y/ds^2 and d^2z/ds^2 for this operation:

$$\frac{d^2 i}{ds^2}=\frac{d}{d\chi}\left(\frac{di}{d\chi}\frac{d\chi}{ds}\right)\frac{d\chi}{ds}=\frac{d^2 i}{d\chi^2}\frac{d\chi}{ds}\frac{d\chi}{ds}+\frac{di}{d\chi}\frac{d^2\chi}{ds^2},$$
$$i=x,y,z \tag{5}$$

$$\frac{di}{d\chi}=\sum_{k=1}^{3}\frac{dN_k}{d\chi}i_k\,, \qquad i=x,y,z \tag{6}$$

$$\frac{d^2i}{d\chi^2}=\sum_{k=1}^{3}\frac{d^2N_k}{d\chi^2}i_k\,, \qquad i=x,y,z \tag{7}$$

4. DETERMINATION THE EQUIVALENT NODAL FORCES

In the general case, the tendon can occupy a general position within 2D or 3D concrete elements. The point in which the tendon is anchored is located on the boundary plane of the concrete element not necessarily at its nodes.

The influence of the prestressed tendon upon the concrete is exerted by two compressive forces at the ends of the tendon and the distributed normal and tangential stress along the tendon.

In accordance with the finite element method approach in Ref. [9] the acting forces, continuously distributed normal and tangential load along the tendon and the two forces concentrated on the anchors, have to be transferred to the nodes of the 2D or 3D concrete element. So, we have to determine equivalent nodal forces.

4.1 Equivalent nodal forces due to anchorage forces

The concentrated forces act in the points where the prestressing forces are applied. The tensile stresses occur in the tendon during the prestressing phase. After the anchorage the tendon tries to return to its original position what causes compression on the concrete element. This influence is modelled by a concentrated compressive force which acts in the point of anchorage. The action force point coordinates are defined in the global and the line coordinate system with the geometry of the tendon. It is necessary to map this point into the local coordinate system of the parent concrete element to determine equivalent nodal forces of the 2D or 3D concrete element

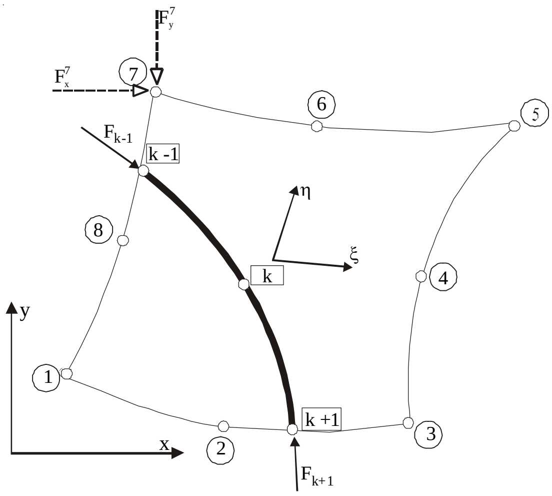
For 2D analysis the equivalent force at node i ($i=1,...,8$), due to the concentrated force on the anchor, can be expressed as:

$$\begin{bmatrix}F_{xi}\\F_{yi}\end{bmatrix}=N_i\left(\xi,\eta\right)\begin{bmatrix}F_x\\F_y\end{bmatrix} \tag{8}$$

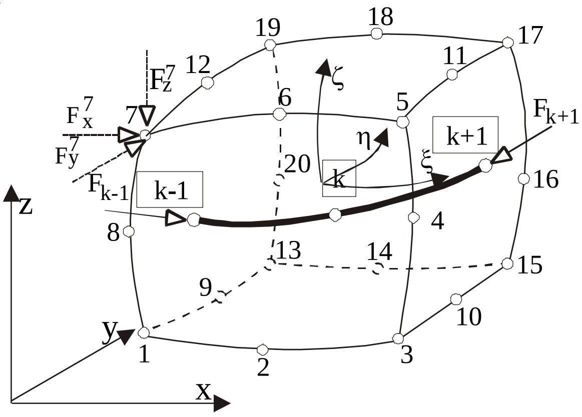
where F_x and F_y are components of the force in the direction of the axes x and y in a global coordinate system, and $N_i(\xi,\eta)$ is the value of the shape function of a two-dimensional 8-nodes element at the point where the force is acting.

For three-dimensional analysis the number of equations is much greater. It is necessary to compute three components F_x, F_y and F_z in the direction of the axis x, y and z in a global coordinate system and we are using shape functions $N_i(\xi,\eta,\zeta)$ for a three-dimensional 20-node element. It can be expressed as:

$$\begin{bmatrix}F_{xi}\\F_{yi}\\F_{zi}\end{bmatrix}=N_i\left(\xi,\eta,\zeta\right)\begin{bmatrix}F_x\\F_y\\F_z\end{bmatrix} \tag{9}$$



a) for two-dimensional analysis

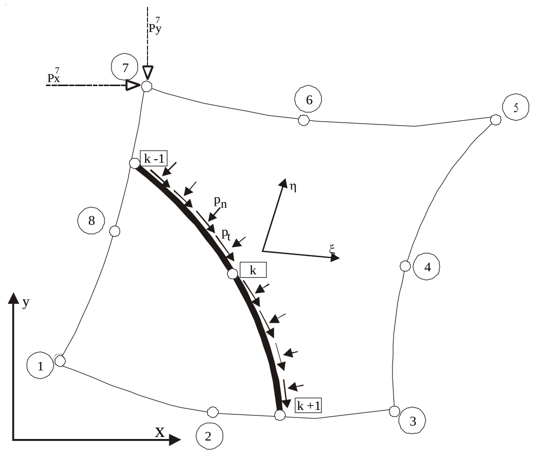


b) for three-dimensional analysis

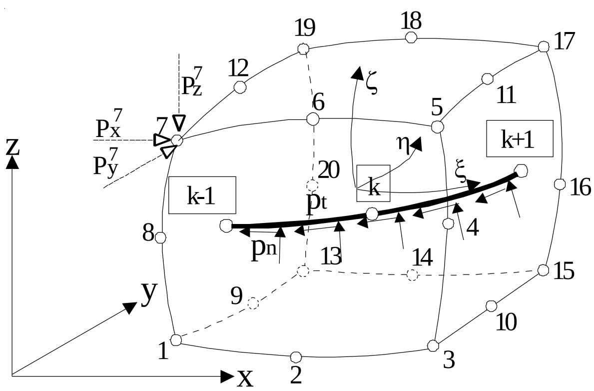
Fig 4. The equivalent nodal forces due to the anchorage forces for two-dimensional and three-dimensional analysis

4.2 Equivalent nodal forces due to distributed load along tendon

Due to the prestressing, beside the concentrated compressive forces at the anchorages, there are forces along the tendon which are modelled as distributed load with its normal and tangential components. These values are defined in the line coordinate system χ. As the problem is solved with the FEM approach these forces have to be transferred into the nodes of the 2D or 3D concrete element (see Figure 5), i.e. we have to determine adequate equivalent nodal forces. It is necessary to map this load from the local line coordinate system into the global coordinate system *x-y* or *x-y-z*, and afterwards to define components of these forces in the direction of *x*, *y* or *x*, *y* and *z* axes.



a) for two-dimensional analysis



b) for three-dimensional analysis

Fig. 5 Equivalent nodal forces due to the distributed load along the tendon for two-dimensional and three-dimensional analysis

The force along the tendon changes during the prestressing, so, we have to determine the increments of the normal and the tangential components in the direction of the global coordinate axes. For two-dimensional analysis it is *dx* and *dy* and for three-dimensional analysis it is *dx*, *dy* and *dz*. The total forces along the tendon in the direction of the global coordinate axes are obtained by the numerical integration. For two-dimensional analysis, these components can be expressed as:

$$P_x = \int_C \left(p_n \frac{\partial y}{\partial \chi} - p_t \frac{\partial x}{\partial \chi} \right) d\chi$$
$$P_y = \int_C \left(-p_n \frac{\partial x}{\partial \chi} - p_t \frac{\partial y}{\partial \chi} \right) d\chi \qquad (10)$$

For three-dimensional analysis the same components are:

$$P_x = \int_K \left(-p_n \frac{1}{\sqrt{\frac{d^2x}{ds^2} + \frac{d^2y}{ds^2} + \frac{d^2z}{ds^2}}} \rho \frac{d^2x}{ds^2} - p_t \frac{dx}{d\chi} \right) d\chi$$

$$P_y = \int_K \left(-p_n \frac{1}{\sqrt{\frac{d^2x}{ds^2} + \frac{d^2y}{ds^2} + \frac{d^2z}{ds^2}}} \rho \frac{d^2y}{ds^2} - p_t \frac{dy}{d\chi} \right) d\chi \qquad (11)$$

$$P_z = \int_K \left(-p_n \frac{1}{\sqrt{\frac{d^2x}{ds^2} + \frac{d^2y}{ds^2} + \frac{d^2z}{ds^2}}} \rho \frac{d^2z}{ds^2} - p_t \frac{dz}{d\chi} \right) d\chi$$

For both models by performing Gauss numerical integration of the Eqs. (10) and (11) one can obtain the values of the distributed load components along the tendon in the Gauss points of the 1D tendon element ($P_x^{g.p.}$, $P_y^{g.p.}$ and $P_z^{g.p.}$).

To determine the influence of this distributed load along 1D tendon element on the concrete element it is necessary to map the coordinates of the Gauss points from the global coordinate system to the local coordinate system of the parent concrete element. Finally, the components of the equivalent nodal forces due to the distributed load along the tendon defined in the global coordinate system can be expressed as:

* for two-dimensional analysis:

$$P_x^i = \sum_{k=1}^{3} N_i(\xi_{g.p.}, \eta_{g.p.}) P_x^{g.p.}$$
$$P_y^i = \sum_{k=1}^{3} N_i(\xi_{g.p.}, \eta_{g.p.}) P_y^{g.p.} \qquad (12)$$

* for three-dimensional analysis:

$$P_x^i = \sum_{k=1}^{3} N_i(\xi_{g.p.}, \eta_{g.p.}, \zeta_{g.p.}) P_x^{g.p.}$$

$$P_y^i = \sum_{k=1}^{3} N_i(\xi_{g.p.}, \eta_{g.p.}, \zeta_{g.p.}) P_y^{g.p.} \qquad (13)$$

$$P_z^i = \sum_{k=1}^{3} N_i(\xi_{g.p.}, \eta_{g.p.}, \zeta_{g.p.}) P_z^{g.p.}$$

where $N_i(\xi_{g.p.}, \eta_{g.p.})$ is the shape function for two-dimensional 8-node element and $N_i(\xi_{g.p.}, \eta_{g.p.}, \zeta_{g.p.})$ is the shape function for three-dimensional 20-node element, Ref. [9].

At the end all influences will be summed up.

4.3 Possibilities of tendon prestressing

Tendons can be prestressed at one end or at both ends. If the tendon is prestressed at one end applying force F_A, then force F_B at the other end of the tendon can be computed according to expression (14) by integration along the entire tendon length:

$$F(s) = F_A e^{\int_A^s k(s)\,ds} \tag{14}$$

The force at any cross-section of the tendon and the distributed load can be obtained directly according to Eqs. (1) and (14). If the tendon is prestressed from one end, and if the forces F_A and F_B at both tendon ends are known, then the friction coefficient can be computed according to expression:

$$\mu = \frac{1}{g(s)} ln \frac{F_A}{F_B}, \quad g(s) = \sum_{i=1}^{3} \frac{ds(\chi_i)}{r(\chi_i)} \tag{15}$$

If the tendon is prestressed on both ends, the force decreases, due to friction between the tendon and concrete, if the distance from the end is increased. In symmetrical prestressing, the problem can be simply solved since the decrease in force is the greatest at the middle of the beam. For a beam with length l the force in the tendon at distance l/2 can be calculated according to Eq. (14), whereas the value $g(s)$ is obtained by integration from the beginning to the middle of the beam.

If the tendon is asymmetric or if the prestressing forces at the tendon ends are not equal ($F_A \neq F_B$), the procedure is more complex. The minimum force will occur at the cross-section which has not been previously known. Let us denote by x the distance of that cross-section from end A, and by l-x the distance from end B. The force at that cross-section F_{min} can be computed according to forces F_A or F_B by using one of the following expressions:

$$F^L_{min} = F_A\, e^{-\mu \int_A^x k(s)ds} \quad ; \quad F^R_{min} = F_B\, e^{-\mu \int_B^{l-x} k(s)ds} \tag{16}$$

The forces should be equal regardless of the ends at which they were computed. By equating terms in Eq. (16) we shall obtain an equation where the integration limit is an unknown value. This equation is solved numerically. In this model, asymmetrical prestressing is performed by taking a cross-section with the greatest decrease in force. The prestressing force in a cross-section assumed according to Eq. (16) is calculated before computing the entire structure. According to the ratio between forces F^L_{min} and F^R_{min}, the assumed cross-section is moved either to the left or to the right. This procedure is repeated for each tendon separately in the phase of input data preparation. Subsequently, the structure is computed and the possible difference between the two forces F_{min} to the left or to the right from the selected cross-section can be neglected.

5. NUMERICAL EXAMPLES

Example I

The described modelling of the reinforcing bars and prestressing tendons in 2D and 3D are implemented in the computer programmes PRECON and PRECON3D. The performance of the proposed models is illustrated by the solution procedure of one example: prestressed non-prismatic girder clamped at one end and extended over the fixed support at the other end, see Figure 6.

The geometrical and material data are taken from Ref. [13]. The modulus of elasticity of the concrete is E_c=*28000 N/mm*^{2}, Poisson ratio is *0.25*, the modulus of elasticity of the tendon is E_s=*22400 N/mm*^{2} and the tendon cross-section area is A_s=*2000 mm*^{2}.

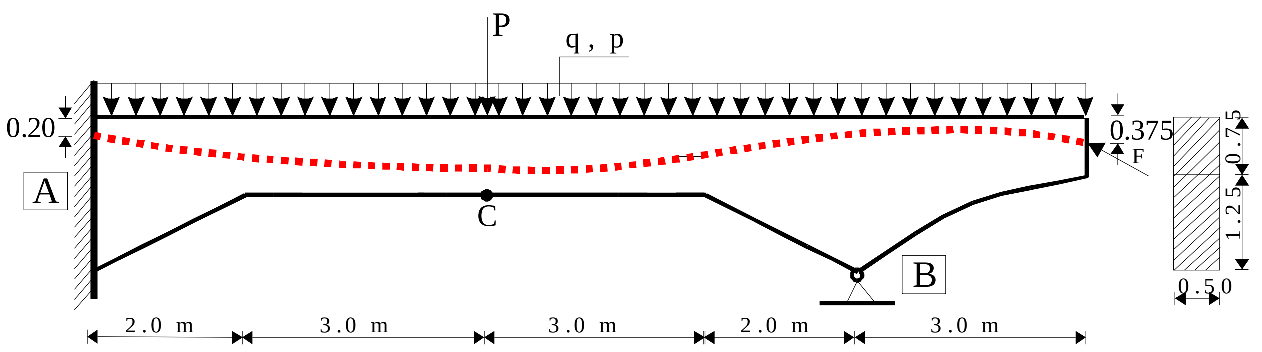


Fig. 6 Geometry of the girder and loadings

The load is considered in three phases:

Phase I

In the first phase the structure was computed taking into account the load it carried before the prestressing of tendons, the girder's own weight $g=25\ kN/m^3$ and a uniformly distributed dead load $q=20\ kN/m$. A concrete or reinforced concrete structure is analyzed herein.

According to the known geometry and load we form the global stiffness matrix $\boldsymbol{K}_I$ for *Phase I* according to expression:

$$\boldsymbol{K}_I=\boldsymbol{K}_C+\boldsymbol{K}_R \tag{17}$$

where $\boldsymbol{K}_C$ is the concrete stiffness matrix and $\boldsymbol{K}_R$ is the reinforcement stiffness matrix, which is obtained by the numerical integration along the reinforcing bar according to expression:

$$\boldsymbol{K}_R=\int_\chi \boldsymbol{B}_R^T E_R \boldsymbol{B}_R \frac{ds}{d\chi} A_R d\chi \tag{18}$$

In Eq. (18) $\boldsymbol{B}_R$ is a strain matrix of the reinforcement element, E_R is the tangential modulus of elasticity of the reinforcement, A_R is the cross-section area of the bar, ds is a differential element of the length and ζ is the independent normalized coordinate.

Global load vector (vector of residual forces) $\boldsymbol{F}_I$ is determined according to expression:

$$\boldsymbol{F}_I=\boldsymbol{F}_C+\boldsymbol{F}_R \tag{19}$$

where $\boldsymbol{F}_C$ is a vector of external forces and residual forces on concrete element, while $\boldsymbol{F}_R$ is vector of residual forces due to reinforcement strain:

$$\boldsymbol{F}_R=\int_\chi \boldsymbol{B}_R^T \sigma_R A_R \frac{ds}{d\chi} d\chi \tag{20}$$

In Eq. (20) σ_R is the normal stress in reinforcement.

Phase II

Generally, in the second phase the tendons are tensioned individually. The prestress force can be applied at once or incrementally, and, thus, gradual prestressing procedures can be simulated. The previously applied force can be subsequently decreased, which is sometimes done in practice, in order to reduce high initial stress in one part of the tendon. During the prestressing phase the respective tendon is not treated as a structural element. Its geometry is used actually to compute the initial influence of prestressing which is modelled as a fictitious distributed load. In subsequent iteration, the tendon functions as a classical reinforcement with a given initial stress. During successive prestressing of tendons, the tendon which is currently being prestressed does not influence the stiffness of the structure, while the previously prestressed tendons take over the stresses as a classical reinforcement.

The global stiffness matrix in this phase can be presented in the following form:

$$\boldsymbol{K}_{II}^i=\boldsymbol{K}_I+\sum_{j=1}^{i-1}\boldsymbol{K}_P^j \tag{21}$$

where:

i - tendon index, i.e. index of a group of tendons which are being prestressed,

$\boldsymbol{K}_{II}^i$ - global stiffness matrix at the moment of prestressing the *i-th* group of tendons,

$\boldsymbol{K}_I$ - global stiffness matrix of *Phase I*,

$\boldsymbol{K}_P^j$ - stiffness matrix of one tendon or group of tendons which started functioning as a classical reinforcement.

The loading vector can be presented in the following form:

$$\boldsymbol{F}_{II}^i=\boldsymbol{F}_I+\sum_{j=1}^{i}\Delta\boldsymbol{F}_{II}^j \tag{22}$$

where:

$\boldsymbol{F}_I^i$ - global vector of loading at the moment of prestressing the i-th group of tendons,

$\boldsymbol{F}_I$ - load vector after *Phase I*,

$\Delta\boldsymbol{F}_{II}^i$ - vector of equivalent load which represents the influence of the prestressing force of a given tendon upon the concrete structure.

When the prestressing force is introduced into the structure gradually, vector $\Delta\boldsymbol{F}_{II}^i$ is applied incrementally and not at once.

In this example, *Phase II* is the prestressing phase and the loading includes all loads from *Phase I* and a prestressing force $F=2000\ kN$ applied at one end of the tendon while the other end is anchored into concrete body.

Phase III

The prestressing of all tendons is followed by the third phase in which the structure is computed taking into account the remaining part of the dead load and the live load. Concrete, reinforcement and all prestressed tendons which function as a classical reinforcement, contribute to the stiffness of the structure. The load is applied incrementally until failure. The stiffness matrix in this phase $\boldsymbol{K}_{III}$ is:

$$\boldsymbol{K}_{III}=\boldsymbol{K}_C+\boldsymbol{K}_R+\sum_{j=1}^{n}\boldsymbol{K}_P^j \tag{23}$$

where *n* is the number of prestressed tendons, i.e. the number of prestressed tendon groups.

The loading vector can be presented as:

$$\boldsymbol{F}_{III}=\boldsymbol{F}_I+\sum_{j=1}^{n}\Delta\boldsymbol{F}_{II}^j+\Delta\boldsymbol{F}_{III} \tag{24}$$

where $\Delta\boldsymbol{F}_{III}$ is the part of the loading vector which resulted from the load taken over by the structure after completed prestressing. When the load is applied incrementally vector $\Delta \mathbf{F}_{III}$ is applied in increments.

In this numerical example *Phase III* is the phase considering service-load conditions and the loading includes all loads from *Phase II*, a uniformly distributed live load $p=20$ kN/m and a concentrated load $P=200$ kN.

This example was previously analysed in Ref. [10] in the linear domain with 2D discretisation then it was analysed in the linear and non-linear domain with 2D discretisation by numerical programme PRECON and finally in the linear domain but with 3D discretisation with the developed computer programme PRECON3D. Table I shows the support reactions in cross-section A calculated by the mentioned three approaches in the linear domain.

The shown results (Table I) for *Phase I* agree well for all three approaches. However, the results for Phases II and III differ, what was expected, because the analysis with the developed approach is three-dimensional, Ref. [3], while the analyses with two other approaches, Refs. [2,3], are two-dimensional (plane stress conditions).

Table 1. Support reactions at cross-section A (Ri [kN], M [kNm])

	PRECON3D [3]	Ref. [13]	PRECON [2]
Phase I	$R_x = 106$	$R_x = 105$	$R_x = 107$
	$R_z = 179$	$R_z = 180$	$R_z = 180$
	$M = 297$	$M = 298$	$M = 296$
Phase II	$R_x = -430$	$R_x = -479$	$R_x = -469$
	$R_z = 275$	$R_z = 273$	$R_z = 273$
	$M = 677$	$M = 647$	$M = 655$
Phase III	$R_x = -210$	$R_x = -260$	$R_x = -251$
	$R_z = 430$	$R_z = 487$	$R_z = 485$
	$M = 1076$	$M = 1093$	$M = 1093$

Non-linear analysis with 2D discretization of the structure is performed with the programme PRECON and the deflection of the point C is observed up to the failure of the structure. With programme PRECON3D only the linear analysis is performed by increasing load intensity factor. Figure 7 shows load-deflection curves of the point C for both approaches and a very good agreement of the obtained results in the linear part can be seen.

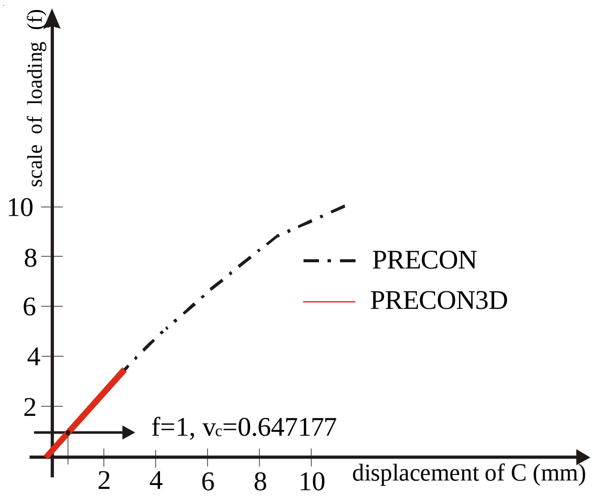


Fig. 7 Load versus deflection at point C

Example 2

Prestressed beams and/or girders used in everyday engineering structures generally have **I**, **T**, **Π** or similar cross-sections. The beams and/or girders with those cross-sections due to apparent three-dimensional stress state cannot be analyzed exactly with the two-dimensional model and code which was one of the reasons for developing 3D model and code, Ref. [3].

In this example, prestressed I-beam taken from Ref. [10] is analyzed. The beam geometry and loading are shown in Figure 8.

The material characteristics of the I-beam according to Ref. [10] are: the modulus of elasticity of the concrete $E_c=35000$ N/mm^2, Poisson's ratio of the concrete $\nu=0.25$, the modulus of elasticity of the prestressed tendon $E_s=210000$ N/mm^2 and the prestressed tendon cross-sectional area $A_s=1962.5$ mm^2.

The I-beam concrete structure is discretised with 550 three-dimensional isoparametric 20-node finite elements and with 55 one-dimensional isoparametric 3-node elements for tendon discretisation.

Figure 9 shows a deformed configuration of the I-beam under prestressing force only while Figure 10 shows a deformed configuration of the I-beam under concentrated force $P=200$ kN acting after prestressing.

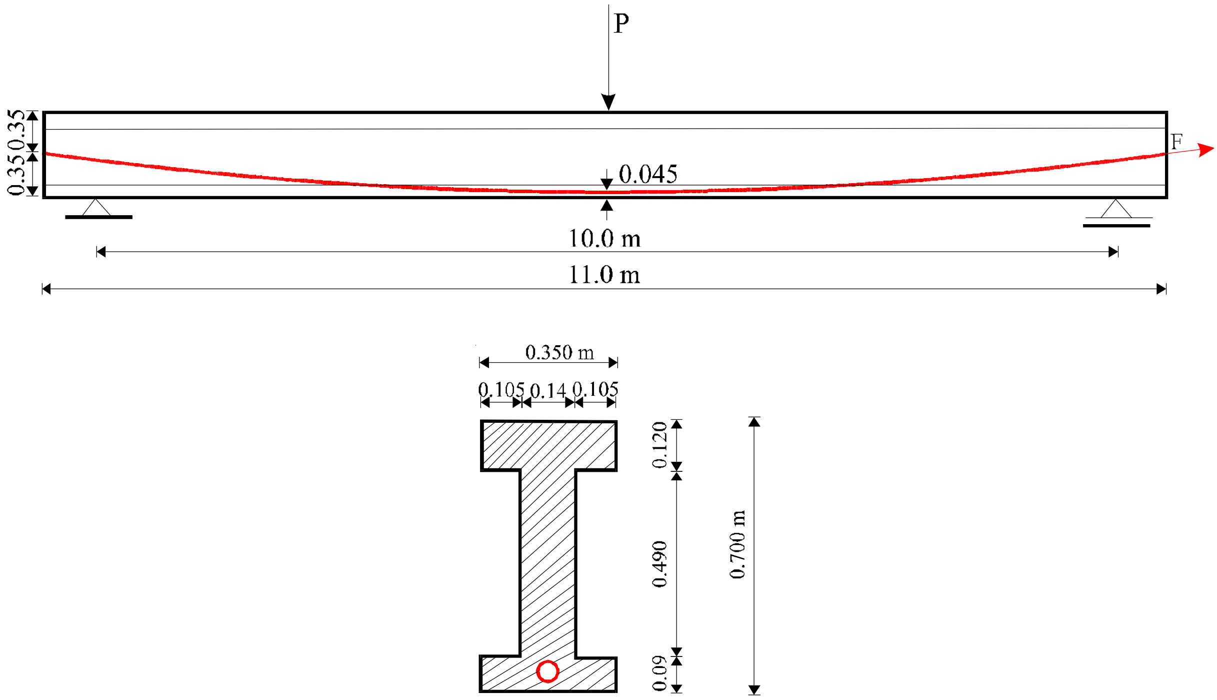


Fig. 8 Geometry of the analysed I-beam

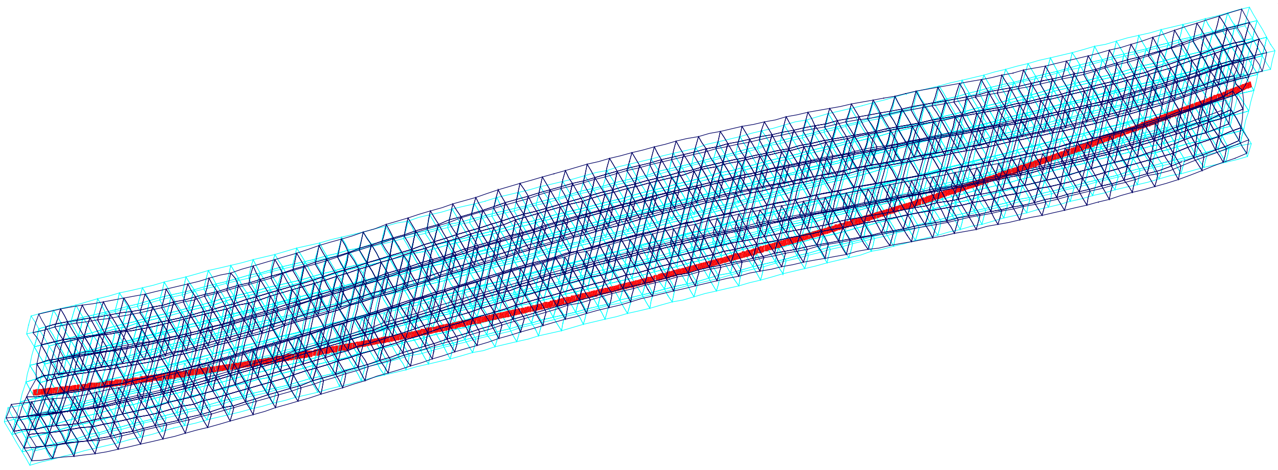


Fig. 9 Deformed configuration of the I-beam under prestressing force

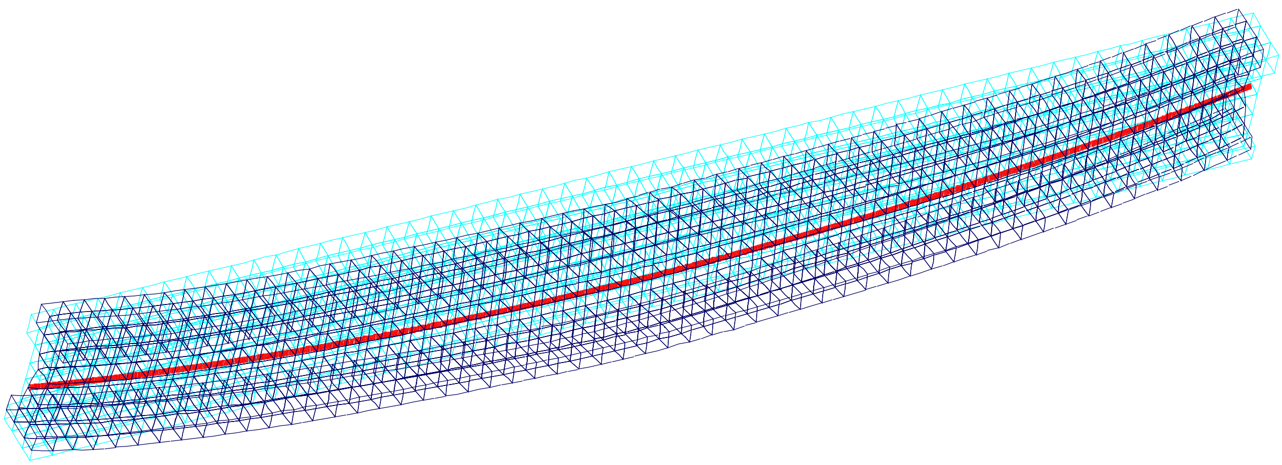


Fig. 10 Deformed configuration of the I-beam under concentrated force acting after prestressing occur

The load-deflection diagrams for three different analyses: (1) numerical analysis according to Ref. [10]; (2) experimental investigations according to Ref. [10]; and (3) numerical analysis according to the presented proposed model and the computer programme PRECON3D, Ref. [3], are shown in Figure 11. These lines present mid-span deflection under the second loading case. A very good agreement of the obtained results for all three analyses in the linear domain is evident.

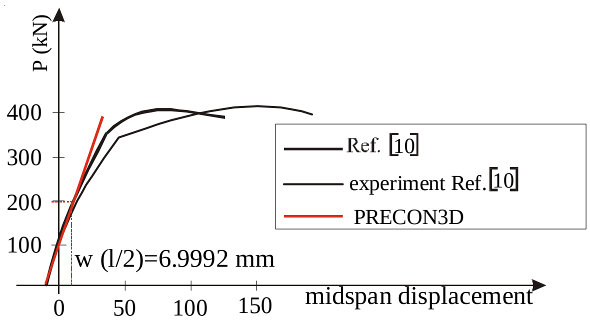


Fig. 11 Load-deflection diagrams for different analyses

6. CONCLUSIONS

This paper presents a numerical treatment of reinforcing bars and prestressing tendons for two-dimensional and three-dimensional numerical modelling of reinforced and prestressed concrete structures. The advantage of the proposed modelling is a complete freedom in prescribing the location and geometry of reinforcing bars and prestressing tendons.

The described modelling of the reinforcing bars and prestressed tendons is implemented in two computer programmes, for two-dimensional analysis PRECON and three-dimensional analysis PRECON3D. The numerical examples, a prestressed non-prismatic girder clamped at one end and extended over the fixed support at the other end and a prestressed I-beam, are given to illustrate the possibilities of the developed models.

A very good agreement of the results obtained by 2D and 3D analyses is evident from the first example. From these results it can be seen that 2D programme is sufficient for describing structures in plane strain or plane stress state. Furthermore, 2D programme requests fewer input data and shorter running time. So, 2D programme is to be recommended when the structure can be appropriately described two-dimensionally, i.e. when the width of the structural element cross sections is constant over the height.

The full advantage of the proposed 3D modelling is evident when the width of the cross-section over the height is not constant, e.g. when we have **I**, **T**, **Π** or similar cross-sections.

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The partial financial support, provided by the Ministry of Science and Technology of the Republic of Croatia under the projects *Numerical Modelling of Engineering and Lightweight Concrete Structures*, Grant No. 083133, *Total Reinforced Lightweight Concrete Structures*, Grant No. 083130, and *Numerical and Experimental Models of Engineering Structures*, Grant No. 0083061, is gratefully acknowledged.

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USPOREDBA DVODIMENZIONALNE I TRODIMENZIONALNE ANALIZE ARMIRANIH I PREDNAPETIH BETONSKIH KONSTRUKCIJA

SAŽETAK

U ovom radu prikazana je usporedba dvodimenzionalne i trodimenzionalne analize armiranih i prednapetih betonskih konstrukcija. Zakrivljeni prednapeti kabel i armatura koji su modelirani kao jednodimenzionalni 3-čvorni linijski element ugrađeni su u odgovarajući dvodimenzionalni 8-čvorni odnosno trodimenzionalni 20 (27) čvorni konačni element. Utjecaj prednapinjanja je modeliran kao jednoliko raspodijeljeno opterećenje duž kabela i koncentrirane sile na sidrima. Analiza prednapete konstrukcije omogućena je u tri faze: prije prednapinjanja, za vrijeme prednapinjanja i nakon prednapinjanja. Prikazana su dva numerička primjera i uspoređeni rezultati dobiveni dvodimenzionalnom i trodimenzionalnom analizom.

Ključne riječi*: MKE, armiranobetonske konstrukcije, prednapete konstrukcije, fazno prednapinjanje.*