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Review of mixed lubrication in concentrated contacts: thinning, mixed and partial films, classical, modern and future modes

Yongbin Zhang
Hangzhou Gear Box Company, Zhejiang Province, P. R. CHINA
e-mail: zyyh0504@yahoo.com

SUMMARY

The mixed lubrication in concentrated contacts, i.e. mixed elastohydrodynamic lubrication (mixed EHL) in both line and point contacts developed in the past time is reviewed. Due to the fluid non-Newtonian shear thinning, contact surface roughness and frictional heating effects, in mixed EHL especially in severe operating conditions, the fluid film can be severely thinned and the fluid film thickness in the Hertzian zone can be reduced to molecular scale and even disappearance. As a result of this, the oxidized chemical boundary layer lubrication or/and dry contact may occur between opposing asperities of the contact surfaces where the fluid film disappears. Actually, the mixed EHL in concentrated contacts may be transited gradually from the mixed EHL of relatively thick overall fluid film to chemical boundary layer lubrication mainly occurring in the Hertzian zone and then further to the destruction of the contact surfaces, i.e. the removal of the oxidized chemical boundary layer from the contact surfaces or dry contact between asperities mainly occurring in the Hertzian zone where the fluid film disappears, when the severity of the operating condition is gradually increased, i.e. the carried load of the contact, the sliding speed of the contact or/and the bulk fluid temperature are gradually increased. Even on the stage of the mixed EHL of relatively thick overall fluid film, in usual operating conditions, small areas of oxidized chemical boundary layer lubrication and instant dry contact can actually locally occur between opposing asperities of the contact surfaces. These boundary lubrication and dry contact respectively have important impacts on the performance and the transition of the mixed EHL of relatively thick overall fluid film and can quickly result in the transition of this mixed EHL to chemical boundary layer lubrication mainly occurring in the Hertzian zone. These show that the lubricating film and dry contact are usually mixed in an actual mixed EHL and even the lubricating films in an actual mixed EHL are also mixed due to different rheological behavior lubricating films respectively, occurring simultaneously in different areas of the mixed EHL contact. These mixed lubricating films occurring simultaneously in an actual mixed EHL include viscoelastic continuum (relatively thick) fluid film, viscoplastic continuum (relatively thick) fluid film, non-continuum fluid film i.e. physical adsorbed layer boundary lubrication film, chemical boundary layer and vanishing lubricating film, i.e. dry contact. Each of these mixed lubricating films may only be locally present and may thus only be partially present in an actual mixed EHL contact. According to the development and the future tendency, the mode of mixed EHL in concentrated contacts is classified as three kinds i.e. classical, modern and future modes of mixed EHL in concentrated contacts respectively. The classical mode of mixed EHL in concentrated contacts refers to the mode of mixed EHL in concentrated contacts where the fluid film between the contact surfaces is overall relatively thick. Thus continuum across the fluid film thickness contacts in the whole mixed EHL. The modern mode of mixed EHL in concentrated contacts refers to the mode of mixed EHL in concentrated contacts where the fluid film between the contact surfaces is molecularly thin. Thus non-continuum across the fluid film thickness on some separate locations of the contact or dry contact of asperity occurs on some separate locations of the contact, while in the other zones of the contact the fluid film is relatively thick and so that continuum across the fluid film thickness and the fluid behavior is Newtonian, (shear-thinning) viscoelastic or viscoplastic depending on the operating condition. The future mode of mixed EHL in concentrated contacts refers to the mode of mixed EHL in concentrated contacts where the fluid film between the contact surfaces may be molecularly thin, i.e. non-continuum across the fluid film thickness on some separate locations of the contact. At the same time chemical boundary layer lubrication and dry contact may occur respectively on some other separate *locations of the contact, while in the other zones of the contact the fluid film is relatively thick and thus continuum across the fluid film thickness and the fluid behavior is shear-thinning viscoelastic or viscoplastic depending on the operating condition. It is found out that the future mode of mixed EHL in concentrated contacts best reflects the mode of mixed EHL in real concentrated contacts among these three modes of mixed EHL in concentrated contacts. It is the direction of the future research of mixed EHL. It is also found out that in the study of the future mode of mixed EHL in concentrated contacts, detailed, fine and time-dependent results for mixed EHL are of purpose and the mixed EHL contact is more technical and real. In this mixed EHL study, the fluid model needs to be non-Newtonian, the contact surface roughness needs to be taken as technical and real, the frictional heating effect both within the fluid film and in dry contact of asperity needs to be incorporated. In the study of the future mode of mixed EHL, the lubrication stage transition occurring in real mixed EHL contacts should be studied under the operating condition. The friction, wear and contact surface destruction respectively, occurring in real mixed EHL contacts in dependence on the operating condition should be studied as well.*

Key words: *Elastohydrodynamic lubrication, mixed lubrication, concentrated contacts, lubrication modes, thinning, mixed films, partial films.*

1. INTRODUCTION

Elastohydrodynamic lubrication (EHL) is the hydrodynamic lubrication occurring in line and point concentrated contacts typically occurring on gears, cams and roller bearings [1]. Conventional theories have been established for this lubrication based on the Newtonian fluid model, the ideally smooth contact surfaces and the isothermal condition [1]. It was basically shown that the fluid viscosity in the EHL inlet zone and the contact surface elastic deformation respectively play important roles in the EHL film formation. The contact surface circumferential speeds, the carried load of the contact, the fluid viscosity-pressure index and the compound curvature radius of the contact surfaces were also shown to have respectively significant influences on the EHL film formation. Typically, in EHL, the contact surfaces are parallel to one another in the Hertzian zone, convergent in the inlet zone and divergent in the outlet zone. The featured central and minimum EHL film thicknesses both occur in the Hertzian zone. It was shown by both theories and experiments that the EHL film thickness in the Hertzian zone is typically on the scale of $0.1\ \mu m$.

Practically, in EHL, the manufactured contact surface roughness is comparable to the film thickness especially in the Hertzian zone. Therefore, the more realistic EHL theory needs to incorporate the contact surface roughness effect on EHL. The elastohydrodynamic lubrication coupled with the contact surface roughness is named as mixed elastohydrodynamic lubrication in theory. Many researches were made on this mode of lubrication in the past time. The analytical approaches to this mode of lubrication as ever taken could be classified as two kinds i.e. stochastic approach and deterministic approach. Christensen [2] may be one of the earliest researchers who introduced the stochastic approach into the study of mixed EHL. Due to the stochastic and random process of the manufacturing of the surface roughness on mechanical components, in EHL, the manufactured contact surface roughness has the stochastic and random properties and features. Since detailed and real contact surface roughness must be measured one by one mechanical component, detailed and accurate results of mixed EHL must be found for each couple of elastohydrodynamic components. It is a very hard work. Always, average and stochastic mixed EHL results for a batch of elastohydrodynamic components which have the same contact surface roughness features, are also of interest in engineering. These results can be obtained by the stochastic approach to mixed EHL. The stochastic approach to mixed EHL only needs a few characterization parameters of the contact surface roughness which are the same for a batch of elastohydrodynamic components. This mixed EHL approach can stochastically give the average fluid film pressure and the average fluid film thickness in mixed EHL for a batch of rough elastohydrodynamic components which have the same values of the contact surface roughness characterization parameters. The interest and value of the stochastic mixed EHL results are limited, mainly used in determination of the scope of mixed EHL. Patir and Cheng [3] applied the stochastic approach in obtaining the average mixed EHL results for point contact mixed EHL in isothermal condition, based on the Newtonian fluid model. Hughes and Bush [4] applied the stochastic approach in finding the combined effect of the fluid shear thinning non-Newtonian property and the contact surface roughness on the average mixed EHL results of line contacts in isothermal condition, based on the non-Newtonian fluid model. They showed that the fluid non-Newtonian property has a strong effect on the average mixed EHL results. However, since the contact surface roughness in mixed EHL practically essentially causes the time-dependent i.e. transient performance of mixed EHL, the stochastic approach to mixed EHL has the shortcoming of being unable to give the instant mixed EHL film pressure and thickness distributions in the contact, the variations of the mixed EHL film pressure and thickness with time and the characteristic values of both the mixed EHL film pressure and thickness during these variations such as the maximum fluid film pressure, the local fluid cavitation (i.e. the minimum fluid film pressure), the maximum fluid film pressure gradient and the minimum fluid film thickness, which are important for the failures of both the mixed EHL film and the contact surfaces. Therefore, the stochastic mixed EHL results are not able enough to describe the performance of mixed EHL.

To give more detailed and precise mixed EHL results and to better describe the performance of mixed EHL, the deterministic approach to mixed EHL was used. In the past time, the contact surface roughness was either artificially defined or measured for one defined couple of elastohydrodynamic components when applying the deterministic approach to mixed EHL. The steady-state or transient mixed EHL results of line or point contacts were usually numerically solved for a given operating condition based on this contact surface roughness by the deterministic approach. These mixed EHL results give the real-time mixed EHL film pressure and thickness during the mixed EHL operation. Therefore, these mixed EHL results detailed and precisely describe the performance of mixed EHL for a given operating condition.

The deterministic approach to mixed EHL is more capable of exploring the mechanism of the contact surface roughness effect on mixed EHL than the stochastic approach to mixed EHL. By the deterministic approach to mixed EHL, based on the Newtonian fluid model, Goglia et al. [5] showed that the contact surface roughness generates considerable fluid film pressure ripples under medium loads in mixed EHL, which almost removes the manufactured contact surface roughness. By the deterministic approach and based on the Newtonian fluid model, Lubrecht et al. [6] showed that the contact surface roughness with large amplitude and short wavelength generates extremely high fluid film pressure ripples in mixed EHL under heavy loads. By the deterministic approach, Greenwood and Johnson [7] showed that in mixed EHL, when the fluid is Newtonian, the manufactured contact surface roughness is almost removed and is replaced by fluid film pressure ripples if the minimum value of the fluid film pressure ripple is not too small. They also theoretically showed that in mixed EHL, when the fluid is shear thinning non-Newtonian, the fluid film pressure ripple and the removal of the manufactured contact surface roughness due to the fluid film pressure ripple both are considerably reduced compared to those for the Newtonian fluid for the same operating condition. Greenwood and Johnson [7] theoretically showed that the fluid shear thinning non-Newtonian effect is qualitatively different from the fluid Newtonian effect both on EHL film pressure and on EHL film thickness in mixed EHL. Later, Chang and Zhao [8] similarly showed that Newtonian and shear thinning non-Newtonian mixed EHL results would be qualitatively different especially for the contact surface roughness of short wavelength. They proposed that the Newtonian fluid model would not be practically appropriate in the modelling of mixed EHL and the mixed EHL results based on the Newtonian fluid model would practically be misleading when the fluid shear thinning effect is significant in mixed EHL for the given operating condition, due to substantial short-wavelength machine element surface roughness. By the deterministic approach, Chang et al. [9] showed that the fluid model holds sensitivity to the mixed EHL film pressure and thickness results. Also, Chang et al. [10] showed that the contact surface roughness effect can reduce the overall EHL film thickness due to increasing the lubricant side leakage and can reduce the local EHL film thickness due to causing the local fluid cavitation in mixed EHL. Johnson and Higginson [11] showed that the fluid viscoplastic shear thinning non-Newtonian effect can increase the lubricant side leakage to reduce EHL film thickness in mixed EHL in the rolling and sliding condition where the fluid shear thinning effect is significant. By the deterministic approach, Zhang [12] used the non-Newtonian fluid model incorporating the contact-fluid interfacial shear strength to study the combined effect of contact surface roughness and contact-fluid interfacial shear strength in mixed EHL. He showed that no fluid film pressure ripples are generated and the manufactured contact surface roughness persists in the sliding mixed EHL where the contact-fluid interfacial slippage widely occurs in the contact, due to the contact-fluid interfacial shear strength effect. He found that another mechanism of the contact surface roughness effect on mixed EHL film thickness is that the contact surface roughness effect increases the extent of the contact-fluid interfacial slippage in mixed EHL to reduce mixed EHL film thickness due to the fluid shear strength non-Newtonian effect or/and the contact-fluid interfacial shear strength effect. He proposed that in mixed EHL, when the contact-fluid interfacial slippage widely occurs in the contact, the contact surface roughness standard deviation in the lubricated contact, to which the ratio of the average mixed EHL film thickness in the Hertzian zone is conventionally defined as the parameter λ to show the mixed EHL stage, needs to be based on the manufactured contact surface roughness and the value of the parameter λ for mixed EHL calculated based on the Newtonian fluid model is much overestimated and misleading. This proposal is contradictory to the mixed EHL results based on the Newtonian fluid model obtained by Venner and Napel [13] which showed that the value of the parameter λ for mixed EHL based on the manufactured contact surface roughness is misleading in describing the mixed EHL stage especially for medium and heavy loads due to the almost complete removal of the manufactured contact surface roughness in the mixed EHL contact due to the generated fluid film pressure ripple.

The above text shows that the fluid rheology has a significant effect on mixed EHL results and different conclusions on mixed EHL would be drawn based on different fluid models. The fluid model is therefore important in the modelling of mixed EHL. It is necessary and of essential interest that the fluid model is taken as non-Newtonian and more realistic in the modelling of mixed EHL, since the fluid in EHL has been found to be generally non-Newtonian especially in severe operating conditions and the mixed EHL film pressure and thickness are so sensitive to the fluid rheological behavior. At least, the stochastic approach results and the deterministic approach results to mixed EHL both show that there are great and qualitative differences between Newtonian and non-Newtonian mixed EHL results in the operating condition where the fluid shear thinning non-Newtonian effect is significant. Therefore, the Newtonian fluid model is generally denied in modelling mixed EHL and the mixed EHL results based on the Newtonian fluid model are dangerous in engineering application.

The contact surface roughness effect on both mixed EHL film pressure and thickness was widely shown to be significant. This effect heavily depends on the fluid rheological behavior. However, no consensus has been reached on the contact surface roughness effect in mixed EHL, since different fluid models were used in modelling mixed EHL and gave qualitatively different mixed EHL results and there is actually no concurrence on the validity of the applied fluid model for mixed EHL for an operating condition. Actually, mixed EHL results based on different fluid models are currently plentiful but confusing. It is necessary and important to return our heads to arrange and evaluate these mixed EHL results for finding an efficient and correct direction into the right way to study mixed EHL.

In the past time, the fluid film thermal effect in mixed EHL due to the fluid film viscous heating was also studied. Christensen [14] assumed that in mixed EHL, the carried load of the contact was usually balanced by both the fluid film pressure and by the asperity contact, which would be softened due to the contact frictional heating and compressed due to the carried load of the asperity contact and this softening. He obtained the mixed EHL modelling results based on this assumption that in mixed EHL, when the carried load of the whole contact was greater than a critical level, the contact surface asperity would be significantly compressed, the fluid film was thus collapsed, and as a result the asperity contact carried nearly all the load of the whole mixed EHL contact. He found that this process was the thermal instability of the fluid film in mixed EHL. Zhang [15, 16] obtained the EHL modelling results that in EHL, due to the combined effect of the fluid film viscous heating, the contact-fluid interfacial shear strength and the contact-fluid interfacial slippage, the EHL film in the Hertzian zone can be extremely thin and even approach to disappearance.

In an actual case, the physical and chemical phenomena occurring in and important to mixed EHL are usually complicated and may be unable to be fully modelled by a simple mixed EHL model. The mixed EHL models and their corresponding mixed EHL theories established in the past time may all be oversimplified and thus unable to model a real mixed EHL. Consequently, these mixed EHL theories may largely deviate from a real mixed EHL. However, mixed EHL experiments are able to directly, correctly and partly show that what really happen in a mixed EHL and are important to a real mixed EHL. Fortunately, mixed EHL experiments were parallel carried out with mixed EHL modelling in the past time.

Czichos and Kirschke [17] investigated the failure of mixed EHL contacts in the condition of steel to steel simple sliding point contacts. They obtained the result of the three-dimensional critical failure surface constructed by three operational parameters i.e. the carried load of the contact, the contact sliding speed and the bulk fluid temperature. They found that sliding mixed EHL contacts of steel were in failure due to the contact surface severe wear in wide operating conditions when each value of these three operational parameters exceeds a critical value. For explaining their observation of the mixed EHL contact failure, they proposed the hypothesis of the critical contact-fluid interfacial energy input composed of thermal energy and mechanical energy which a given mixed EHL contact can withstand. According to their observations and analytical results, they proposed that in a mixed EHL, for high thermal energy input i.e. high bulk fluid temperature and high fluid film viscous heating through high sliding speed, the maximum endurable contact-fluid interfacial mechanical stresses including the maximum endurable contact-fluid interfacial shear stress can only be low, whereas for high mechanical stresses at the contact-fluid interface the thermal capacity of the mixed EHL contact can only be low. However, their observation of the failure phenomena occurring in the mixed EHL contact is unable to be explained by conventional EHL theory and by any of the previous mixed EHL theories. Their proposed factors which appear important to a mixed EHL also were not included in any of the previous mixed EHL modelling.

Later, Begelinger and Gee [18] carried out similar experiments on sliding mixed EHL contacts of steel to more detailed reveal the transition of the mixed EHL stage and the contact failure both occurring in these mixed EHL contacts for wide operating conditions. They also found the critical failure surface constructed by the operational parameters of the mixed EHL contact same as found by Czichos and Kirschke [17]. More detailed, they obtained that there are usually two primary transitions and one secondary transition of the mixed EHL stage in sliding mixed EHL contacts of steel when the severity of the operating condition is gradually increased i.e. the carried load of the contact, the sliding speed of the contact or/and the bulk fluid temperature are gradually increased. These two primary transitions of the mixed EHL stage are respectively the transition from the mixed EHL stage of relatively thick overall mixed EHL film to the mixed EHL stage of oxidized chemical boundary layer lubrication mainly and widely occurring in the Hertzian zone (named as the first primary transition of the mixed EHL stage) and the transition from the mixed EHL stage of oxidized chemical boundary layer lubrication mainly and widely occurring in the Hertzian zone to the severe removal of the oxidized chemical boundary layer from the contact surfaces i.e. the destruction of the contact surfaces occurring after the first primary transition of the mixed EHL stage (named as the second primary transition of the mixed EHL stage). These two primary transitions of the mixed EHL stage occur in sequence in sliding mixed EHL contacts of steel when the severity of the operating condition is gradually increased. Before the first primary transition of the mixed EHL stage i.e. on the mixed EHL stage of relatively thick overall mixed EHL film, the friction coefficient of the mixed EHL contact is around *0.1* and the wear rate of the mixed EHL contact is small. After the first primary transition but before the second primary transition of the mixed EHL stage i.e. on the mixed EHL stage of oxidized chemical boundary layer lubrication mainly and widely occurring in the Hertzian zone, the friction coefficient of the mixed EHL contact is around *0.35* and the wear rate of the mixed EHL contact is still small. However, after the second primary transition of the mixed EHL stage i.e. on the stage of the severe removal of the oxidized chemical boundary layer from the contact surfaces, the friction coefficient of the mixed EHL contact is around *0.35* and the wear rate of the mixed EHL contact is large. The secondary transition of the mixed EHL stage in sliding mixed EHL contacts of steel is the transition from the mixed EHL stage of the severe removal of the oxidized chemical boundary layer from the contact surfaces to the mixed EHL stage of the newly formed oxidized chemical boundary layer lubrication mainly and widely occurring in the Hertzian zone with the friction coefficient *0.1* of the mixed EHL contact and a very small wear rate of the mixed EHL contact.

The experiments of Begelinger and Gee [18] at least provide us several important information usually occurring in a real mixed EHL contact. One is that the contact frictional heating is usually severe in the mixed EHL contact especially on the stage of oxidized chemical boundary layer lubrication mainly and widely occurring in the Hertzian zone due to the considerable and even high friction coefficient of the mixed EHL contact. The second is that it is sure that wear occurs although is light both on the mixed EHL stage of relatively thick overall mixed EHL film and on the mixed EHL stage of oxidized chemical boundary layer lubrication mainly and widely occurring in the Hertzian zone. Actually, the experiments of Begelinger and Gee showed that even on the mixed EHL stage of relatively thick overall mixed EHL film, small areas of oxidized chemical boundary layer lubrication and instant dry contact between opposing asperities usually respectively locally occur and these small areas of local boundary lubrication and dry contact have significant effects on the mixed EHL performance and on the first primary transition of the mixed EHL stage. The third is that the failure form of the contact surfaces in mixed EHL may actually usually be the removal of the oxidized chemical boundary layer from the contact surfaces but not dry a contact between opposing asperities as conventionally speculated. The fourth is that it is necessary to study the final stage of a practical mixed EHL with the destruction of the contact surfaces i.e. the removal of the oxidized chemical boundary layer from the contact surfaces in the modelling of mixed EHL. The fifth is that it may be of interest to study the secondary transition of the mixed EHL stage in a practical mixed EHL contact and the mixed EHL stage of the newly formed oxidized chemical boundary layer lubrication mainly and widely occurring in the Hertzian zone after the removal of the oxidized chemical boundary layer from the contact surfaces occurring in a practical mixed EHL contact both from experiments and from theoretical modelling.

Unfortunately, the previous mixed EHL modelling and theories were not found to incorporate these important information practically occurring in a real mixed EHL contact as found by Begelinger and Gee [18]. It is hard to believe that those mixed EHL modelling and theories are correct and would have application values to the engineering design of EHL contact. Therefore, a realistic and reasonable mixed EHL modelling and theory must incorporate these important factors practically occurring in a real mixed EHL contact.

Begelinger and Gee [19] also studied the effects of the contact surface roughness and the oxygen content of the fluid respectively on the transition of the mixed EHL stage and the destruction of the contact surfaces in real sliding mixed EHL point contacts of steel. They found that before the first primary transition of the mixed EHL stage i.e. on the mixed EHL stage of relatively thick overall mixed EHL film, in a real mixed

EHL contact, if the oxidization of the contact surfaces keeps ahead of the formation of new asperity contacts, the asperity contact in oxidized chemical boundary layer lubrication is locally steadily present due to the oxidization of the newly formed local asperity contact and the local asperity contact of boundary lubrication and the EHL fluid film in the other non-asperity contact areas of the contact both separate the contact surfaces and ensure the mixed EHL contact operating on the low friction and small wear rate state. Oppositely, they found that before the first primary transition of the mixed EHL stage i.e. on the mixed EHL stage of relatively thick overall mixed EHL film, in a real mixed EHL contact, if the oxidization of the contact surfaces can not keep ahead of the formation of new local asperity contacts, the oxidized chemical boundary layer is unable to be formed within the newly formed local asperity contact, the newly formed local asperity contact is thus in dry fresh metal to metal contact, and the whole contact surfaces will then quickly be in failure due to their severe scuffing or/and wear originated from these local dry asperity contacts. They found that in a real mixed EHL contact, increasing the roughness of the contact surfaces and reducing the oxygen content of the fluid both significantly reduce the load-carrying capacity of the mixed EHL contact, lead the mixed EHL stage transited into that of more thinning, mixed and partial lubricating films in the mixed EHL contact, and lead to easier destruction and failure of the contact surfaces. It was again shown by their experiments [19] that in a real mixed EHL contact, the oxidized chemical boundary layer lubrication occurring in local asperity contacts and the instant dry contact of local asperity are usually able to occur before the first primary transition of the mixed EHL stage i.e. on the mixed EHL stage of relatively thick overall mixed EHL film and have significant effects on the first primary transition of the mixed EHL stage. They showed that in a real mixed EHL contact, especially for low sliding speeds of the contact, the roughness of the contact surfaces has significant effects on the first primary transition of the mixed EHL stage and thus on the second primary transition of the mixed EHL stage. It is clear that this contact surface roughness effect in mixed EHL is not caused by the transient process of the mixed EHL due to the contact surface roughness, since their experiments were almost of non-transient mixed EHL due to the very low roughness of one contact surface of the contact. This contact surface roughness effect in mixed EHL is also greatly different from the mixed EHL modelling results based on the Newtonian fluid model as can be found from the previous mixed EHL modelling. From this, the applicability of the Newtonian fluid model in the modelling of a real mixed EHL contact is also rapped. This contact surface roughness effect in mixed EHL may therefore be due to the fluid non-Newtonian behavior in a real mixed EHL contact. Actually, their experiments also showed that in a real mixed EHL contact, for larger roughness of the contact surfaces, more contact surface asperities penetrate into the fluid film and the depth of the penetration of the contact surface asperity into the fluid film is larger. These experimental results are agreeable with the modelling results of mixed EHL by Zhang [12] based on the shear strength non-Newtonian fluid model. The oxygen content of the fluid was also shown by their experiments to have significant effects on the first primary transition of the mixed EHL stage in a real mixed EHL contact.

However, these important contact surface roughness and fluid oxygen content effects to a real mixed EHL contact as shown by Begelinger and Gee [19] were not modelled by the mixed EHL modelling and theories established in the past time. The validity and applicability of the previous mixed EHL modelling and theories to a practical mixed EHL contact are again questioned. It is of great importance and interest to incorporate these important effects to a real mixed EHL contact in the modelling of mixed EHL. This should be considered by the future mixed EHL modelling and the research on this subject should be of significant advance into the study of mixed EHL.

Tabor [20] carried out similar experiments on sliding mixed EHL point contacts of steel as Begelinger and Gee [18, 19] had done. He found that in sliding mixed EHL point contacts of steel, the first primary transition of the mixed EHL stage depends on the contact surface roughness, the oxygen content of the fluid, the bulk viscosity of the fluid and the chemical composition of the fluid. He suggested that in a real mixed EHL, the bulk fluid viscosity and the fluid chemical composition can be used to evaluate the performance of the fluid in the mixed EHL on the stage of relatively thick overall mixed EHL film. For a given sliding speed of the contact, he obtained that in a real mixed EHL contact, the load-carrying capacity of the mixed EHL contact i.e. the maximum Hertzian contact pressure of the mixed EHL contact on the first primary transition of the mixed EHL stage is expressed as the following function of the bulk fluid viscosity and the fluid chemical composition: $p_1=\beta log\eta+\alpha$ where η is the bulk fluid viscosity, α is the constant depending on the fluid chemical composition and β is a constant. This load-carrying capacity expression of the mixed EHL contact on the first primary transition of the mixed EHL stage is unable to be described by not only conventional EHL theories but also any of the previous mixed EHL modelling and theories. The validity of the previous mixed EHL modelling and theories is here again suspicious. It is of necessity to develop new and fresh mixed EHL theories to describe the performance of a real mixed EHL for practically wide operating conditions, by incorporating the operational parameters such as the bulk fluid viscosity, the fluid chemical composition, as well as the contact surface roughness and the fluid oxygen content as found by Tabor [20] and Begelinger and Gee [18, 19]. The mathematical modelling and description of the transition of the mixed EHL stage can be developed by these new mixed EHL theories. This would be of significant practical interest to the design of the mixed EHL contact, to the prediction of the mixed EHL stage and to completing the mixed EHL theory.

Although experiments on mixed EHL especially in the condition of sliding steel to steel point contacts have been well made, the mathematical modelling of a real mixed EHL contact is necessary to be developed. This is of importance not only to understanding the mechanism of a real mixed EHL contact but also to establishing the criterion of the design of elastohydrodynamic lubrication in practical concentrated contacts. Before a more realistic modelling of mixed EHL is developed, it may be necessary to review the researches by others on EHL especially in recent years which show what more and new phenomena would happen in a real EHL and are important to a real EHL. Those newly found phenomena occurring in EHL need to be considered when developing the new modelling and theory of mixed EHL.

Zhang [21] proposed a mixed rheologies regime in isothermal EHL of ideally smooth line contacts. He found that in this EHL, viscoelastic continuum, viscoplastic continuum and non-continuum fluids are distributed from the inlet zone to the Hertzian contact zone in order for severe operating conditions when the contact-fluid interfacial shear strength is low in the inlet zone. He showed that the molecularly thin fluid film occurs in the Hertzian contact zone in this condition and due to the non-continuum characteristics of this film across the film thickness, the rheological behavior of this film is qualitatively different from those of the viscoelastic continuum and viscoplastic continuum fluids, which respectively simultaneously occur in different and other areas of this EHL contact in this condition for the relative high fluid film thickness and thus the continuum characteristics of these fluids film across the film thickness in these areas. He revealed that in EHL, due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect, the overall EHL film can be severely thinning and lubricating films with different rheological behaviors can respectively simultaneously occur in different areas of the contact and thus be mixed in the whole contact. His research may show a new mode of mixed EHL in the mixed EHL theoretical modelling, where mixed rheologies occur in the contact. According to his research, mixed EHL can occur even in the EHL of smooth concentrated contacts due to the occurrence of mixed rheologies in these EHL contacts. For an actual mixed EHL contact with contact surface roughness, the concept of mixed EHL may thus need to be revised, extended and generalized as the mixed EHL where lubricating films with different rheological behaviors can respectively simultaneously occur in different areas of the contact between the rough contact surfaces and may respectively locally occur in more irregular areas of the contact due to the local fluid film thinning and thickening in more irregular areas of the contact respectively due to the ridge penetration into and the furrow denting out of the fluid film of the contact surfaces, according to Zhang's research [21]. This more generalized mixed EHL concept well fits the experimental results of mixed EHL respectively obtained by Begelinger and Gee [18, 19] and by Tabor [20]. It is of importance to the future modelling of mixed EHL and may help to define the configuration of the future mode of mixed EHL.

Zhang [22] obtained the isothermal pure rolling line contact EHL results for ideally smooth contact surfaces from the EHL inlet zone analysis based on the Newtonian fluid model. His results show that for a given relatively heavy load W of dimensionless form, the central EHL film thickness in the Hertzian contact zone in this EHL contact approaches to zero when the dimensionless rolling speed of the contact is lower than the dimensionless characteristic rolling speed $U_{ch}=0.0372W^{1.50}/G$ (where G is the dimensionless material parameter of the contact [1]). This is not predictable from conventional EHL theories. The central EHL film thickness in this condition is usually much lower than predicted by conventional EHL theories as Zhang [22] showed. Zhang's results [22] show that the EHL film thickness in the Hertzian zone is at least on the nanometer scale and the fluid is at least non-continuum across the fluid film thickness in the Hertzian contact zone at relatively heavy loads in this EHL contact when the dimensionless rolling speed is lower than the dimensionless characteristic rolling speed U_{ch}, due to the central EHL film thickness approaching to zero in this condition. Zhang [22] proposed that in this condition, mixed rheologies occur in the whole EHL contact due to the non-continuum fluid across the fluid film thickness in the Hertzian contact zone and the continuum fluid across the fluid film thickness in the other areas of the contact and different fluid rheological models need to be used to respectively describe the behaviors of these fluids in different areas of the lubricated contact when modelling this elastohydrodynamic lubrication. He explained the necessity of this EHL modelling in this condition from the view of the transportation and flow of the elastohydrodynamic fluid through the contact when the fluid film is molecularly thin in the Hertzian zone in the EHL contact. In this study [22], Zhang revealed in another operating condition the occurrence of the qualitatively same mode of mixed EHL in the EHL of ideally smooth line contacts as found by himself before in the same EHL contact for a different operating condition as described above. Zhang's further finding in this study [22] show that mixed rheologies and the new mode of mixed EHL proposed by himself before in reference [21] actually occur in ideally smooth line contact EHL for sufficiently a low rolling speed or sufficiently a heavy load even when the elastohydrodynamic fluid is Newtonian. The correctness of the above revised, extended and generalized mode of mixed EHL for an actual mixed EHL contact with contact surface roughness is therefore further confirmed and is therefore definite. This more generalized mode of mixed EHL should be taken in the future modelling of mixed EHL.

Besides this more generalized mode of mixed EHL, the other findings of the phenomena occurring in EHL both from theories and from experiments especially in recent years are necessary to be reviewed. Zhang et al. [23] analytically found that the overall EHL film is usually severely thinning in isothermal EHL of ideally smooth line contacts in the condition of medium/heavy loads and large slide-roll ratios, due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect. They theoretically explained that this overall EHL film thinning is caused by the great reduction of the total fluid flow through the EHL contact in severe operating conditions due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect [23, 24]. They analytically found that the EHL film thickness in the Hertzian zone in this EHL contact is usually extremely reduced and on the nanometer scale in the condition of medium/heavy loads and large slide-roll ratios due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect [23, 24]. They showed that mixed rheologies usually occur in the whole EHL contact in this operating condition due to the non-continuum fluids across the fluid film thickness occurring in the Hertzian zone and the continuum fluids across the relatively high fluid film thickness in the other areas of the contact and thus lubricating fluid films with different rheological behaviors are usually mixed in the EHL contact in this operating condition. They showed that the more generalized mode of mixed EHL proposed above occur in this EHL contact of ideally smooth contact surfaces due to the severe overall EHL film thinning due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect. When the contact surfaces are rough, the more generalized mode of mixed EHL proposed above should comprehensively occur in this EHL contact in this operating condition. They later obtained the critical compound curvature radius of the contact for the occurrence of the non-continuum fluids across the fluid film thickness i.e. the occurrence of the more generalized mode of mixed EHL proposed above in the isothermal pure rolling ideally smooth elastohydrodynamic line contacts, considering the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect [25]. When the compound curvature radius of the contact is smaller than this critical radius, in isothermal pure rolling EHL of ideally smooth line contacts, mixed rheologies and thus mixed EHL occur according to their theory. They later further obtained and plotted the operational scope i.e. the ranges of the dimensionless carried load and rolling speed of the contact for the occurrence of the non-continuum fluids across the fluid film thickness i.e. the occurrence of the more generalized mode of mixed EHL proposed above in the isothermal pure rolling EHL of ideally smooth line contacts, for wide values of the contact-fluid interfacial shear strength and different values of the compound curvature radius of the contact surfaces [26]. Zhang et al. [27, 28] studied the mode of molecularly thin i.e. non-continuum fluid film lubrication in one-dimensional problem. They developed the flow factor for the total non-continuum fluid flow through the contact in this mode of lubrication due to the combined effect of the inhomogeneity and discontinuity across the fluid film thickness of the non-continuum fluid on this fluid flow. They theoretically found from this flow factor that the fluid flow through the contact and thus the overall fluid film thickness in this mode of lubrication are increased due to the inhomogeneity and discontinuity across the fluid film thickness of the non-continuum fluid. Later, Zhang et al. [29, 30, 31] modelled the isothermal EHL of ideally smooth line contacts in the operating condition where molecularly thin i.e. non-continuum fluid films occur in the Hertzian zone i.e. the mixed EHL in the isothermal ideally smooth line contact in the operating condition where mixed rheologies occur in the whole contact studied and shown before by Zhang et al. [23, 24] and mentioned above, by incorporating the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect and this flow factor. By this modelling, they showed that the molecularly thin fluid film suddenly locally disappears in this EHL when the minimum EHL film thickness is of the scale of *1 nm*, due to the effect of the attractive surface pressures i.e. the van der Waals and solvation pressures between and acting on the contact surfaces on the elastic deformation of the contact surfaces which causes the two contact surfaces locally in adhesion with one another due to the local very large attractive surface pressures at very low local EHL film thickness. This result shows that dry contact or oxidized chemical boundary layer lubrication locally occur in this EHL as a result of the disappearance of the local molecularly thin fluid film in the condition of medium/heavy loads and large slide-roll ratios. This result shows that the dry contact or oxidized chemical boundary layer lubrication, the non-continuum fluid across the fluid film thickness and the continuum fluid across the fluid film thickness respectively simultaneously locally occur in different areas of the contact in this EHL, and the lubricating film or the contact regime are more mixed in this EHL. When the contact surface roughness is further incorporated, this fits the experimental results of mixed EHL respectively obtained by Begelinger and Gee [18, 19] and by Tabor [20]. This result further confirms the correctness of the more generalized mode of mixed EHL proposed above for an actual mixed EHL contact. According to this result, it is necessary to incorporate local dry contact into this more generalized mode of mixed EHL as a new local contact regime between the rough contact surfaces for an actual case.

In the previous researches of EHL, the severe overall and local EHL film thinning were also found from theoretical analysis and by experimental verification, respectively due to the fluid shear thinning non-Newtonian the contact surface roughness and the fluid film thermal effects. Greenwood and Kauzlarich [32] found that the central EHL film thickness percentage reduction due to the fluid shear thinning effect of power-law shear thinning fluids can be *80%*. Zhang [22] theoretically found that in isothermal pure rolling EHL of ideally smooth line contacts, for a given rolling speed, at sufficiently heavy loads, the central EHL film thickness approaches to zero and is far lower than predicted by conventional EHL theories for the Newtonian fluid. In this condition, the overall EHL film of the Newtonian fluid is severely thinning. For shear thinning fluids, the overall EHL film is more severely thinning in this condition. Zhang [12] analytically found that in isothermal EHL of rough line contacts in the condition of medium/heavy loads and large slide-roll ratios, the average fluid film thickness between the rough contact surfaces in the Hertzian zone usually is greatly reduced while the contact surface roughness is not deformed by the generated EHL film pressure, due to the combined effect of the contact-fluid interfacial shear strength, the contact-fluid interfacial slip and the contact surface roughness. In this case, the EHL film in the Hertzian contact zone can be locally severely thinning due to the persisting contact surface roughness penetration into the average fluid film between the rough contact surfaces in the Hertzian contact zone which has been great thinning. Zhang [15, 16] analytically found that in pure rolling EHL of ideally smooth line contacts in severe operating conditions i.e. for medium/heavy loads, high rolling speeds and high bulk fluid temperatures, the EHL film can be overall severe thinning due to the combined effect of the fluid film viscous heating, the contact-fluid interfacial shear strength and the contact-fluid interfacial slippage. For sliding contact surfaces, the overall EHL film is more severe thinning in these conditions, due to the more severe fluid film viscous heating.

In some previous researches of mixed EHL, the dry contact between the contact surfaces was studied. Cheng [33] modelled the mixed EHL contact by using the Newtonian fluid model in the whole lubricated area of the contact. He suggested that in the whole mixed EHL contact the Newtonian fluid film was mixed with the dry contact. According to his suggestion, the Newtonian fluid film occurs in the lubricated area of the whole mixed EHL contact while the dry contact locally occurs between the opposing asperities of the contact surfaces. Therefore, his mixed EHL model showed that the whole mixed EHL contact consisted of the Newtonian fluid film lubricated area and the dry contact between the opposing asperities of the contact surfaces. Although his mixed EHL model was of more progress into the modelling of mixed EHL compared to his earlier one, it may be still over simplified compared to the experimental and theoretical findings of the phenomena in mixed EHL in the past time as mentioned above.

In the future modelling of mixed EHL by taking the more generalized mode of mixed EHL proposed above for an actual mixed EHL contact, the features of the overall and local fluid film thinning, the mixed contact regimes between the contact surfaces and the partial lubricating films all occurring in a mixed EHL contact need to be incorporated.

According to the researches of mixed EHL carried out in the past time mentioned above, the modes of mixed EHL taken in the past and to be taken in the future in the theoretical modelling of mixed EHL can be classified as three kinds i.e. the classical mode of mixed EHL, the modern mode of mixed EHL and the future mode of mixed EHL. The classical mode of mixed EHL was taken in the early time of the theoretical modelling of mixed EHL. The features of this mode of mixed EHL are that the fluid film between the rough contact surfaces in the whole mixed EHL contact is relatively thick and the fluid film is continuum across the fluid film thickness in the whole mixed EHL contact due to the relatively high fluid film thickness. In this mode of mixed EHL, the fluid film thickness in the whole mixed EHL contact is often higher than $0.05\ \mu m$. This mode of mixed EHL was taken in both the stochastic and deterministic approaches to mixed EHL in the previous modelling of mixed EHL. The modes of mixed EHL respectively modelled by Patir and Cheng [3], Hughes and Bush [4], Goglia et al. [5], Lubrecht et al. [6], Greenwood and Johnson [7], Chang and Zhao [8], Chang et al. [9, 10], Johnson and Higginson [11], Zhang [12], and Venner and Napel [13] mentioned above belong to this mode of mixed EHL. The configurations of the modern mode of mixed EHL were detailed proposed by Zhang [21, 22]. The essential features of this mode of mixed EHL are that the contact regimes between the contact surfaces are different in different parts of the whole mixed EHL contact and thus mixed in the whole mixed EHL contact. These mixed contact regimes between the contact surfaces are the lubricating fluid films with the different rheological behaviors in different parts of the whole mixed EHL contact which are respectively viscoelastic continuum, viscoplastic continuum and non-continuum, according to the researches of Zhang

[21, 22]. Incorporating the features of the researches of Zhang [21, 22] and also incorporating the feature of the dry contact, the defined mixed contact regimes between the contact surfaces of the modern mode of mixed EHL i.e. the features of the modern mode of mixed EHL are the lubricating shear-thinning viscoelastic continuum, viscoplastic continuum and non-continuum fluid films and the dry contact which respectively simultaneously locally occur in different areas of the whole mixed EHL contact in this mode of lubrication depending on the operating condition. In the modern mode of mixed EHL, the fluid film thickness in the whole mixed EHL contact is usually widely varied and usually varies from the value over *1 μm* in the inlet zone to the value of the scale of *1 nm* between the opposing asperities of the contact surfaces in the Hertzian contact zone and even to the value of zero i.e. the occurrence of the dry contact between the local opposing asperities of the contact surfaces. The deterministic approach was usually taken to the modern mode of mixed EHL in the previous study of this mode of lubrication by modelling. The modes of mixed EHL previously respectively proposed by Zhang [21, 22] and modelled by Zhang et al. [29, 30, 31] mentioned above may belong to the modern mode of mixed EHL. The future mode of mixed EHL is proposed in the present paper both based on the experimental results of mixed EHL in simple sliding steel to steel point contacts respectively obtained by Begelinger and Gee [18, 19] and by Tabor [20] based on the features of the modern mode of mixed EHL. This mode of mixed EHL is defined as the mixed EHL where the shear-thinning viscoelastic continuum, viscoplastic continuum and non-continuum fluid films, the oxidized chemical boundary layer lubrication and the dry contact respectively simultaneously locally occur in different areas of the whole mixed EHL contact depending on the operating condition, i.e. the more generalized mode of mixed EHL proposed above for an actual mixed EHL contact. The essential features of this mode of mixed EHL are that in itself, in this mode of mixed EHL, the contact regimes between the contact surfaces are very mixed in the whole contact and are more mixed than those of the modern mode of mixed EHL due to incorporating the oxidized chemical boundary layer lubrication as a new local contact regime between the rough contact surfaces. In the future mode of mixed EHL, the fluid film thickness in the whole mixed EHL contact is also usually widely varied and usually varies from the value over *1 μm* in the inlet zone to the value of the scale of *1 nm* between the opposing asperities of the contact surfaces in the Hertzian contact zone and even to the value of zero i.e. the occurrences of the local oxidized chemical boundary layer lubrication or the dry contact between the local opposing asperities of the contact surfaces. Therefore, the future mode of mixed EHL is able to be used in the study of the mixed EHL in wide and severe operating conditions where the stage of mixed EHL can be transited from the mixed EHL stage of full hydrodynamic fluid films in the whole mixed EHL contact and local relatively thick continuum fluid films to that of full hydrodynamic fluid films in the whole mixed EHL contact and local molecularly thin i.e. non-continuum fluid films and further to that of partial hydrodynamic fluid films in the whole mixed EHL contact and local oxidized chemical boundary layer lubrication or/and local dry contact between the opposing asperities of the contact surfaces when the severity of the operating condition is gradually increased. If the modelling of the process of the removal of the local oxidized chemical boundary layer from the contact surfaces is included in the future mode of mixed EHL, the first and second primary transitions of the mixed EHL stage experimentally found mentioned above can be theoretically modelled by the future mode of mixed EHL. The effect of the contact surface roughness in mixed EHL can also be comprehensively studied by the future mode of mixed EHL. Therefore, the future mode of mixed EHL the best reflects and fits the real mode of mixed EHL among the above three classified modes of mixed EHL taken in the modelling of mixed EHL. The future mode of mixed EHL should be taken in the future modelling of mixed EHL and should be studied soon.

It is to realize the necessity and importance of collecting, analyzing and appraising the literatures of the researches of mixed EHL in the past time and finding the correct and shortest road onto the efficient and effective study of mixed EHL for an actual case. In the present paper, the characteristic and development of mixed EHL in the industry in the past time are commented and both the characteristic of mixed EHL and the trend of the development of mixed EHL in the industry in the future time are expected. Accordingly, the representative researches of mixed EHL both by experiments and by theory in the past time are appraised, the characteristic and development of the researches of mixed EHL in the past time are commented, and both the characteristic of the research of mixed EHL and the direction of the research of mixed EHL in the academic activity in the future are suggested. Especially, the modes of mixed EHL taken in the modelling of mixed EHL in the past time are appraised and classified. Combining the features of the previous modes of mixed EHL taken in the modelling of this lubrication and according to the characteristic and direction of the mixed EHL research in the future academic activity, the future mode of mixed EHL is proposed for the modelling of mixed EHL in the future. The theoretical modelling of mixed EHL in the future by taking this proposed mode of mixed EHL would be of substantial advance in the study of mixed EHL by theory. This would strongly propel the progress in the research and understanding of mixed EHL in actual cases.

2. DEVELOPMENT AND CHARACTERISTIC OF MIXED EHL IN THE INDUSTRY AND IN THE ACADEMIC RESEARCH IN THE PAST TIME

2.1 Development and characteristic of mixed EHL in the industry in the past time

2.1.1 Typically thinner and even more mixed EHL films in the EHL contact in the industry with proceeding time

The lubricating media of elastohydrodynamic lubrication occurring in gears, cams and roller bearings are commonly fluid and grease. Sometimes, they may be special lubricating media with special physical and chemical structures and properties such as magnetic fluids and ER (electrorheological) fluids. The present paper only addresses the most popularly used lubricating media in EHL in the industry – common fluids.

The development of EHL and mixed EHL in the industry in the past time shows the following characteristic of EHL and mixed EHL in the industry in the past time: The EHL and mixed EHL films in the industry in the past time were thinner and even more mixed when time proceeded. One of the reasons of this characteristic is that the severity of the operating condition of the EHL contact on the conventional typical elastohydrodynamic components of gears, cams and roller bearings was typically increased with proceeding time and thus the EHL film thickness on these components was typically reduced with proceeding time in the industry in the past time. In the industry of the early time of the 20th century, typically, the carried load of the EHL contact was relatively light and was no more than *1.0 GPa* in its generated maximum Hertzian contact pressure, the circumferential speeds of the EHL contact surfaces were medium, however the compound curvature radius of the EHL contact surfaces was relatively large. As a result, the EHL film was relatively thick and its thickness at the Hertzian contact center was on the scale of $0.5\ \mu m$ in that industry. As time proceeded, with the development of the industry, the carried load of the EHL contact was significantly increased, the circumferential speeds of the EHL contact surfaces were also significantly increased, and the compound curvature radius of the EHL contact surfaces was smaller. On the other hand, with the development of the industry, the sliding to rolling ratio i.e. the slide-roll ratio between the EHL contact surfaces was significantly increased, and the bulk fluid temperature within the EHL contact was significantly higher. It was found that the EHL film thickness actually is not slightly varied with heavy loads as conventional EHL theory describes but actually is sensitively reduced with load increase when the carried load of the EHL contact is heavy. It was also found that the EHL film thickness is actually unable to be increased by the rolling speed of the EHL contact by the exponential law as conventional EHL theory describes, but actually has a limit at a critical rolling speed of the EHL contact and is independent on the rolling speed of the EHL contact when the rolling speed is higher than this critical rolling speed of the EHL contact, due to the fluid film viscous heating or/ and the effect of the contact-fluid interfacial shear strength within the EHL contact. It is well known from conventional EHL theory that for a given operating condition, the EHL film thickness at the Hertzian contact center is linearly reduced with the reduction of the compound curvature radius of the EHL contact surfaces. It was found that the EHL film thickness is sensitively reduced with the increase of the slide-roll ratio between the EHL contact surfaces due to the fluid film viscous heating and the effect of the contact-fluid interfacial shear strength within the EHL contact. It was also found that the EHL film thickness is sensitively reduced with the increase of the bulk fluid temperature within the EHL contact due to the rapid reduction of the bulk fluid viscosity in the EHL inlet zone by this fluid temperature increase. Due to these factors, as a result, the EHL film was typically significantly thinner with the development of the industry in the past time due to the considerable increase of the severity of the operating condition of the EHL contact and its thickness at the Hertzian contact center fell to the scale of $0.1\ \mu m$ in the EHL contact of severe operating conditions in the late decades of last century. Experiments [18, 19, 20] on EHL and mixed EHL in the 1970s showed that the EHL stage in severe operating conditions is actually transited from the mixed EHL of relatively thick overall EHL film where the local EHL film thickness is on the scale of the contact surface roughness and is even of molecular scale to the mixed EHL stage of oxidized chemical boundary layer lubrication mainly and widely occurring in the Hertzian contact zone until the failure of the EHL contact surfaces, when the severity of the operating condition of the EHL contact is increased, as described above. The theoretical analysis [29, 30, 31] and theory [21] of EHL in the beginning of the 21th century showed that the EHL film in the Hertzian contact zone can be molecularly thin and even locally disappear in severe operating conditions due to the combined effect of the contact-fluid interfacial shear strength and the surface pressure. The EHL theories [21, 22] in that period showed that the molecularly thin fluid film and the oxidized chemical boundary layer film in the Hertzian contact zone and the continuum fluid film in the EHL inlet zone can simultaneously occur and thus be mixed in an EHL contact in severe operating conditions due to the severe thinning of the overall EHL film due to the contact-fluid interfacial shear strength effect and even for the Newtonian fluid. These theoretical results are qualitatively agreeable with the experimental results [18, 19, 20] of EHL for severe operating conditions obtained in 1970s. Experiments on EHL [18, 19, 20] and the theoretical EHL analysis [29, 30, 31] both showed that the dry contact between the contact surfaces can locally occur and thus be mixed with the lubricating fluid film and oxidized chemical boundary layer film respectively, occurring simultaneously locally in other areas of the whole contact in the EHL contact of severe operating conditions. Therefore, with the development of the industry in the past time, due to the increase of the severity of the operating condition of the EHL contact, the EHL film was typically thinner and can be overall severe thinning it can be locally molecularly thin and locally vanished on the conventional typical elastohydrodynamic components in severe operating conditions in the modern industry and especially in today's industry. In the modern and today's industries, the lubricating films including the molecularly thin fluid film, the oxidized chemical boundary layer film and the continuum fluid film and the local dry contact between the contact surfaces more simultaneously locally occur in the EHL contact in severe operating conditions and the lubricating films and the contact regimes between the contact surfaces are both thus more mixed in this EHL contact. These are actually typical features of EHL in the practical contact of conventional typical elastohydrodynamic components in severe operating conditions in current industries.

Another reason of the characteristic of the thinner and even more mixed EHL films with proceeding time in the industry in the past time is that the development of the nano-technology made the mechanical component micro in MEMS (Micro Electromechanical Systems) in the development of the industry in the end of last century and the EHL film on this mechanical component is extremely thinning and molecularly thin. The molecularly thin lubricating film is a typical feature of the EHL occurring on the component of MEMS.

Therefore, with the development of the industry, the EHL film has a tendency of being thinner and even more mixed in the EHL contact. At present, the new and typical feature of the EHL film in practical most popular concentrated contacts is the local extremely thinning and molecularly thin fluid and even oxidized chemical boundary layer films mainly occurring in the Hertzian contact zone and mixed lubricating films with different rheological behaviors respectively, occurring simultaneously locally in different areas of the whole contact. Previously established EHL theories are incapable of describing this mode of EHL. New EHL theories need to be developed for this mode of EHL to satisfy the requirement of the present industry. The study on this mode of EHL is a necessary and important task in the research of EHL now and in the following years.

2.1.2 Typically more partial EHL films in the whole EHL contact in the industry with proceeding time

Another characteristic of the development of EHL and mixed EHL in the industry in the past time is that the EHL film was more partial in the whole EHL contact in the industry in the past time when time proceeded. In the industry of the early time of the 20th century, the EHL film thickness was typically relatively high as described in the above section. In that industry, the EHL film thickness in the whole EHL contact was more frequently higher than the roughness of the EHL contact and was often high enough to fully separate the contact surfaces in the whole EHL contact. In that industry, the local collision and thus the local dry contact between the contact surfaces relatively scarcely occurred in the whole EHL contact. In that industry, the EHL film was relatively scarcely partial in the whole EHL contact and the EHL contact was usually lubricated by relatively thick continuum fluid films fully formed in the whole EHL contact. As reviewed in the above section, the EHL film was typically significantly thinner with proceeding time due to the increase of the severity of the operating condition of the EHL contact and now is typically locally extremely thinning and molecularly thin in practical most popular EHL contacts in severe operating conditions. The molecularly thin fluid film, the oxidized chemical boundary layer film and the local dry contact between the contact surfaces actually usually simultaneously occur locally in different areas of the EHL contact, thus they are actually usually mixed in the EHL contact in severe operating conditions in the modern and today's industries. Although the smoothness of the manufactured contact surfaces of EHL was improved with proceeding time in the industry in the past time, now, the RMS roughness of the manufactured contact surface of EHL is usually on the scale of $0.1\ \mu m$ to the most extent by super finishing of the contact surface. In the modern and today's industries, the EHL film thickness in part of the Hertzian contact zone is usually much comparable to and even much lower than this manufactured roughness of the contact surface of EHL in severe operating conditions due to the severe thinning of the overall EHL film. The experiment [20] on mixed EHL and the theoretical analysis [12] of mixed EHL indicated that the manufactured contact surface roughness may almost persist in mixed EHL in the condition of large slide-roll ratios. In the modern and today's industries, the local collision and thus the local dry contact between the opposing asperities of the contact surfaces may therefore usually occur in the whole EHL contact in severe operating conditions due to the local penetration of these contact surface asperities into the fluid film. Due to this, in the modern and today's industries, the EHL film may usually be partial in the whole EHL contact in severe operating conditions and the EHL contact may usually only be lubricated in part of the contact by the lubricating films only partially formed in the whole EHL contact. As described in the above section, in the modern and today's industries, the lubricating films with different rheological behaviors usually simultaneously locally occur in different areas of the whole EHL contact, thus they are usually mixed in the EHL contact in severe operating conditions. Therefore, each kind of lubricating film with a certain rheological behavior is also usually partial in the whole EHL contact in severe operating conditions in the modern and today's industries. Therefore, the EHL film was more partial with proceeding time in the industry in the past time due to the increase of the severity of the operating condition of the EHL contact. The lubricating films are usually very mixed and thus very partial in the EHL contact in severe operating conditions in the modern and today's industries due to the mixed rheologies and the local dry contact between the contact surfaces respectively occurring simultaneously in this EHL contact. The more partial EHL films with proceeding time is another important feature of the development of EHL and mixed EHL in the industry in the past time. The partial lubricating films are another important feature of the EHL and mixed EHL in severe operating conditions in current industries.

2.2 Development and characteristic of the academic research of mixed EHL in the past time

2.2.1 Development of the academic research of mixed EHL in the past time

The academic research of EHL and mixed EHL in the past time was purposed to satisfy the requirement of the industry at that time. In the past time, the academic research of EHL and mixed EHL appeared to often lag after the development of EHL and mixed EHL in the industry.

2.2.1.1 Development of the theoretical modelling of mixed EHL in the past time

2.2.1.1.1 Development of EHL theories for isothermal rolling and sliding EHL of ideally smooth line and point contacts based on the Newtonian fluid model

In the late decades of the 19th century, with the development of the industry, machine and mechanical components entered the ordinary life of human being including the industry. Consequently, elastohydrodynamic lubrication (EHL) occurred on the typical mechanical components in machines of gears, cams and roller bearings to reduce the friction and wear of these components. An integral EHL theory applicable to the industry had not been seen until in 1940s, Grubin [34] developed the EHL theory for isothermal EHL of ideally smooth line contacts under heavy loads based on an EHL inlet zone analysis and the Newtonian fluid model considering the elastic deformation of the contact surfaces and obtained the formula for predicting the EHL film thickness at the Hertzian contact center as functions of the dimensionless carried load, the dimensionless rolling speed and the dimensionless material parameter of the EHL contact which was applicable to the industry. In 1960s, Dowson and Higginson [1] established the EHL theory for isothermal EHL of ideally smooth line contacts for wider operating conditions based on numerical calculation with computer and the Newtonian fluid model considering the elastic deformation of the contact surfaces and regressed out the formulae respectively for predicting the featured minimum EHL film thickness and the EHL film thickness at the Hertzian contact center in the EHL contact similar to Grubin's central EHL film thickness formula for line contact EHL [34]. Their research was more progressive than Grubin's theory because they obtained the EHL film thickness and pressure distributions in line contact EHL and the minimum EHL film thickness formula for line contact EHL which was more applicable to the industry than the central EHL film thickness formula for line contact EHL. In 1970s, Dowson and Hamrock [35] similarly established the EHL theory for isothermal EHL of ideally smooth elliptical contacts based on numerical calculation with computer and the Newtonian fluid model as Dowson and Higginson [1] did and regressed out the central and minimum EHL film thickness formulae for elliptical contact EHL for varying ellipticity ratio of the EHL contact which had predictive and applicable values to the industry. However, both Grubin's EHL theory and Dowson's EHL theory were found to have only predictive values in rather limited scopes of the operational parameters of the EHL contact and fail for the EHL of line or point contacts in severe operating conditions i.e. for heavy loads, high rolling speeds and high bulk fluid temperatures in pure rolling or rolling and sliding conditions [36, 37, 38].

2.2.1.1.2 Development of EHL theories for isothermal EHL of line and point contacts for rolling and sliding incorporating the roughness of the EHL contact, based on the Newtonian fluid model

Actually, the EHL contact on mechanical components is rough. The roughness of the EHL contact was usually significant on the mechanical components in the industry before 1950s and especially in the early period of the 20th century. Although the EHL film thickness in those periods was typically relatively high due to the relatively light operating condition, the roughness of the EHL contact can actually have significant effects on EHL film pressure and EHL film thickness on the mechanical components in those periods. However, an integral EHL theory considering the EHL contact roughness i.e. an integral mixed EHL theory was not seen before 1960s. Christensen [2] was one of the earliest researchers to study mixed EHL in isothermal condition based on the Newtonian fluid model by the stochastic approach. In 1970s, Patir and Cheng [3] developed a stochastic mixed EHL theory for isothermal line contact EHL based on the Newtonian fluid model for predicting the contact surface roughness effect on the average EHL film pressure and the average EHL film thickness in this EHL. In 1980s and 1990s, Goglia et al. [5], Lubrecht et al. [6], Greenwood and Johnson [7], Chang and Zhao [8], Chang et al. [9] and Chang et al. [10] respectively studied the mixed EHL in isothermal line or point contacts in steady or transient conditions based on the Newtonian fluid model by the deterministic approach. As mentioned in the section of Introduction, these mixed EHL researches are unable to predict the EHL film breakdown occurring in the real mixed EHL contact in severe operating conditions, since they showed that the contact surface roughness effect on EHL film thickness is rather modest based on the Newtonian fluid model. These mixed EHL researches may only give starting and preliminary insights into the contact surface roughness effect on mixed EHL. In the end of last century, the mixed EHL theory being of predictive value and applicable to the real mixed EHL contact in wide operating conditions was still far from having been established.

2.2.1.1.3 Development of EHL theories for EHL of line and point contacts for rolling and sliding incorporating the roughness of the EHL contact and the fluid film viscous heating, based on the Newtonian and non-Newtonian fluid models

Actually, the fluid film viscous heating is usually significant in a practical EHL contact. The fluid film thermal effect is actually usually considerable in a practical EHL contact. In the late decades of last century, the theoretical researches of EHL and mixed EHL also incorporated the fluid film thermal effect on EHL and mixed EHL. Hsiao and Hamrock [39] studied the thermal rolling and sliding EHL of ideally smooth line contacts by using a shear-thinning non-Newtonian fluid model. They showed that the fluid film thermal effect on EHL film thickness was usually modest and may often be modest even in severe operating conditions when the fluid model was taken as a practical non-Newtonian. Cioc et al. [40] studied the thermal mixed EHL of practical line contacts in the condition of medium and heavy loads, high rolling speeds and large slide-roll ratios by taking the measured profiles of the EHL contact roughness respectively based on the Newtonian and shear-thinning non-Newtonian fluid models. They also showed that the fluid film thermal effect on EHL film thickness was typically modest in relatively severe operating conditions in their mixed EHL modelling. The mixed EHL modelling incorporating the fluid film thermal effect in last century was not seen to be satisfactory in predicting the EHL film breakdown usually occurring in the real mixed EHL contact in severe operating conditions. It seems that the fluid film thermal effect is not the only factor to cause this EHL film breakdown in a real mixed EHL according to these referenced thermal EHL and mixed EHL modelling. Other important factors still needed to be incorporated into the mixed EHL modelling for obtaining the mixed EHL model of predicative values to the real mixed EHL contact for practical wide operating conditions.

2.2.1.1.4 Development of EHL theories for EHL of line and point contacts for rolling and sliding incorporating the roughness of the EHL contact, the fluid film viscous heating and the fluid non-Newtonian behavior

It was experimentally found by various researchers [41, 42, 43] in 1970s that the fluid rheological behavior is actually usually non-Newtonian and shear thinning in a real EHL contact especially in severe operating conditions. From those experimental researches, Johnson and Tavaarwerk [42] obtained the Ree-Eyring fluid model for expressing the fluid shear thinning non-Newtonian behavior in practical EHL contacts in certain scopes of the operational parameters of the EHL contact. Bair and Winer [43] obtained the shear-thinning non-Newtonian fluid model incorporating the fluid shear strength parameter for expressing the fluid non-Newtonian behavior in practical EHL contacts they observed in considerably wide scopes of the operational parameters of the EHL contact. According to these obtained non-Newtonian fluid models respectively for practical EHL contacts, the EHL and mixed EHL theories based on the Newtonian fluid model developed before actually still did not basically correctly reflect the reality of the EHL contact in the industry. After 1970s, the fluid rheological effect also was considered to be an important factor probably influencing the performance of EHL and mixed EHL in the EHL modelling. In 1980s, Conry et al. [44] studied the isothermal rolling and sliding EHL of ideally smooth line contacts for practical operational parameters of the EHL contact by using the Ree-Eyring fluid model. They showed that the fluid shear-thinning non-Newtonian effect on EHL film thickness expressed by the Ree-Eyring fluid model is typically only slight. In 1990s, Hsiao and Hamrock [39] studied the thermal rolling and sliding EHL of ideally smooth line contacts by using the circular shear-thinning non-Newtonian fluid model incorporating the fluid shear strength parameter similar to the Bair and Winer's non-Newtonian fluid model [43]. They showed that the fluid non-Newtonian effect on EHL film thickness expressed by the circular fluid model was rather modest in thermal condition for their studied scopes of the operational parameters of the EHL contact which however was not of severe operating conditions. In 2000s, Cioc et al. [40] studied the thermal mixed EHL of practical line contacts in relatively severe operating conditions by using the Hsiao and Hamrock's circular non-Newtonian fluid model [39]. They also showed that the fluid non-Newtonian effect on EHL film thickness expressed by the circular fluid model was very limited in thermal condition for their chosen operational parameters of the EHL contact. It appeared that the mixed EHL modelling preliminarily incorporating the combined effect of the fluid non-Newtonian behavior, the contact surface roughness and the fluid film viscous heating was still seen not to be satisfactory in predicting the EHL film breakdown typically occurring in the practical mixed EHL contact in severe operating conditions. However, the fluid non-Newtonian behavior, the contact surface roughness and the fluid film viscous heating may all actually be very complicated in a practical mixed EHL contact. The mixed EHL theories developed in the past time incorporating the combined effect of these factors may all be still oversimplified due to not accurately understanding and not appropriately modelling these factors. Actually, these three important factors in EHL still need to be further studied and more throughly understood. With the progress of the understanding of these important factors, the finer and better mixed EHL theory based on the more accurate and better modelling of these factors would be developed to be of more predictive and applicable values to the real mixed EHL contact in wider operating conditions and even in severe operating conditions.

2.2.1.1.5 Development of the mode of mixed EHL with thinning, mixed and partial EHL films in the mixed EHL modelling

Cheng [33] developed a mixed EHL model which assumed the fluid film to occur simultaneously in the contact with the local dry contact occurring between the opposing asperities of the contact surfaces. His mixed EHL model was qualitatively different from the mixed EHL models developed before. It should be of more progress in the theoretical modelling of mixed EHL due to proposing the mode of mixed EHL which agreed better with the experimental results of mixed EHL than the modes of mixed EHL proposed before as described in the section of Introduction. However, his mixed EHL model assumed the lubricating film in the whole mixed EHL contact to be Newtonian fluid film. This assumption is very simple and actually qualitatively largely deviates from the experimental results of the fluid rheology and the occurring lubricating films in a practical mixed EHL contact. It may not be allowable in the theoretical modelling of mixed EHL for a real mixed EHL contact.

Later, Zhang [21] proposed a new mode of mixed EHL based on the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect in EHL. He showed that the molecularly thin fluid film, the viscoplastic continuum fluid film and the viscoelastic continuum fluid film can respectively simultaneously occur locally in different areas of the EHL contact and thus be mixed in the whole EHL contact in severe operating conditions i.e. for heavy loads, high rolling speeds, high bulk fluid temperatures and large slide-roll ratios due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect. According to Zhang's results [21], this mode of mixed EHL can occur in the EHL of ideally smooth concentrated contacts. This mode of mixed EHL was new compared to the modes of mixed EHL proposed before in the theoretical modelling of mixed EHL. It further agreed better with the experimental results of mixed EHL than the modes of mixed EHL proposed before as described in the section of Introduction, if the roughness of the EHL contact and the frictional heating of the EHL contact were incorporated into this mode of mixed EHL. Zhang [21] showed in his mode of mixed EHL that the lubricating films with different rheological behaviors were mixed in the whole mixed EHL contact due to the severe thinning of the overall EHL film. His mode of mixed EHL showed that each kind of lubricating film with a certain rheological behavior was actually partial in the whole mixed EHL contact. He also proposed in his mode of mixed EHL that an accurate fluid rheological model was important to the mixed EHL results obtained by theoretical modelling and the fluid rheological model in the modelling of mixed EHL should generally be non-Newtonian and fit the fluids in different areas of the mixed EHL contact which respectively had different rheological behaviors. These characteristic of his mode of mixed EHL well agree with the experimental results of the fluid rheology and the occurring lubricating films in a practical mixed EHL contact as respectively obtained by Bair and Winer [43], Begelinger and Gee [18, 19] and Tabor [20].

Based on this new mode of mixed EHL, Zhang et al. [29, 30, 31] further modelled the isothermal simple sliding EHL of ideally smooth line contacts with the molecularly thin fluid film, the viscoplastic continuum fluid film and the viscoelastic continuum fluid film mixed in the whole EHL contact by incorporating the combined effect of the contact-fluid interfacial shear strength, the contact-fluid interfacial slip, the surface pressure and the non-continuum property of the molecularly thin fluid film and by using the practical generalized shear-thinning non-Newtonian fluid model which were implemented to all the fluid film lubricated areas of the EHL contact i.e. the whole EHL contact. They showed that the molecularly thin fluid film can actually locally disappear in the EHL contact due to this combined effect in the condition of medium/heavy loads and large slide-roll ratios. As a result of this molecularly thin fluid film local disappearance, the dry contact or the oxidized chemical boundary layer lubrication locally occurred in this EHL contact between the contact surfaces. Combining the features of these theoretical EHL results obtained by Zhang and his colleagues, the whole configuration of Zhang's mode of mixed EHL is described as the mode of mixed EHL in which the molecularly thin fluid film, the viscoplastic continuum fluid film, the viscoelastic continuum fluid film, the oxidized chemical boundary layer lubrication and the dry contact between the contact surfaces respectively simultaneously occur locally in different areas of the whole EHL contact and are thus mixed in the whole EHL contact depending on the operating condition. If the contact surface roughness is incorporated into Zhang's mode of mixed EHL, Zhang's mode of mixed EHL becomes the more generalized mode of mixed EHL as described in the section of Introduction where these lubricating films with different rheological behaviors respectively simultaneously occur locally in more irregular areas of the whole mixed EHL contact and are thus more mixed in the whole mixed EHL contact depending on the operating condition due to the perturbation of the contact surface roughness to the local fluid film thickness in mixed EHL. The features of Zhang's mode of mixed EHL are thinning, mixed and partial lubricating films respectively simultaneously occurring locally in different areas of the whole mixed EHL contact. These features can be very significant in a mixed EHL contact depending on the operating condition. These features fairly agree with the experimental results of mixed EHL respectively obtained by Begelinger and Gee [18, 19] and by Tabor [20] as described in the section of Introduction. Although Zhang's mode of mixed EHL may still have deviations from the real mixed EHL, his mode of mixed EHL was more substantially progressive and better than the modes of mixed EHL proposed before in the theoretical modelling of mixed EHL due to its features more agreeing with the features of the real mixed EHL occurring in the industry in the past time as described in section 2.1.

Zhang's mode of mixed EHL proposed in reference [21] and complemented in references [29, 30, 31] drew detailed and substantial configurations of the modern mode of mixed EHL. Although it may be still oversimplified compared to the real mixed EHL, this mode of mixed EHL significantly progressed toward establishing the mode of mixed EHL for a real mixed EHL contact compared to the modes of mixed EHL developed before in the mixed EHL modelling.

2.2.1.2 Development of the experimental research of mixed EHL in the past time

In last century, with the development of the theoretical modelling of mixed EHL, the experimental study of mixed EHL actually was carried out parallel. The most typical, practical and influencing experiments on mixed EHL in last century may be the experimental study of lubricant failure in concentrated steel contacts launched in 1970s by the International Research Group on Wear of Engineering Materials (IRG). The representative results of these experimental studies were respectively obtained by Begelinger and Gee [18, 19] and by Tabor [20]. Their experimental results were purposed to reveal the mechanism of the lubricant failure in practical EHL contacts occurring in the industry in that time. Their experiments were standardized with the same experimental technique, design, condition and materials. Their experimental results were highly reproducible, convincing and indicative. The critical contents of their experimental results have been described in the section of Introduction. In this section, their experimental results are only necessarily summarized.

The experiments were respectively carried out by Begelinger and Gee [18, 19] and by Tabor [20] on the mixed EHL in (52100) steel to (52100) steel simple sliding point contacts with widely varying carried load and sliding speed of the EHL contact and representative low and high temperatures of the bulk fluid. The experimental results respectively obtained by them were basically identical and may respectively be with emphasis in detail on different aspects of mixed EHL in a practical contact. The following paragraphs mainly demonstrate the identical experimental results of mixed EHL respectively obtained by them.

2.2.1.2.1 Transitions of the mixed EHL stage

Begelinger and Gee [18, 19] and Tabor [20] all found from experiments that three primary transitions of the mixed EHL stage typically occurred in a practical mixed EHL contact. These three primary transitions of the mixed EHL stage were respectively named as the first, the second and the third primary transitions of the mixed EHL stage.

(1) First primary transition of the mixed EHL stage

The first primary transition of the mixed EHL stage was the transition from the mixed EHL stage of relatively thick overall mixed EHL film to the mixed EHL stage of oxidized chemical boundary layer lubrication mainly and widely occurring in the Hertzian zone. On the mixed EHL stage of relatively thick overall mixed EHL film, the lubricating films in the mixed EHL contact are mainly the hydrodynamic fluid films. These fluid films may respectively be locally continuum and non-continuum across the fluid film thickness in different zones of the fluid film lubricated area in the whole mixed EHL contact. On this mixed EHL stage, the oxidized chemical boundary layer lubrication or/and the dry contact between the rough contact surfaces can respectively locally occur in different and small areas of the whole mixed EHL contact depending on the operating condition. This oxidized chemical boundary layer lubrication occurred due to the local disappearance of the fluid films between the opposing asperities of the contact surfaces in small areas of the whole mixed EHL contact and the subsequent oxidization of the fresh metal surfaces of the contact of these small areas. The local dry contact between the contact surfaces occurring on this mixed EHL stage was between the fresh metal surfaces, instant and quickly replaced by the oxidized chemical boundary layer lubrication between these contact surfaces due to the oxidization of this dry contact by the oxygen within the fluid. However, these small areas of oxidized chemical boundary layer lubrication and dry contact occurring in the mixed EHL contact had significant effects on the first primary transition of the mixed EHL stage.

On the mixed EHL stage of relatively thick overall mixed EHL film, the friction coefficient of the whole mixed EHL contact is on the scale of *0.1* and the wear rate of the whole mixed EHL contact is low. These values of the friction coefficient and wear rate well reflect the lubrication regimes occurring within the mixed EHL contact of this mixed EHL stage and are the distinguishing features of this mixed EHL stage. This friction coefficient usually generates significant frictional heating within the mixed EHL contact of this mixed EHL stage. The fictional heating effect is usually also significant in the mixed EHL contact of this mixed EHL stage. This wear rate is usually unable to cause in a short time the failure of the mixed EHL contact of this mixed EHL stage.

The mixed EHL stage of oxidized chemical boundary layer lubrication mainly and widely occurring in the Hertzian contact zone was transited from the mixed EHL stage of relatively thick overall mixed EHL film when the hydrodynamic fluid films locally disappeared between the opposing asperities of the contact surfaces in wide areas of the Hertzian contact zone and subsequently the oxidization of the fresh metal surfaces of the contact and then the oxidized chemical boundary layer lubrication occurred in these areas on the mixed EHL stage of relatively thick overall mixed EHL film. On the mixed EHL stage of oxidized chemical boundary layer lubrication mainly and widely occurring in the Hertzian contact zone. In the Hertzian contact zone, the oxidized chemical boundary layer lubrication in wide areas mainly occurs simultaneously and thus is mainly mixed with the hydrodynamic fluid film lubrication in small areas where the fluid may be continuum or non-continuum across the fluid film thickness depending on the fluid film thickness between the rough contact surfaces. On this mixed EHL stage, in the inlet zone the continuum hydrodynamic fluid film lubrication mainly occurs due to the relatively high fluid film thickness between the rough contact surfaces. In this inlet zone, the non-continuum fluid film lubrication or/and the oxidized chemical boundary layer lubrication and even the dry contact between the opposing asperities of the contact surfaces may simultaneously locally occur in considerably small areas between the opposing asperities of the contact surfaces and thus be mixed with the continuum hydrodynamic fluid films in the other areas.

On the mixed EHL stage of oxidized chemical boundary layer lubrication mainly and widely occurrs in the Hertzian contact zone, the friction coefficient of the whole mixed EHL contact is typically on the scale of *0.35* and the wear rate of the whole mixed EHL contact is still relatively low. These values of the friction coefficient and wear rate well reflect the lubrication regimes occurring within the mixed EHL contact of this mixed EHL stage and they are the distinguishing features of this mixed EHL stage. This friction coefficient usually causes severe frictional heating and thus severe energy loss within the mixed EHL contact of this mixed EHL stage. The frictional heating effect is usually significant in the mixed EHL contact of this mixed EHL stage. This wear rate is usually unable to cause in a short time the failure of the mixed EHL contact of this mixed EHL stage.

(2) Second primary transition of the mixed EHL stage

The second primary transition of the mixed EHL stage was the transition from the mixed EHL stage of oxidized chemical boundary layer lubrication mainly and widely occurring in the Hertzian contact zone to the severe removal of the oxidized chemical boundary layer from the contact surfaces i.e. the destruction of the contact surfaces occurrs after the first primary transition of the mixed EHL stage. The process and mechanism of the removal of the oxidized chemical boundary layer from the contact surfaces on the second primary transition of the mixed EHL stage are still not clear. For a given mixed EHL contact, the second primary transition of the mixed EHL stage usually occurs in much more severe operating conditions i.e. for much heavier carried loads or much higher sliding speeds of the contact or much higher bulk fluid temperatures than the first primary transition of the mixed EHL stage.

On the mixed EHL stage of the severe removal of the oxidized chemical boundary layer from the contact surfaces i.e. the destruction of the contact surfaces, the friction coefficient of the whole mixed EHL contact is typically on the scale of *0.35* and the wear rate of the whole mixed EHL contact is typically very high (and about one hundred times higher than that before the occurrence of this mixed EHL stage). These values of the friction coefficient and wear rate well reflect the phenomena occurring within the mixed EHL contact of this mixed EHL stage and they are the distinguishing features of this mixed EHL stage. This friction coefficient usually causes severe frictional heating and severe energy loss within the mixed EHL contact of this mixed EHL stage. This wear rate usually causes quickly the failure of the mixed EHL contact of this mixed EHL stage.

(3) Third primary transition of the mixed EHL stage

The third primary transition of the mixed EHL stage was the transition from the mixed EHL stage of relatively thick overall mixed EHL film to the severe removal of the oxidized chemical boundary layer from the contact surfaces i.e. the destruction of the contact surfaces. This mixed EHL stage transition occurs for relatively high sliding speeds of the mixed EHL contact. This mixed EHL stage transition is caused by the local fluid film disappearance in wide areas of the Hertzian contact zone and the metallurgical transformation of the contact surfaces in these areas at high contact surface temperatures.

2.2.1.2.2 Effects of the factors within the mixed EHL contact on the mixed EHL stage transitions and the destruction of the mixed EHL contact

Begelinger and Gee [18, 19] found that the roughness of the mixed EHL contact and the oxygen content of the fluid both have significant effects on the mixed EHL stage transitions and the destruction of the mixed EHL contact. Tabor [20] found that the roughness of the mixed EHL contact, the oxygen content of the fluid, the bulk fluid viscosity and the chemical composition of the fluid all have significant effects on the mixed EHL stage transitions and the destruction of the mixed EHL contact. The effects of these factors in the mixed EHL contact are summarized as follows according to their experiments.

(1) Effect of the roughness of the mixed EHL contact on the mixed EHL stage transitions and the destruction of the mixed EHL contact

It was found by Begelinger and Gee [18, 19] that in the mixed EHL contact, if the oxidization of the contact surfaces can not keep ahead of the formation of the contact of the opposing asperities of the contact surfaces, the newly formed contact of the opposing asperities of the contact surfaces will be in dry contact instead of in oxidized chemical boundary layer lubrication due to the non oxidization of the fresh metal surfaces within this asperity contact. This dry contact of asperity will quickly cause the severe wear or/and scuffing and then the failure of the mixed EHL contact. It was found by them [18, 19] that the increase of the manufactured contact surface roughness of the mixed EHL contact or/and the reduction of the oxygen content of the fluid both result in the transition of the mixed EHL into the stage of thinner, more mixed and more partial lubricating films in the mixed EHL contact and stimulate the destruction of the mixed EHL contact. It is found from their experiments that the local dry contact or/and the local oxidized chemical boundary layer lubrication between the opposing asperities of the contact surfaces can respectively occur simultaneously in the mixed EHL contact and thus be mixed with the hydrodynamic fluid films in other areas of the contact in the mixed EHL contact due to the manufactured contact surface roughness of the mixed EHL contact and the oxygen contained within the fluid. Therefore, because of the oxidization of the fresh metal surfaces and the oxidized chemical boundary layer formed on the contact surfaces due to this oxidization occurring locally in a practical mixed EHL contact, the mode of a real mixed EHL is actually the mode of mixed EHL in which the hydrodynamic fluid film, the oxidized chemical boundary layer lubrication and the dry contact between the opposing asperities of the contact surfaces respectively occur locally and simultaneously in different areas of the whole mixed EHL contact. They are thus mixed in the whole mixed EHL contact depending on the operation condition, i.e. with mixed lubricating films, instead of the conventional defined mode of mixed EHL [33], in which only the Newtonian hydrodynamic fluid film in the lubricated contact was present. It is clear that Zhang's mode of mixed EHL [21, 29, 30, 31] reviewed in section 2.2.1.1.5 Essentially agrees with the mode of a real mixed EHL. Therefore it is also directly proven that the more generalized mode of mixed EHL i.e. the future mode of mixed EHL proposed in the present paper in the section of Introduction essentially fits the mode of a real mixed EHL. The potential value of the more generalized mode of mixed EHL proposed in the present paper is obvious. It must turn at the moment the direction of the theoretical modelling of mixed EHL to this valuable mode of mixed EHL.

It was concluded from their experiments that the contact surface roughness has significant effects on the first and the third primary transitions of the mixed EHL stage of a real mixed EHL contact, while it has no effect on the second primary transition of the mixed EHL stage of a real mixed EHL contact.

Also, as reviewed and commented in the section of Introduction, the contact surface roughness effect on the mixed EHL stage transitions in a real mixed EHL contact significant especially for low sliding speeds of the mixed EHL contact is unable to be predicted from the mixed EHL modelling based on the Newtonian fluid model and may be due to the fluid non-Newtonian behavior in a real mixed EHL contact.

(2) Effect of the oxygen content of the fluid on the mixed EHL stage transitions and the destruction of the mixed EHL contact

The contribution of the oxygen content of the fluid to a mixed EHL is practically mainly the oxidization of the fresh metal surfaces within the dry contact of asperity by the oxygen contained within the fluid and thus the consequent oxidized chemical boundary layers are formed on the contact surfaces due to this oxidization which locally occurrs in a real mixed EHL contact. The formation of this oxidized chemical boundary layer and thus its lubrication between the opposing asperities of the contact surfaces locally occurring in a real mixed EHL contact causes the friction, especially the wear of these opposing asperities is much lower than of these asperities if these opposing asperities were in dry contact with one another. By this way, this formed oxidized chemical boundary layer lubrication between the local opposing asperities of the contact surfaces in a real mixed EHL contact keeps the friction and especially the wear of the whole mixed EHL contact in a tolerable level and thus maintains the operation of the whole mixed EHL contact in continued and considerably longer time. However, in spite of this boundary lubrication formed within a real mixed EHL contact, the frictional heating and thus the energy loss in this mixed EHL contact are usually significant especially when this boundary lubrication widely occurs in wide areas of the Hertzian contact zone. The oxygen content of the fluid has significant effects on the formation of this boundary lubrication in a real mixed EHL contact. Thus effecting the performances of this boundary lubrication in a real mixed EHL contact and effecting the whole mixed EHL contact.

It was concluded from the experiments of Begelinger and Gee [18, 19] and Tabor [20] that the oxygen content of the fluid has significant effects on the first and the third primary transitions of the mixed EHL stage of a practical mixed EHL contact, while it has no effect on the second primary transition of the mixed EHL stage of a practical mixed EHL contact.

(3) Effect of the bulk fluid viscosity on the mixed EHL stage transitions and the destruction of the mixed EHL contact

It is well known that the fluid viscosity in the EHL inlet zone has a strong effect on the overall EHL film thickness, since the EHL inlet zone determines the overall EHL film thickness. Therefore, the bulk fluid viscosity (at ambient condition) has strong effects on the overall EHL film thickness and the EHL load-carrying capacity. Since the inlet zone of the mixed EHL contact still determines the overall fluid film thickness of the mixed EHL contact and in most areas of the inlet zone of the mixed EHL contact is present the continuum hydrodynamic fluid film when the first primary transition of the mixed EHL stage starts to occur due to the hydrodynamic fluid film disappearance which locally occurrs between the opposing asperities of the contact surfaces mainly in the Hertzian contact zone and then quickly ends in the mixed EHL contact. The bulk fluid viscosity still has an important and determinative effect on the load-carrying capacity of the mixed EHL contact on the first primary transition of the mixed EHL stage and on the first primary transition of the mixed EHL stage of a practical mixed EHL contact. Tabor [20] obtained that in a practical mixed EHL contact, for a given sliding speed of the contact, the load-carrying capacity of the mixed EHL contact, i.e. the maximum Hertzian contact pressure of the mixed EHL contact on the first primary transition of the mixed EHL stage has the following relationship with the bulk fluid viscosity: $p_1=\beta log\eta+\alpha$ where η is the bulk fluid viscosity, α is the constant depending on the fluid chemical composition and β is a constant. This load-carrying capacity expression of the mixed EHL contact on the first primary transition of the mixed EHL stage is however unable to be described by not only conventional EHL theories but also by any of the previous mixed EHL modelling and theories. This shows that the mechanism of the bulk fluid viscosity effect on a practical mixed EHL is new and special. New mixed EHL theories therefore need to be developed to describe the bulk fluid viscosity effect on a practical mixed EHL for practically wide operating conditions.

It was concluded from the experiment of Tabor [20] that the bulk fluid viscosity has significant effects on the first and the third primary transitions of the mixed EHL stage of a practical mixed EHL contact, while it has no effect on the second primary transition of the mixed EHL stage of a practical mixed EHL contact. It was also concluded by Tabor [20] that the bulk fluid viscosity can be used as one of the criteria to judge the performance of the fluid in a practical mixed EHL contact.

(4) Effect of the chemical composition of the fluid on the mixed EHL stage transitions and the destruction of the mixed EHL contact

The contribution of the chemical composition of the fluid to a mixed EHL is also mainly the formation of the chemical boundary layer on the contact surfaces within the dry contact of asperity due to the chemical composition of the fluid and the consequent chemical boundary layer lubrication between these opposing asperities of the contact surfaces by this formed chemical boundary layer locally occurring in a real mixed EHL contact. The property of this formed chemical boundary layer and the performance of this boundary lubrication locally occurring in a real mixed EHL contact heavily rely on the chemical composition of the fluid. It was found from the experiment of Tabor [20] that the chemical boundary layer lubrication between the opposing asperities of the contact surfaces locally occurring in a real mixed EHL contact formed by the chemical composition of the fluid practically has significant effects on the performance of the mixed EHL contact on the mixed EHL stage of relatively thick overall mixed EHL film and the first and the third primary transitions of the mixed EHL stage of a practical mixed EHL contact. It was also found experimentally that the chemical boundary layer on the contact surfaces between the opposing asperities of the contact surfaces formed by the chemical composition of the fluid has significant effects on the performance of the mixed EHL contact, on the mixed EHL stage of chemical boundary layer lubrication, which mainly and widely occurr in the Hertzian contact zone occurring after the first primary transition of the mixed EHL stage of the mixed EHL contact and the second primary transition of the mixed EHL stage of a practical mixed EHL contact, i.e. the removal of this chemical boundary layer from the contact surfaces and the destruction of the contact surfaces in a practical mixed EHL contact. Therefore, the chemical composition of the fluid always has important effects on the performance of a practical mixed EHL contact and on the transition of the mixed EHL stage of a practical mixed EHL contact.

As shown in the previous section, Tabor obtained from his experiment [20] that the load-carrying capacity i.e. the maximum Hertzian contact pressure of the mixed EHL contact on the first primary transition of the mixed EHL stage is directly linearly proportional to the parameter α which depends on the chemical composition of the fluid. This load-carrying capacity expression of the mixed EHL contact on the first primary transition of the mixed EHL stage is obviously unable to be explained by any of the previous EHL and mixed EHL theories. New mixed EHL theories therefore need to be developed to describe the performance of a practical mixed EHL contact by incorporating the effect of the chemical composition of the fluid for a general operating condition.

It was concluded by Tabor [20] that the chemical composition of the fluid can be used as another criterion to judge the performance of the fluid in a practical mixed EHL contact.

(5) Effect of other factors within the mixed EHL contact on the performance of a practical mixed EHL contact

Even in the mixed EHL contact on the mixed EHL stage of relatively thick overall mixed EHL film, although the contact surfaces of the mixed EHL contact are in running-in in the macroscopic view, the wear of these contact surfaces practically occurs locally or in the microscopic area of the whole mixed EHL contact. In another word, the wear of the contact surfaces usually occurs locally in a practical mixed EHL contact. Therefore, the wear debris due to this wear usually occurs within a practical mixed EHL contact. It is necessary to study this kind of mixed EHL, the wear of the contact surfaces in this mode of mixed EHL and the effect of the debris of the wear of the contact surfaces of mixed EHL.

2.2.2 Characteristic of the academic research of mixed EHL in the past time

2.2.2.1 Characteristic of the theoretical modelling of mixed EHL in the past time

Before 1990s, the theoretical modelling of mixed EHL as represented by the references cited in the section of Introduction was qualitatively preliminary and on a relatively low level. The mixed EHL modelling and theories in that period obviously have large deviations from the reality of a practical mixed EHL whenever their approaches are stochastic or deterministic. This is especially obvious since those mixed EHL modelling and theories all neglects modelling the phenomena of the friction and wear usually significantly occurring in a practical mixed EHL contact, which are important to a practical mixed EHL contact. The main reason for this is however actually that the modes of lubrication taken in those mixed EHL modelling and theories are oversimplified compared to the mode of a practical mixed EHL. Those modes of mixed EHL taken in the mixed EHL modelling are in the present paper classified as one kind of mode of mixed EHL and defined as the classical mode of mixed EHL in the following section. The essential features of those modes of mixed EHL i.e. the theoretical modelling of mixed EHL before 1990s are that the hydrodynamic fluid film occurs in the entire area of the whole mixed EHL contact and is relatively thick. The hydrodynamic fluid film in those modes of mixed EHL is therefore also continuum across the fluid film thickness due to the relatively thick fluid film thickness in the entire area of the whole mixed EHL contact. The lubricating films in those modes of mixed EHL are actually qualitatively not thinning, mixed or partial lubricating films in a mixed EHL contact. Those lubricating films are therefore not those typically occurring in a practical mixed EHL contact which are usually thinning, mixed and partial in the whole mixed EHL contact, as reviewed and described in sections 2.1 and 2.2.1.2. The features of the theoretical modelling of mixed EHL before 1990s do therefore qualitatively not fit the essential features of a practical mixed EHL contact as described in sections 2.1 and 2.2.1.2. The theoretical results obtained from those mixed EHL modelling are therefore rather limited in their predictive and application values to a practical mixed EHL contact, though they may be of indicative values. The classical mode of mixed EHL taken in the mixed EHL modelling before 1990s should be concluded as a historical mark of the theoretical modelling of mixed EHL.

In the beginning of this century, the mode of mixed EHL taken in the EHL modelling may be the mode of mixed EHL proposed and modelled by Zhang and his colleagues [21, 29, 30, 31] in the theoretical study of EHL. The theoretical modelling of mixed EHL in that period may be the researches of the mixed EHL by those authors by mathematical modelling as reviewed and described in section 2.2.1.1.5. The features of the mixed EHL modelling by those authors in that period have the features of the modern mode of mixed EHL. The essential features of the modern mode of mixed EHL are that the lubricating films with different rheological behaviors and the dry contact between the contact surfaces respectively occur locally simultaneously in different areas of the whole mixed EHL contact. They are thus mixed in the whole mixed EHL contact depending on the operating condition due to the severe thinning of the overall and local lubricating films and the disappearance of the local lubricating film in a mixed EHL contact. Therefore, in the modern mode of mixed EHL, each kind of lubricating film with a certain rheological behavior is in essence also partial in the whole mixed EHL contact depending on the operating condition. The configuration of the modern mode of mixed EHL well agrees with the mode of a practical mixed EHL as reviewed and described in sections 2.1 and 2.2.1.2. The modern mode of mixed EHL is of substantial progress in the theoretical study of mixed EHL compared to the classical mode of mixed EHL. The theoretical results of mixed EHL obtained from the modern mode of mixed EHL are consequently of substantial progress compared to the theoretical results of mixed EHL obtained from the classical mode of mixed EHL. However, the effects of the contact surface roughness, the frictional heating of the whole mixed EHL contact and the lubricating film non-Newtonian rheological behavior in a mixed EHL contact all are not well incorporated by the modern mode of mixed EHL.

In concluding remarks, the theoretical modelling of mixed EHL and the modes of mixed EHL taken in this modelling developed in the past time are not mature. The modern mode of mixed EHL also has considerable deviations from the reality of a practical mixed EHL contact due to neglecting the important factors to a practical mixed EHL contact verified by experiments and reviewed and described in section 2.2.1.2.2. The mode of mixed EHL taken in the mixed EHL modelling now needs to be further developed into the more generalized mode of mixed EHL proposed in the present paper in the above section, i.e. the future mode of mixed EHL by incorporating the features of the modern mode of mixed EHL as discussed in the above sections. In the future mode of mixed EHL, the combined effects of the contact surface roughness, the frictional heating of the whole mixed EHL contact and the lubricating film non-Newtonian rheological behavior in a mixed EHL contact need to be considered in detail and well incorporated.

2.2.2.2 Characteristic of the experimental research of mixed EHL in the past time

Although the experimental researches of mixed EHL in the past time are much fewer than the theoretical modelling of mixed EHL in the past time, the experimental results of mixed EHL in simple sliding point contacts of steel typically obtained in 1970s by Begelinger and Gee [18, 19] and by Tabor [20] are of substantial progress and particular interest in the study of mixed EHL. Their experimental results of mixed EHL are indicative to both the theoretical modelling of mixed EHL for a real mixed EHL contact and the experimental study of mixed EHL in the following time. However, the operating condition of mixed EHL in an experiment is particular and usually oversimplified compared to that of a practical mixed EHL. The experiments of mixed EHL of Begelinger and Gee [18, 19] and of Tabor [20] were standardized with the restricted same experimental conditions. The experimental results of mixed EHL obtained in the past time representatively by Begelinger and Gee [18, 19] and by Tabor [20] only reflect one part of the reality of a practical mixed EHL contact. On the other hand, the experimental results of mixed EHL obtained in the past time on their addressed subjects may need to be further refined and improved in the future. The experimental results of mixed EHL in the past time may still be unable to fully reveal the important phenomena occurring in and to a practical mixed EHL contact. The experiment of mixed EHL is a weak area of the research of mixed EHL now. It needs to be carried out much more extensively and intensively with thrusting efforts in the future to reveal the mode of a real mixed EHL contact and to verify the mode of mixed EHL proposed in the theoretical modelling of mixed EHL in proceeding time.

2.3 Commenting remarks

Unfortunately, the theoretical modelling, the experimental study and the industrial development of mixed EHL in the past time were not found to well collaborate and cooperate with each other. In the past time, the theoretical modelling of mixed EHL left the experimental verification and obviously had great deviations from the reality of the mixed EHL developed in the industry. It more appeared to be an art practising academic skills in a long time with complicated mathematics which was difficult to understand, accept and apply but was indeed of very limited value. This is not helpful to the progress of both the theoretical and experimental researches of mixed EHL. It should be taken as a lesson in the theoretical research of mixed EHL in the future. In the future, the theoretical modelling of mixed EHL should more concentrate on a real mixed EHL contact rather than the artificial mixed EHL contact fabricated by the construction and design from one's minds but of no proof. It is necessary that the theoretical modelling of mixed EHL in the future is mandated to look for the verification from the reality.

In the past time, the experimental study of mixed EHL representatively carried out by Begelinger and Gee [18, 19] and by Tabor [20] avoided the theoretical explanation and proof of its obtained phenomena and results of mixed EHL. The understanding, recognition and value of the mixed EHL results obtained by those experimental studies are therefore considerably limited. The experimental study of mixed EHL ought to seek its obtained results explained and substantiated by the theoretical modelling of mixed EHL. The experimental study of mixed EHL and the theoretical modelling of mixed EHL should be carried out parallel and correlated with one another.

In the past time, as a whole, the industrial development and results of mixed EHL were not well respected and considered by the academic study of mixed EHL and especially by the theoretical modelling of mixed EHL. In the past time, the experimental study of mixed EHL also did not well involve a lot of the important phenomena and cover all these phenomena occurring in the mixed EHL contact in the industry. The theoretical modelling of mixed EHL and the experimental study of mixed EHL both should be based on the observation and results of mixed EHL in the industry and satisfy the requirement of the industry to directly improve the performance of the mixed EHL contact in the industry. The industrial development of mixed EHL should be the core that the theoretical modelling and the experimental study of mixed EHL both only concentrate on and serve.

3. MODES OF MIXED EHL IN THE PAST TIME

As described in section 2.2.2.1, the modes of mixed EHL taken in the theoretical modelling of mixed EHL in the past time are classified as two kinds of modes of mixed EHL, i.e. the classical mode of mixed EHL and the modern mode of mixed EHL. These two classified modes of mixed EHL are respectively more detailed described in the following sections.

3.1 Classical mode of mixed EHL

As described in section 2.2.2.1, the modes of mixed EHL taken in the theoretical modelling of mixed EHL before 1990s all belong to the classical mode of mixed EHL. The classical mode of mixed EHL is defined as the mode of mixed EHL in which the hydrodynamic fluid film occurs in the entire area of the whole mixed EHL contact and is relatively thick and a continuum across the fluid film thickness in the whole mixed EHL contact. This mode of mixed EHL does not hold any of the essential features of a practical mixed EHL contact which are thinning, mixed and partial lubricating films in the whole mixed EHL contact as reviewed and described in sections 2.1 and 2.2.1.2. This mode of mixed EHL therefore does not fit qualitatively the mode of a practical mixed EHL. It is oversimplified compared to a practical mixed EHL and should be rejected in the mixed EHL modelling now. Figure 1 representatively pictures the classical mode of mixed EHL and its essential features.

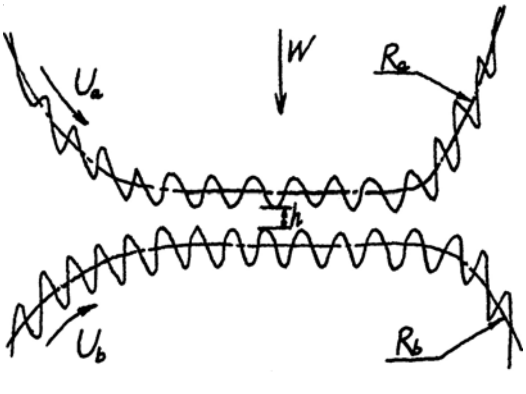

Fig. 1 Representative description of the Classical mode of mixed EHL (h is EHL film thickness between the rough contact surfaces on any location in the lubricated contact; $h > h_{cr,ncf}$ is the critical thickness of the non-continuum fluid film. The fluid film between the rough contact surfaces is relatively thick and thus continuum across the fluid film thickness in the whole mixed EHL contact)

In the classical mode of mixed EHL, the operating condition of a mixed EHL contact is thermal or isothermal, the fluid is Newtonian or non-Newtonian. In this mode of mixed EHL, the contact surface roughness is artificial or real. The mixed EHL contact is line contact or point contact.

3.2 Modern mode of mixed EHL

As described in section 2.2.2.1, the representative modes of mixed EHL taken in the EHL modelling in the beginning of this century are the modern mode of mixed EHL. The modern mode of mixed EHL is a mode of mixed EHL proposed and taken in the EHL modelling in that period by Zhang and his colleagues [21, 29, 30, 31] as reviewed and described in section 2.2.1.1.5. The features of the modes of mixed EHL of the authors in that period form the features of the modern mode of mixed EHL. The features of the modern mode of mixed EHL have been described in section 2.2.2.1 and here are not repeated for simplicity. As reviewed and described in sections 2.1 and 2.2.1.2, the configuration of the modern mode of mixed EHL well agrees with the mode of a practical mixed EHL. The modern mode of mixed EHL is thus much better than the classical mode of mixed EHL. As commented in section 2.2.2.1, the modern mode of mixed EHL also has its limitations of applicability to a practical mixed EHL contact due to its shortcomings. It actually needs to be revised as the future mode of mixed EHL for better fitting the mode of a practical mixed EHL.

Figure 2 representatively pictures the modern mode of mixed EHL and its essential features where the operating condition of a mixed EHL contact is thermal or isothermal, and the fluid is Newtonian or non-Newtonian. In this mode of mixed EHL, the contact surface roughness is artificial or real. The mixed EHL contact is line contact or point contact.

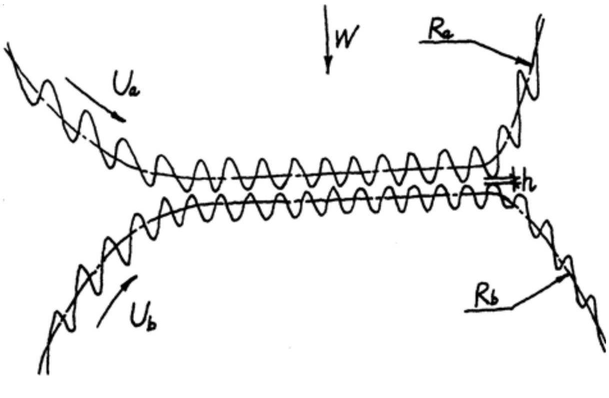

Fig. 2 Modern mode of mixed EHL (h is EHL film thickness between the rough contact surfaces on any location in the lubricated contact. The fluid film between the rough contact surfaces is molecularly thin on some separate locations of the contact when $0<h<h_{cr,ncf}$ on these locations, here $h_{cr,ncf}$ is the critical thickness of the non-continuum fluid film; or, dry contact occurs on some separate locations of the contact when h=0 on these locations. In the other zones of the contact, where $h>h_{cr,ncf}$, the fluid film is relatively thick and thus continuum across the fluid film thickness and the fluid rheological behavior is Newtonian, (shear-thinning) viscoelastic or viscoplastic depending on the operating condition)

4. INDICATIVE RESULTS OF THE MIXED EHL RESPECTIVELY OBTAINED BY THE THEORETICAL MODELLING OF MIXED EHL AND BY THE EXPERIMENTAL STUDY IN THE PAST TIME

Although the theoretical modelling of mixed EHL and the experimental study of mixed EHL in the past time both are oversimplified compared to a practical mixed EHL, some of the results of mixed EHL obtained by those studies may be indicative and useful to the researches of mixed EHL in the future time. This section reviews the fundamental conclusions on mixed EHL drawn by those studies concerning the effects of the fluid rheology, the contact surface roughness and the frictional heating of the mixed EHL contact in a mixed EHL. The thinning, mixed and partial fluid films in a mixed EHL due to these effects found by those studies are also reviewed in this section. The final review in this section is given to the dry contact between the contact surfaces in a mixed EHL contact obtained by the experimental study of mixed EHL and modelled by the theoretical modelling of mixed EHL in the past time.

4.1 Effects of the fluid rheology, the contact surface roughness and the fluid film viscous heating on mixed EHL

4.1.1 Effect of the fluid rheology on mixed EHL

As reviewed in the section of Introduction, the fluid rheology has significant effects on mixed EHL film pressure and film thickness. The fluid Newtonian effect usually causes significant fluid film pressure ripples and consequently almost removes the manufactured contact surface roughness in a mixed EHL. Oppositely, the fluid shear-thinning non-Newtonian effect reduces and even removes fluid film pressure ripples and consequently makes the contact surface roughness retain and even all persist. It was analytically shown by references [7, 8] that the Newtonian and shear-thinning non-Newtonian mixed EHL results are qualitatively different. It has actually been understood from the theoretical modelling of mixed EHL in the past time as reviewed in the section of Introduction that in a mixed EHL, the reactions of the fluid film pressure and then the fluid film thickness distribution to the manufactured roughness of the contact surfaces intimately rely on the fluid rheology. Therefore, a more realistic and more precise fluid rheological model is important to the theoretical modelling of mixed EHL. Since the fluid is usually essentially shear-thinning non-Newtonian in EHL contacts, the Newtonian fluid model has been concluded to be usually inappropriate for the mixed EHL modelling. In the mixed EHL modelling, the primary task is to find the correct fluid model for the modelled case instead of to arbitrarily choose the Newtonian fluid model for any cases regardless of the operating condition in the modelled case.

Johnson and Tavaarwerk [42] experimentally found that the viscous behavior of most fluids in EHL contacts usually follows the fluid Ree-Eyring model. Incorporating the elastic behavior of shear, they obtained the Ree-Eyring fluid model for typical and wide elastohydrodynamic fluids based on primary laboratory data. It was found that their obtained Ree-Eyring fluid model is accurate for low and medium shear strain rates, however is significantly erroneous for high shear strain rates. Later, Bair and Winer [43] found that the fluid shear strength plays and determines the fluid rheological behavior at high shear strain rates of the fluid in EHL contacts. Incorporating the fluid shear strength parameter, they obtained the logarithmic shear-thinning fluid model accurate for low, medium and high shear strain rates for typical and wide elastohydrodynamic fluids based on primary laboratory data. However, it was found from numerical calculation that the Bair and Winer's fluid model is not easy to handle when it is used in the EHL modelling. Later, Zhang [29] combined the features of the Johnson and Tavaarwerk's and Bair and Winer's fluid models to develop a new fluid model for typical and wide elastohydrodynamic fluids for the EHL modelling. The fluid model obtained by Zhang [29] has the feature of the fluid shear thinning at low and medium shear strain rates of the fluid obeying the Ree-Eyring fluid model. It has the feature of the fluid shear thinning at high shear strain rates of the fluid determined by the fluid shear strength obeying the Bair and Winer's fluid model. It was found that the fluid model obtained by Zhang [29] is applicable for low, medium and high shear strain rates of the fluid in EHL. The fluid model obtained by Zhang [29] is therefore usually utilizable in the modelling of EHL for wide operating conditions. Zhang's fluid model [29] has the advantage of the convenience of being handled when it is used in the EHL modelling. Furthermore, Zhang [29] extended their fluid model to the fluid of molecular-scale fluid film thickness in EHL. The fluid model obtained by Zhang [29] is therefore utilizable not only for the relatively thick fluid film but also for the molecularly thin fluid film in EHL. Zhang's fluid model [29] is utilizable in modelling the EHL of mixed rheologies where non-continuum fluid, viscoelastic continuum fluid and viscoplastic continuum fluid respectively simultaneously locally occur in different areas of the whole contact. Presently, Zhang's fluid model [29] is especially of particular application value to the mixed EHL of mixed rheologies where the fluid film is locally molecularly thin while is relatively thick in other areas of the whole contact, i.e. the modern and future modes of mixed EHL as described in the above section, since there are presently no other valid fluid models for this mode of mixed EHL while the modelling of mixed EHL usually has a high demanding on the accuracy of the fluid model.

Due to its application values to the modelling of the classical, modern and future modes of mixed EHL, the fluid model obtained by Zhang [29] is here written as:

$$\begin{cases} \dot{\gamma} = \dfrac{1}{G_{ncf}^{eff}(p,h)}\dfrac{d\tau}{dt} + \dfrac{1}{\eta_{ncf}^{eff}(p,h)}\tau_0(p,h)\sinh[\dfrac{\tau}{\tau_0(p,h)}] & \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for } |\tau| < \tau_l(p,h) & \\ \tau = \mathrm{sign}(\dot{\gamma})\tau_l(p,h) & \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for } |\tau| \geq \tau_l(p,h) & \end{cases} \tag{1}$$

where $\dot{\gamma}$ is the fluid film shear strain rate, τ is the fluid film shear stress, t is time, p is the fluid film pressure, h is the fluid film thickness, G_{ncf}^{eff}, η_{ncf}^{eff} and $\tau_{l,ncf}$ are respectively the shear modulus of elasticity, the viscosity and the shear strength of the fluid which is either non-continuum or continuum across the fluid film thickness, $\tau_0(p,h)=\tau_{l,ncf}(p,h)/2.8$, within the fluid film $\tau_l(p,h)=\tau_{l,ncf}(p,h)$, and at the bulk fluid-contact surface adhering layer interface $\tau_l(p,h)=\tau^t_{l,ncf}(p,h)$ and $\tau^t_{l,ncf}$ is the shear strength of this interface. The detailed interpretations of the parameters appearing in Eq. (1) can be found in reference [29]. The fluid model formulated by Eq. (1) is valid for both the continuum fluid and non-continuum fluid across the fluid film thickness in EHL.

Due to the sensitivity of mixed EHL results obtained by the theoretical modelling of mixed EHL to the fluid model used in this modelling, equation (1) is recommended to be used in the modelling of the present more extended and generalized mode of mixed EHL for a practical mixed EHL contact in which the molecularly thin fluid film, the continuum viscoelastic fluid film, the continuum viscoplastic fluid film, the oxidized chemical boundary layer lubrication and the dry contact between the contact surfaces respectively simultaneously locally occur in different areas of the whole contact and are thus mixed in the whole contact depending on the operating condition, i.e. the modern and future modes of mixed EHL as described in the above section. Equation (1) can be used to model all the fluids with different rheological behaviors respectively simultaneously locally occurring in different areas of the whole contact in these modes of mixed EHL. This would settle the fluid model used in the modelling of the modern and future modes of mixed EHL.

4.1.2 Effect of the contact surface roughness on mixed EHL

According to the theoretical modelling of mixed EHL in the past time as reviewed in the section of Introduction, the contact surface roughness has significant effects on mixed EHL film pressure or/and mixed EHL film thickness. The contact surface roughness effect in mixed EHL intimately relies on the fluid rheology. As reviewed in section 4.1.1, for the Newtonian fluid, the contact surface roughness effect usually causes significant fluid film pressure ripples while greatly removes the manufactured contact surface roughness in a mixed EHL. For the shear-thinning fluid, the contact surface roughness effect usually causes much reduced fluid film pressure ripples while makes the manufactured contact surface roughness retain and even all persist in a mixed EHL. It has been settled that the fluid in EHL is usually essentially shear-thinning non-Newtonian and the contact surface roughness effect in a practical mixed EHL is that for the shear-thinning fluid. However, for the usual shear-thinning fluid, the contact surface roughness effect usually still causes considerable fluid film pressure ripples while makes the manufactured contact surface roughness largely persist in a mixed EHL especially in severe operating conditions according to the mixed EHL results obtained from the theoretical modelling by deterministic approach. As have been known from the researches on the failures of EHL films and EHL contact surfaces in the past time, these generated fluid film pressure ripples and persisting contact surface roughness are important and even critical to the occurrence of the failures of EHL films and EHL contact surfaces in a mixed EHL. As reviewed in the section of Introduction, the stochastic approach to a mixed EHL is unable to give the transient detailed fluid film pressure ripples and the transient detailed persisting contact surface roughness due to the contact surface roughness effect in a mixed EHL contact. The deterministic approach to a mixed EHL is however able to give the transient fluid film pressure ripples and the transient persisting contact surface roughness due to the contact surface roughness effect for any given instant in a mixed EHL contact. However, the accuracy and details of both the fluid film pressure ripples and the transient persisting contact surface roughness obtained by the deterministic approach for a mixed EHL are determined by the accuracy of the contact surface roughness simulated in the computer modelling of mixed EHL. For predicting the performance of an actual mixed EHL contact, the accuracy and details of both the transient fluid film pressure ripples and the transient contact surface roughness in an actual mixed EHL contact are highly demanded for any instant. Therefore, principally, in the computer modelling of mixed EHL for an actual mixed EHL contact, the input data of the manufactured contact surface roughness of the mixed EHL contact are required to be detailed, fine and real.

Unfortunately, in the theoretical modelling of mixed EHL in the past time as reviewed in the section of Introduction, the contact surface roughness was usually taken as simple, artificial, idealistic and fabricated due to the limitations of the computer in that time on the storage of the input data of the manufactured contact surface roughness of the mixed EHL contact and on the computation capacity. The contact surface roughness taken in the simulation of mixed EHL in the past time was usually continuous and at least first-order smooth expressed by mathematical equations in the whole mixed EHL contact. The form of the contact surface roughness in that mixed EHL simulation was usually simple, artificial and expressed by mathematical functions. The direction of the contact surface roughness in that mixed EHL simulation was usually line contact (two dimensional and transverse) surface roughness or point contact (three dimensional transverse, longitudinal or both transverse and longitudinal) surface roughness mathematically formulated and fabricated. The contact surface roughness taken in the past simulation of mixed EHL usually had the advantage of being easily handled in the computer storage and computation. However, it was unable to give detailed, fine and even realistic mixed EHL film pressure and mixed EHL film thickness distributions in that mixed EHL simulation. Therefore, the theoretical modelling of mixed EHL in the past time was usually unable to give detailed, fine and even realistic mixed EHL results, which contain the detailed information about the local EHL film pressure and its gradient, the local EHL film thickness and the local EHL fluid cavitation very important for the local EHL film failure and the local EHL contact surface failure and are thus important to the performance of a mixed EHL contact. The direction of the development of the theoretical modelling of mixed EHL must therefore be toward obtaining detailed, fine and realistic mixed EHL simulation results. The primary condition for realizing this direction is to use detailed, fine and real contact surface roughness in the modelling of mixed EHL.

The real manufactured roughness of the contact surfaces of a mixed EHL contact is always continuous, random, but not smooth. This roughness is usually unable to be expressed by mathematical equations. It usually can only be described by using its values on discretized points. In the simulation of mixed EHL, for obtaining detailed, fine and realistic mixed EHL results, discretized and a great number of points of roughness at the contact surfaces are required to be taken as input data for describing detailed, fine and real roughness of the contact surfaces of the mixed EHL contact. This however heavily increases the computer storage of these discretized roughness values and the difficulty of the computation of the simulation. One of the works of interest in the modelling of mixed EHL in the future time is to solve these conflicts.

4.1.3 Effect of the frictional heating of the mixed EHL contact on mixed EHL

In the mixed EHL where the hydrodynamic fluid film is fully generated in the whole mixed EHL contact, the frictional heating of the mixed EHL contact is caused by the fluid film viscous heating within the whole mixed EHL contact. For the mixed EHL where full hydrodynamic fluid films are generated in the whole mixed EHL contact, using the Newtonian fluid model and taking the contact surface roughness as measured, Cioc et al. [40] found from numerical simulation that the fluid film viscous heating effects on the mixed EHL film thickness and the mixed EHL film pressure both are modest in relatively severe operating conditions, while the fluid film viscous heating effects on the temperature rises both within the fluid film and at the contact surfaces in the mixed EHL are significant in relatively severe operating conditions. For this mixed EHL, using the shear-thinning circular fluid model and taking the measured contact surface roughness, they [40] also showed by numerical simulation that the fluid film viscous heating effect on the mixed EHL film pressure for the typical shear thinning fluid is considerably stronger than that for the Newtonian fluid in relatively severe operating conditions, however the fluid film viscous heating effect on the mixed EHL film thickness for the typical shear thinning fluid is only slightly different from that for the Newtonian fluid in relatively severe operating conditions. Zhang [15, 16] used the shear thinning non-Newtonian fluid model incorporating the contact-fluid interfacial shear strength and contact-fluid interfacial slip to analytically study the combined effect of the contact-fluid interfacial shear strength, the contact-fluid interfacial slippage and the fluid film viscous heating in thermal pure rolling EHL of ideally smooth line contacts lubricated by full hydrodynamic fluid films. He showed that this combined effect greatly reduces the overall EHL film thickness and the fluid film thermal instability even occurs and causes the disappearance of the fluid film in the Hertzian contact zone in this EHL in the condition of medium loads, medium rolling speeds and high bulk fluid temperatures when the contact-fluid interfacial shear strength in the inlet zone is significantly low. He also showed that in this EHL, the temperature rises both within the fluid film and at the contact surfaces caused by the fluid film viscous heating are considerably reduced by the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect compared to those for the Newtonian fluid. These references show that the fluid film viscous heating effect in EHL and mixed EHL intimately relies on the fluid rheological behavior in these EHL. Therefore, the real fluid model is important for studying the fluid film viscous heating effect in EHL and mixed EHL in the theoretical modelling of these EHL. According to this, the fluid model expressed by Eq. (1) is recommended as a more applicable fluid model for studying the more generalized mode of mixed EHL proposed in the present paper i.e. the future mode of mixed EHL in thermal condition by mathematical modelling than the other fluid models proposed in the previous time due to the best fitting to this mode of mixed EHL of this fluid model.

On the other hand, in a mixed EHL, the roughness of the contact surfaces also has a significant influence on the local fluid film viscous heating due to influencing the local fluid film thickness and then the local fluid film shear strain rate, shear stress and heat production rate especially in the condition of large slide-roll ratios. This influence of the contact surface roughness on the local fluid film viscous heating in a mixed EHL has a significant effect on the local fluid film pressure and the local fluid film thickness in a mixed EHL especially in severe operating conditions. The numerical simulation results of the mixed EHL in thermal line contacts of large slide-roll ratios obtained by Cioc et al. [40] substantiates these drawn points. Therefore, the real roughness of the contact surfaces of a mixed EHL contact is important for studying the fluid film viscous heating effect in a mixed EHL in the mixed EHL theoretical modelling. In this view point, in the modelling of the mixed EHL in thermal condition in the past time, the simple, artificial and idealistic roughness of the contact surfaces of the mixed EHL contact taken may give inaccurate thermal mixed EHL results for an actual mixed EHL contact especially for severe operating conditions. Oppositely, in the modelling of thermal mixed EHL for an actual mixed EHL contact, detailed, fine and real roughness of the contact surfaces of the mixed EHL contact are important for obtaining detailed and accurate results for this mixed EHL especially for severe operating conditions.

As found from experiments and suggested by the modern and future modes of mixed EHL reviewed in sections 1, 2.2.1.2 and 3.2, in the actual mixed EHL contact in relatively severe operating conditions, the oxidized chemical boundary layer lubrication and the dry contact between the opposing asperities of the contact surfaces usually respectively simultaneously locally occur in different areas of the whole mixed EHL contact depending on the operating condition. In the areas of a mixed EHL contact where this oxidized chemical boundary layer lubrication or this dry contact occur, the friction coefficient is high (and more than *0.35*). These local contact areas are thus important and even main friction source and consequently important and even main frictional heating source in a mixed EHL contact. In an actual mixed EHL contact, the frictional heating effects in these local contact areas may be strong on the contact pressure, the lubricating film thickness and even the wear. The temperatures of the contact surfaces of these local contact areas are important and even critical for the adhesion wear and scuffing of the contact surfaces of these local contact areas. These contact surface temperatures may also have strong influences on the temperatures of both the contact surfaces and the fluid film in the hydrodynamic fluid film lubricated zone, on the contact-fluid interfacial temperature, on the contact-fluid interfacial thermal desorption, on the shear strength of the contact-fluid interface, and thus on the fluid film collapse, the fluid film thermal instability and the fluid film disappearance in the hydrodynamic fluid film lubricated zone according to the analytical results of EHL in thermal condition obtained by Zhang [15, 16]. In a mixed EHL contact, the frictional heating and temperatures of the contact surfaces in the local contact areas where the oxidized chemical boundary layer lubrication or the dry contact between the opposing asperities of the contact surfaces occur, might thus be able to cause the local wider occurrence of the dry contact between the rough contact surfaces, the higher temperature rises from both the contact surfaces in the whole mixed EHL contact and the hydrodynamic fluid film, the more severe wear and scuffing of the whole mixed EHL contact and then the failure of the whole mixed EHL contact. This occurs more easily in more severe operating conditions. The thermal performances of the local oxidized chemical boundary layer lubrication or/and the local dry contact between the rough contact surfaces in a mixed EHL contact are therefore important. Studying the frictional heating effects in the local contact areas in a mixed EHL contact where the oxidized chemical boundary layer lubrication or the dry contact between the opposing asperities of the contact surfaces occur is therefore of significant interest. In the modelling of the modern and future modes of mixed EHL taking into account the effect of the frictional heating of the mixed EHL contact in a mixed EHL, the frictional heating effects in the local contact areas in a mixed EHL contact where the oxidized chemical boundary layer lubrication or/and the dry contact between the rough contact surfaces occur is necessary to incorporate. This should be done by using a more realistic fluid model, taking detailed, fine and real roughness of the contact surfaces of the mixed EHL contact and incorporating the fluid film viscous heating effect in a mixed EHL.

4.2 Thinning, mixed and partial fluid films in a mixed EHL due to the effects of the fluid rheology, the contact surface roughness and the fluid film viscous heating obtained in the theoretical modelling of EHL and mixed EHL in the past time

This section reviews the results of the thinning, mixed and partial fluid films in a mixed EHL contact obtained by the theoretical modelling of EHL and mixed EHL in the past time. These mixed EHL results may be indicative, instructive and important for developing the more realistic model for the mixed EHL for an actual concentrated contact in the future.

4.2.1 Thinning fluid films in a mixed EHL

4.2.1.1 Due to the fluid rheological effect

(1) Thinning EHL films due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect and the critical compound curvature radius of the contact surfaces in EHL for the non-continuum hydrodynamic fluid film regime caused by the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect

a. For pure rolling

Zhang [45, 46] obtained from numerical simulation that the overall EHL film is severely thinning due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect in the condition of medium/heavy loads, high rolling speeds, high bulk fluid temperatures and large slide-roll ratios, especially in the concentrated contact of small compound curvature radii of the contact surfaces. He [45] obtained that the percentage reduction of the fluid film thickness at the Hertzian contact center in isothermal pure rolling EHL of ideally smooth line contacts can be over *80%* in severe operating conditions due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect. For larger slide-roll ratios, this EHL film thickness reduction is greater [46].

Zhang [25] also found from numerical simulation that due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect, the fluid film in the Hertzian contact zone in isothermal pure rolling EHL of ideally smooth line contacts is severely thinning and thus non-continuum across the fluid film thickness when the compound curvature radius of the contact surfaces of this EHL is smaller than a critical value. He showed that in this case mixed rheologies occur and thus the hydrodynamic fluid films with different rheological behaviors are mixed in the whole EHL contact due to the relatively thick and thus continuum fluid film in the inlet zone and the molecularly thin and thus non-continuum fluid film in the Hertzian contact zone respectively simultaneously occurring in the EHL contact, since these two kinds of fluid films respectively have qualitatively different rheological behaviors. For isothermal pure rolling EHL of ideally smooth line contacts, Zhang [25] obtained the critical compound curvature radius of the contact surfaces of the EHL contact for the occurrence of the non-continuum fluid film in the Hertzian contact zone due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect, for the compound curvature radius of the contact surfaces of this EHL below which the non-continuum fluid film occurs in the Hertzian contact zone in this EHL. This critical compound curvature radius of the contact surfaces of the EHL contact for the occurrence of the non-continuum fluid film in the Hertzian contact zone due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect is expressed as [25]:

$$R_{cr0} = 8.716 \times 10^{-8} \frac{W^{0.2}}{\left(U_{b1} + \frac{K_1}{W^{n_1}}\right)^{0.74}} mm \quad , \qquad (2)$$

for pure rolling and G=4500

where W is the dimensionless load and defined as $W=w/E'R$, E' is the compound Young's modulus of the contact surfaces, R is the compound curvature radius of the contact surfaces, w is the carried load of the whole contact per unit axial length, G is the material parameter and defined as $G=\alpha E'$, α is the fluid viscosity-pressure index, and the parameters U_{b1}, K_1 and n_1 are respectively functions of the dimensionless shear strength C_p $(=\tau_{l0}/(kEG^{0.4})$, where τ_{l0} is the shear strength of the contact-fluid interface in the inlet zone and k is constant) of the contact-fluid interface in the inlet zone and are respectively formulated as:

$$U_{bl} = 27.8249C_p^2 + 2.115843 \times 10^{-5} C_p - 1.11009 \times 10^{-12},$$

$$K_1 = 5.46235 \times 10^{-10}\ C_p + 5.391 \times 10^{-17}$$

and:

$$\begin{cases} n_1 = 1.02806 \times 10^{18} C_p^3 - 2.4495 \times 10^{12} C_p^2 + \\ \quad + 1.91326 \times 10^6 C_p + 0.8582 \\ \qquad\qquad\qquad for \quad C_p \le 3.302 \times 10^{-7} \\ n_1 = 2.347 \times 10^5 C_p + 1.1825 \\ \qquad\qquad\qquad for \ C_p > 3.302 \times 10^{-7} \end{cases}$$

Equation (2) shows that the value of the critical compound curvature radius R_{cr0} of the contact surfaces of the EHL contact in isothermal pure rolling EHL of ideally smooth line contacts for the occurrence of the non-continuum fluid film in the Hertzian contact zone due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect does not depend on the rolling speed of the EHL contact. Therefore, for high rolling speeds of the EHL contact, the fluid film thickness in the Hertzian contact zone in pure rolling line contact EHL is far lower than predicted by classical EHL theory [1] due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect. In this condition, the fluid film in the Hertzian contact zone in pure rolling line contact EHL is severely thinning due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect. Figure 3 typically shows the values of the critical compound curvature radius R_{cr0} of the contact surfaces of the EHL contact in isothermal pure rolling EHL of ideally smooth line contacts for the occurrence of the non-continuum fluid film in the Hertzian contact zone due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect for varying carried loads of the EHL contact and various values of the dimensionless shear strength of the contact-fluid interface in the inlet zone, for $G=4500$. In Figure 3, the dimensionless shear strength of the contact-fluid interface in the inlet zone is expressed by the parameter C, which is defined as $C=\tau_{l0}/(E`G^{0.4})$.

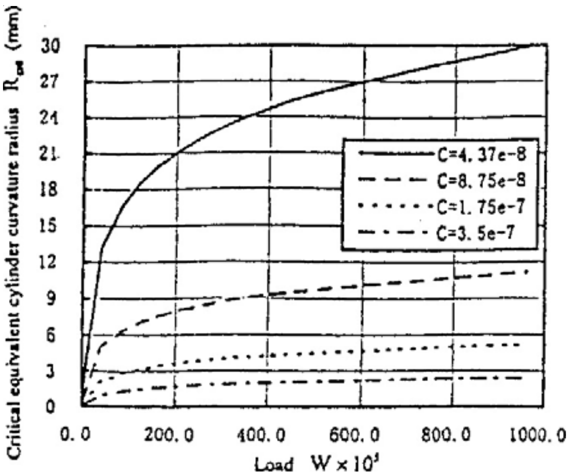

Fig. 3 Plot of the value of the critical compound curvature radius of the contact surfaces of the EHL contact in isothermal pure rolling EHL of ideally smooth line contacts for the occurrence of the non-continuum fluid film in the Hertzian contact zone due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect against the dimensionless carried load of the whole EHL contact for various values of the dimensionless shear strength of the contact-fluid interface in the inlet zone, for G=4500 and pure rolling [25]

b. For rolling and sliding

Zhang [46, 24] showed by numerical simulation that in isothermal EHL of ideally smooth line contacts, when sliding occurs between the contact surfaces, the fluid film thickness at the Hertzian contact center is sensitively reduced with the increase of the slide-roll ratio due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect especially under heavy loads. The overall EHL film is therefore further severely thinning with the slide-roll ratio increase of the EHL contact due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect. For isothermal EHL of ideally smooth line contacts, Zhang [21] obtained the following formula for the fluid film thickness at the Hertzian contact center as function of the slide-roll ratio of the EHL contact due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect:

$$H_c^{v-p}(S)=\begin{cases}(1-\dfrac{S}{2})H_{c,pr}^{v-p}, & \text{for } p_h>0.50GPa \text{ and } S\geq 0\\ H_{c,pr}^{v-p}, & \text{for } p_h\leq 0.50GPa \text{ and } S\geq 0\end{cases} \tag{3}$$

where $H_{c,pr}^{v-p}$ is the fluid film thickness at the Hertzian contact center in this EHL for pure rolling i.e. $S=0$ but the other same operating conditions due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect, p_h is the maximum Hertzian contact pressure of this EHL contact and S is the slide-roll ratio of this EHL contact. Equation (3) shows that in isothermal EHL of ideally smooth line contacts, when the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect is incorporated, the slide-roll ratio influence on the fluid film thickness at the Hertzian contact center is negligible for light loads. While the fluid film thickness at the Hertzian contact center in this EHL is linearly reduced with the increase of the slide-roll ratio of this EHL contact for medium and heavy loads due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect. Therefore, in this EHL, the overall EHL film is severely and even extremely thinning for medium/heavy loads and large slide-roll ratios due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect.

Based on Eq. (3), Zhang [21] plotted the lubrication regime charts due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect consisting of the operational parameters of the dimensionless rolling speed and the dimensionless carried load of the EHL contact for isothermal EHL of ideally smooth line contacts for wide slide-roll ratios, various values of the dimensionless shear strength of the contact-fluid interface in the inlet zone and different compound curvature radii of the contact surfaces of this EHL contact. In these lubrication regime charts, he described the operational scopes for the occurrence of the non-continuum hydrodynamic fluid film in the Hertzian contact zone in isothermal EHL of ideally smooth line contacts due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect for typical and wide slide-roll ratios for given values of the compound curvature radius of the contact surfaces of this EHL contact and given values of the dimensionless shear strength of the contact-fluid interface in the inlet zone. In isothermal EHL of ideally smooth line contacts, in these operational scopes, the fluid film in the Hertzian contact zone is extremely thinning and thus non-continuum across the fluid film thickness due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect, and the relatively thick and thus continuum fluid film in the inlet zone and the molecularly thin and thus non-continuum fluid film in the Hertzian contact zone thus respectively simultaneously occur in the EHL contact and are thus mixed in the whole EHL contact due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect.

Figure 4 typically shows these lubrication regime charts due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect plotted by Zhang [21] for isothermal EHL of ideally smooth line contacts for wide slide-roll ratios. Due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect, the whole operational scope of isothermal EHL of ideally smooth line contacts is divided into three sub-zones in these lubrication regime charts depending on the slide-roll ratio, i.e. the viscoelastic-fluid regime zone, the viscoplastic-fluid regime zone and the non continuum-fluid regime zone. In these rheological regime zones, the fluid regime in the EHL is respectively viscoelastic, viscoplastic and non-continuum due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect. In the non continuum-fluid regime zone, the fluid film in the Hertzian contact zone in the EHL is molecularly thin and thus non-continuum across the fluid film thickness. Figure 4 shows that the slide-roll ratio increase significantly constricts the viscoelastic-fluid regime zone but significantly enlarges the viscoplastic-fluid regime zone. The slide-roll ratio increase also significantly enlarges the non continuum-fluid regime zone. For medium and heavy loads, the non-continuum fluid film more easily occurs in the EHL contact for a larger slide-roll ratio. The increase of the operational scope of the non continuum-fluid regime zone by the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect heavily depends on the slide-roll ratio. Therefore, for large slide-roll ratios, due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect, the fluid film thickness in the Hertzian contact zone in line contact EHL can be far lower than predicted by classical EHL theory, which predicts that the slide-roll ratio influence on EHL film thickness is negligible. The fluid film in the Hertzian contact zone in line contact EHL can therefore be severely thinning for large slide-roll ratios due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect.

(2) Thinning EHL films due to the shear thinning effect of power-law shear thinning fluids

Greenwood and Kauzlarich [32] analytically showed that in isothermal pure rolling EHL of ideally smooth line contacts, the percentage reduction of the fluid film thickness at the Hertzian contact center can be *80%* in severe operating conditions due to the fluid shear thinning non-Newtonian effect for power-law shear thinning fluids. Their analytical results well agreed with experiments. The overall EHL film is therefore also severely thinning in severe operating conditions due to the fluid shear thinning effect for significantly shear thinning fluids.

(3) Thinning EHL films due to the fluid Newtonian effect

Zhang [22] obtained the results of isothermal EHL of ideally smooth line contacts from the EHL inlet zone analysis based on the Newtonian fluid model. He showed that in this EHL, under a given relatively heavy carried load of the EHL contact, when the rolling speed of the EHL contact is lower than a certain value, the fluid film thickness at the Hertzian contact center is far lower than predicted by classical EHL theory [1] and approaches to zero due to the fluid Newtonian effect. Therefore, in isothermal EHL of ideally smooth line contacts, under a given relatively heavy carried load of the EHL contact, when the rolling speed of the EHL contact is lower than a certain value, the fluid film in the Hertzian contact zone is severely and even extremely thinning for the Newtonian fluid. For shear thinning fluids, the EHL film in the Hertzian contact zone is more severely thinning for this condition due to the fluid shear thinning effect. Figure 5 typically shows the extremely thinning EHL film in the Hertzian contact zone in isothermal EHL of ideally smooth line contacts for the Newtonian fluid accurately obtained compared to the classical EHL theory predictions for relatively heavy carried loads of the EHL contact when the rolling speed U of the EHL contact satisfies $U=0.0372W^{1.50}/G$ and is low enough.

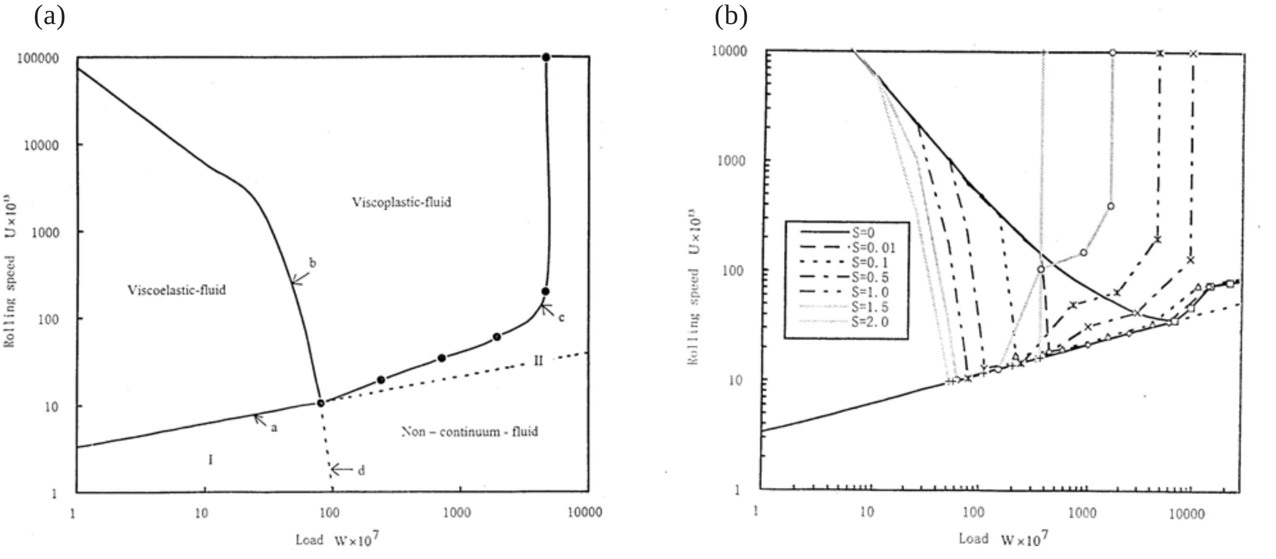

Fig. 4 The U (dimensionless rolling speed)-W (dimensionless load) lubrication regime charts for wide values of the slide-roll ratio S, for G=4500. (a) R=6 mm, C_0=4.78E-6, S=1.0; (b) R=6 mm, C_0=4.78E-6; $C_0=\tau_{l0}/E'$ [21]

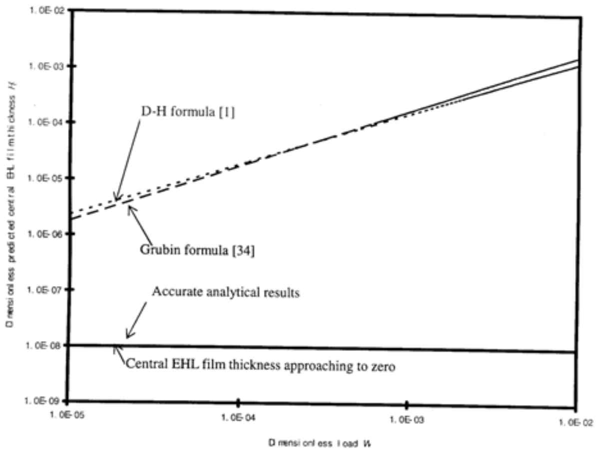

Fig. 5 Typical comparisons between the central EHL film thickness in isothermal ideally smooth line contacts respectively predicted by conventional EHL theories and accurately predicted from the EHL inlet zone analysis when the dimensionless rolling speed U satisfies U=0.0372W1.50/G, based on the Newtonian fluid model. The value of the material parameter G is 4500. D-H formula denotes the Dowson-Higginson formula [22]

4.2.1.2 Due to the contact surface roughness effect

(1) Local thinning EHL films due to the contact surface roughness effect when the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect is incorporated

Zhang [12] showed by numerical simulation that in isothermal simple sliding EHL of rough line contacts, when the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect is incorporated, usually the EHL film pressure distribution is smooth and similar to that in the case of ideally smooth contact surfaces and the contact surface roughness is almost not deformed by the generated EHL film pressure due to the combined effect of the contact surface roughness, the contact-fluid interfacial shear strength and the contact-fluid interfacial slip. In this condition, even the average fluid film thickness between the contact surfaces in the Hertzian contact zone is greatly reduced due to the wide occurrence of the contact-fluid interfacial slip in the lubricated area. Therefore, in this condition, the EHL film in the Hertzian contact zone can be locally severely thinning due to the persisting contact surface roughness penetrating into the average fluid film between the contact surfaces in the Hertzian contact zone which has been greatly thinning.

(2) Overall thinning EHL films due to the fluid side leakage caused by the contact surface roughness

For Newtonian fluids, it was proposed that the fluid side leakage can result in fluid film disappearance in rough EHL contacts due to the contact surface roughness effect [47]. Based on the Newtonian fluid model, the fluid side leakage in three dimensional rough EHL contacts was shown by Chang et al. [10] to cause EHL film breakdown due to the contact surface roughness effect. Johnson and Higginson [11] analytically found that the fluid side leakage is increased by the fluid viscoplastic behavior in point contact EHL. The combined effect of the contact surface roughness and the fluid viscoplastic behavior or the contact-fluid interfacial slip may significantly increase the fluid side leakage and consequently result in severe overall fluid film thinning in EHL of rough contacts and especially in elliptical contact EHL of rough contacts with small ellipticity ratios. This combined effect is important for EHL and may need to be studied with emphasis in the future research of mixed EHL by taking more realistic engineering contact surface roughness and taking the fluid as viscoplastic and the contact-fluid interfacial slip. The combined effect on the fluid side leakage of the contact surface roughness and the fluid viscoplastic behavior or the contact-fluid interfacial slip is very possible to be able to cause severe fluid film thinning in real EHL contacts.

(3) Overall thinning EHL films due to the extension of the contact-fluid interfacial slip region and the reduction of the total fluid flow entrained into the EHL contact by the fluid side leakage caused by the contact surface roughness

Zhang [12] showed by numerical simulation that in isothermal simple sliding EHL of rough line contacts under medium loads, the contact surface roughness can increase the extent of the contact-fluid interfacial slip, the increase of which reduces the total fluid flow entrained into the EHL contact and thus the overall fluid film thickness in the EHL contact. Therefore, the contact surface roughness can result in overall thinning fluid films in the EHL contact due to increasing the contact-fluid interfacial slip and thus reducing the total fluid flow entrained into the EHL contact. This effect may occur in EHL of rough contacts in rolling and sliding under medium and heavy loads.

(4) Local thinning EHL films due to the loss of the local fluid film pressure and thus the local elastic rebounding of the contact surfaces due to the elastic deformation of the contact surfaces, due to the contact surface roughness effect

Zhang [12] showed by numerical simulation that in isothermal simple sliding EHL of rough line contacts under medium loads, when the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect is incorporated, usually the loss of the local EHL film pressure in the Hertzian contact zone is great compared to the EHL film pressure in those locations for no interfacial shear strength and interfacial slip effect due to the wide occurrence of the contact-fluid interfacial slip in the Hertzian contact zone. Consequently, in this condition, the contact surfaces locally elastically rebound in the Hertzian contact zone due to this fluid film pressure loss and the elastic deformation of the contact surfaces. In this condition, when the contact surface roughness is present in the Hertzian contact zone, this contact surface roughness elastically rebounds and can almost not be deformed by the fluid film pressure due to this fluid film pressure loss. Therefore, in this condition, the fluid film can be locally severely thinning in the Hertzian contact zone when the persisting contact surface roughness locally deeply penetrates into the fluid film in the Hertzian contact zone due to this fluid film pressure loss.

In EHL of rough contacts, the local high contact surface roughness may also cause the value of the local fluid film pressure below zero and thus the local fluid cavitation in the Hertzian contact zone due to the Newtonian response of the local fluid film pressure to this contact surface roughness. In this locally cavitated lubricated area, the fluid film pressure becomes zero and has severe loss compared to the fluid film pressure in this lubricated area for smooth contact surfaces for the same operating condition. Samely, in this case, the contact surfaces locally elastically rebound around this cavitated lubricated area due to this fluid film pressure loss and the elastic deformation of the contact surfaces. Therefore, in this case, the fluid film around this cavitated lubricated area can be locally severely thinning due to this local elastic rebounding of the contact surfaces.

4.2.1.3 Due to the fluid film thermal effect

(1) Overall thinning EHL films due to the fluid film thermal effect when the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect is incorporated

In EHL contacts, in severe operating conditions, i.e. for high rolling speeds, heavy loads, large slide-roll ratios and high bulk fluid temperatures, the fluid film viscous heating is usually severe. In this condition, the temperature rises both within the fluid film and at the contact surfaces are usually high in EHL contacts. Conventional EHL theory [48] showed that for Newtonian fluids, the fluid film thermal effect considerably reduces the overall EHL film thickness however is overall modest in this condition. In this condition, the contact-fluid interfacial shear strength may be low where the contact surface temperature is high [17]. On the other hand, the contact-fluid interfacial shear strength may be considerably reduced with the contact surface temperature rise [16]. Zhang [15, 16] analytically showed that due to these effects, when the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect is incorporated, the EHL film in the Hertzian contact zone can be extremely thin and even approach to disappearance due to the combined effect of the fluid film viscous heating, the contact-fluid interfacial shear strength and the contact-fluid interfacial slip in this condition. Therefore, in EHL contacts in severe operating conditions, the fluid film may be overall severely thinning due to the combined effect of the fluid film viscous heating, the contact-fluid interfacial shear strength and the contact-fluid interfacial slip for an actual case. Figure 6 shows the severely fluid film thinning and even the fluid film disappearance in the Hertzian contact zone in thermal pure rolling EHL of ideally smooth line contacts due to the combined effect of the fluid film viscous heating, the contact-fluid interfacial shear strength and the contact-fluid interfacial slip when the contact-fluid interfacial shear strength at ambient pressure is reduced and significantly low, obtained by Zhang [15, 16].

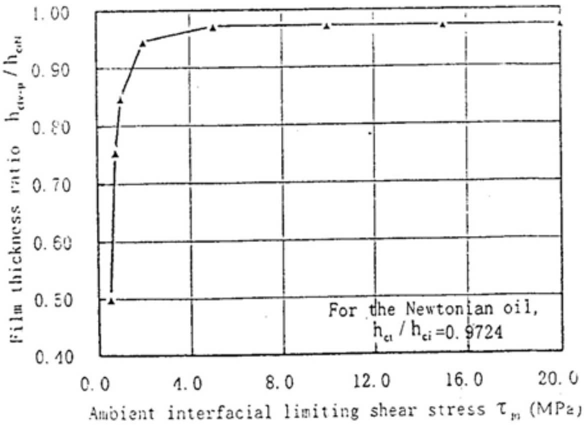

Fig. 6 Plot of the dimensionless central EHL film thickness against the ambient contact-fluid interfacial shear strength for a steady-state, thermal and pure rolling EHL of ideally smooth line contact, where h_{ctv-p} is central EHL film thickness in thermal condition based on the assumption of the contact-fluid interfacial shear strength; and h_{ciN} is central EHL film thickness in isothermal condition based on the Newtonian fluid model [16]

(2) Overall thinning EHL films due to the fluid film thermal effect on the fluid film stability

Christensen [14] proposed that in a mixed EHL, the carried load of the whole mixed EHL contact was usually balanced by both the fluid film pressure and the asperity contact. He showed by theoretical analysis that in a mixed EHL, when the lubricated area and the asperity contact both were frictional heating, the asperity would be thermally softened and thus compressed under its carried load, this caused more severe frictional heating both in the lubricated area and in the asperity contact and thus the reduction of the load-carrying capacity of the fluid film, and the portion of the load carried by the asperity contact was therefore increased. He found from theoretical analysis that in a mixed EHL, when the carried load of the whole mixed EHL contact was greater than a critical level, the asperity would be significantly compressed, the fluid film was thus collapsed, and as a result the asperity contact carried nearly all the load of the whole mixed EHL contact. This was found to be the thermal instability of the fluid film in a mixed EHL. Therefore, in a mixed EHL, due to this thermal instability of the fluid film, the fluid film will be overall severely thinning when the carried load of the whole mixed EHL contact is greater than a critical level.

(3) Local thinning EHL films due to the contact frictional heating effect on the thermo-elastic deformation of the contact surfaces

In an actual mixed EHL, in severe operating conditions, the temperature rise of the contact surfaces can locally be very high due to the contact frictional heating especially when part of the concentrated contact is in boundary lubrication or dry contact. On the location where this high contact surface temperature rise occurs, the component of the elastic deformations of the contact surfaces due to the contact surface thermal expansion may be considerable and comparable to the fluid film thickness especially when this fluid film thickness is low. Therefore, in a mixed EHL, the fluid film may be locally considerably thinning due to the contact surface thermo-elastic deformation when the contact surface temperature rise is locally high due to the contact frictional heating. This local fluid film thinning may be especially significant in the mixed EHL with local dry contact between the opposing asperities of the contact surfaces or local boundary lubrication in severe operating conditions, where the fluid film thickness is locally very low around the local dry contact or/and local boundary lubrication areas however the contact surface thermo-elastic deformation around the local dry contact or local boundary lubrication areas is usually much comparable to this low local fluid film thickness due to the severe contact frictional heating and the high contact surface temperature rise both locally occurring around the local dry contact or local boundary lubrication areas. This effect may need to be studied with emphasis in the future research of mixed EHL.

4.2.2 Mixed fluid films in a mixed EHL

4.2.2.1 Mixed rheologies in elastohydrodynamic lubrication

Zhang [21] found from numerical simulation and theoretical analysis that in isothermal EHL of ideally smooth line contacts, viscoelastic continuum, viscoplastic continuum and non-continuum fluids are distributed from the inlet zone to the Hertzian contact zone in order in severe operating conditions, i.e. for heavy loads, high rolling speeds, high bulk fluid temperatures and large slide-roll ratios due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect and thus the extreme overall EHL film thinning in this EHL when the contact-fluid interfacial shear strength is low in the inlet zone. He showed that the molecularly thin fluid film occurs in the Hertzian contact zone in this condition and the rheological behavior of this film is qualitatively different from those of the viscoelastic continuum and viscoplastic continuum fluids respectively simultaneously occurring in different and other areas of this EHL contact in this condition due to the non-continuum and continuum characteristics across the fluid film thickness of these fluids respectively. Also, in this EHL contact, the rheological behavior of the viscoelastic continuum fluid locally occurring is viscoelastic and qualitatively different from that of the viscoplastic continuum fluid locally occurring in other area of the EHL contact, which is viscoplastic. He showed that in EHL, due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect, the overall EHL film can be extremely thinning and fluid films with different rheological behaviors can respectively simultaneously occur in different areas of the EHL contact and thus be mixed in the whole EHL contact. Figure 7 shows the mixed fluid films with different rheological behaviors in isothermal EHL of ideally smooth line contacts in severe operating conditions due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect obtained by Zhang [21], which respectively simultaneously occur in different areas of the whole EHL contact.

In isothermal EHL of ideally smooth line contacts, the regime of these mixed fluid films occurs in fairly a wide operational scope i.e. in the operational scope of the non continuum-fluid regime zone depending on the contact-fluid interfacial shear strength in the inlet zone, the compound curvature radius of the contact surfaces and the slide-roll ratio, as typically shown in Fig. 4. More details on this operational scope can be found in Ref. [21]. In isothermal EHL of ideally smooth line contacts, in this operational scope, the molecularly thin fluid film occurs in the Hertzian contact zone while the viscoelastic continuum and viscoplastic continuum fluids respectively simultaneously occur in different and other areas of the whole EHL contact. Thus, mixed fluid films with different rheological behaviors respectively simultaneously locally occur in this EHL contact for this operational scope.

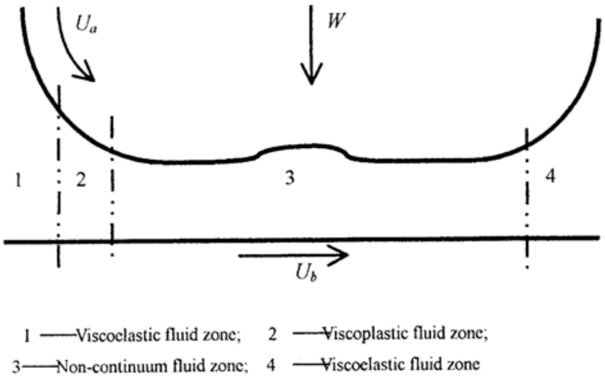

Fig. 7 Mixed rheologies in isothermal EHL of ideally smooth line contacts due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect in the condition of medium/heavy loads, high bulk fluid temperatures and large slide-roll ratios [21]

4.2.3 Partial fluid films in a mixed EHL

(1) Conventional definition

In conventional mixed EHL theory [33], the partial fluid film in a mixed EHL was defined as the Newtonian fluid film in a mixed EHL contact which only occurs in part of the whole mixed EHL contact and is mixed in the whole mixed EHL contact with the dry contact between the opposing asperities of the contact surfaces simultaneously occurring in the other part of the whole mixed EHL contact. In conventional concept, the mode of mixed EHL with partial fluid film was defined as the mode of mixed EHL where the fluid film disappeared in local areas of the whole mixed EHL contact, i.e. between the local opposing asperities of the contact surfaces and then the dry contact between the rough contact surfaces occurs there while the Newtonian fluid film was only able to be generated in the other areas of the whole mixed EHL contact and was thus partial in the whole mixed EHL contact as a consequence. This conventional mode of mixed EHL with partial fluid film was attempted to more fit the mode of mixed EHL in a real mixed EHL contact. However, the mechanism of the partial fluid film in a mixed EHL contact, i.e. the mechanism of the fluid film disappearance in local areas of a whole mixed EHL contact was not clear in this defined conventional mode of mixed EHL with partial fluid film. This was obviously caused by the oversimplified modelling and theory for a mixed EHL when developing this conventional mode of mixed EHL with partial fluid film, as can be found in reference [33].

(2) Zhang's definition

Zhang [21] showed by numerical simulation and theoretical analysis that in isothermal EHL of ideally smooth line contacts, the molecularly thin fluid film, the viscoplastic continuum fluid film and the viscoelastic continuum fluid film respectively simultaneously occur in different areas of the whole EHL contact and are thus mixed in the whole EHL contact in severe operating conditions due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect. He showed that these fluid films respectively occur from the Hertzian contact zone to the EHL inlet entrance in order in the EHL contact. According to his analytical results of EHL, the mode of EHL or mixed EHL with partial fluid film is more generally defined as the mode of EHL or mixed EHL where a kind of fluid film with a certain rheological behavior is only present in part of the whole EHL or mixed EHL contact. In this defined mode of EHL or mixed EHL with partial fluid film, the fluid film may be present in the whole EHL or mixed EHL contact but have different rheological behaviors in different areas of the whole EHL or mixed EHL contact, i.e. fluid films with different rheological behaviors may respectively simultaneously occur in different areas of the whole EHL or mixed EHL contact and thus be mixed in the whole EHL or mixed EHL contact. Or, in this defined mode of mixed EHL with partial fluid film, the fluid film may be separated by the dry contact between the opposing asperities of the contact surfaces locally occurring on some separate locations of the whole mixed EHL contact and thus only be present in part of the whole mixed EHL contact. According to this mode of EHL or mixed EHL with partial fluid film, the partial fluid film in a mixed EHL is more generally defined as the fluid film with a certain rheological behavior in a mixed EHL which only occurs in part of the whole mixed EHL contact and is thus partial in the whole mixed EHL contact due to the fluid films with different rheological behaviors respectively simultaneously locally occurring in the mixed EHL contact or/and the dry contact between the opposing asperities of the contact surfaces locally occurring in the whole mixed EHL contact. According to this definition of the partial fluid film in a mixed EHL, in a mixed EHL, when these mixed rheologies and the dry contact between the opposing asperities of the contact surfaces both locally occur, the fluid film is complicated and very partial and is more partial than the partial fluid film conventionally defined as described in the fore sub-section. The mode of mixed EHL with partial fluid film defined according to Zhang's results of EHL [21] and described here much better fits the experimental results of mixed EHL respectively obtained by Begelinger and Gee [18, 19] and by Tabor [20] as described above than the conventionally defined mode of mixed EHL with partial fluid film as described in the fore sub-section.

4.3 Dry contact between the contact surfaces i.e. fresh metal to metal contact is actually usually not the direct cause of the failure of the contact surfaces and is thus actually usually not the steady stage experienced by the mixed EHL in a practical mixed EHL contact

The experiments of mixed EHL respectively carried out by Begelinger and Gee [18, 19] and by Tabor [20] showed that in a practical mixed EHL contact, the steady-state dry contact between the contact surfaces does actually usually not occur in the whole operation longevity of the mixed EHL contact including when the contact surfaces are on destruction or failure, due to the oxidization of the fresh metals of the contact surfaces by the oxygen within the mixed EHL contact. Their experiments showed that in a practical mixed EHL contact, the instant dry contact between the opposing asperities of the contact surfaces however actually usually locally occurs on all the mixed EHL stages due to the fluid film disappearance between these opposing asperities and the oxidization of the contact surfaces unable to keep ahead of the formation of the new asperity contact between the contact surfaces. Therefore, in a practical mixed EHL contact, the dry contact between the contact surfaces actually usually locally occurs in fairly small areas of the whole mixed EHL contact and is actually usually instant. However, this dry contact was shown by the experiments of Begelinger and Gee [18, 19] and Tabor

[20] to have significant effects on the performance of the mixed EHL contact and on the transition of the mixed EHL stage. The previous mode of mixed EHL proposed in the theoretical modelling of mixed EHL such as shown in reference [33] which assumed the steady-state and wide areas of dry contact between the rough contact surfaces locally occurring in a mixed EHL contact however conflicts with the mode of a practical mixed EHL and is therefore not realistic. The predictive and application values of that mode of mixed EHL to a real mixed EHL contact are thus very limited. Instead, the instant and small areas of dry contact between the rough contact surfaces locally occurring in a practical mixed EHL contact needs to be added into the mode of mixed EHL and needs to be studied in the theoretical modelling of mixed EHL.

5. CONTACT-FLUID INTERFACIAL SHEAR STRENGTH AND CONTACT-FLUID INTERFACIAL SLIPPAGE IN EHL AND MIXED EHL

This section reviews the important phenomena of the contact-fluid interfacial shear strength and the contact-fluid interfacial slippage occurring in a practical EHL or mixed EHL contact, which are important for the performance of EHL or mixed EHL and for the transition of the EHL or mixed EHL stage. Due to the limitation of the length of the present paper, experimental results and theoretical modelling of these phenomena both are only briefly reviewed in this section as follows.

5.1 Contact-fluid interfacial shear strength in EHL and mixed EHL

Rozeanu and Snarsky [49] proposed that there are three shear strengths in a hydrodynamic lubrication, i.e. the bulk fluid shear strength, the shear strength of the contact surface adhering layer and the shear strength between this layer and the bulk fluid. In EHL or mixed EHL, since the contact surface adhering layer is very thin compared to the EHL film thickness, the contact-fluid interfacial shear strength is one of these three shear strengths which is the smallest.

It was suggested by Zhang et al. [46, 50] based on experimental results and theoretical derivation that the bulk fluid shear strength be predicted by the following equation:

$$\tau_l = \begin{cases} \tau_{l0} + \alpha_{\tau l}(\,p - p_s\,), & for\ p \geq p_s \\ \tau_{l0}\,, & for\ p < p_s \end{cases} \qquad (4)$$

where p is the fluid film pressure, p_s is the fluid solidification pressure, τ_{l0} is the bulk fluid shear strength at ambient pressure, and $\alpha_{\tau l}$ is the bulk fluid shear strength-pressure proportionality.

The shear strength between the contact surface adhering layer and the bulk fluid was shown by Rozeanu and Snarsky [49] by theoretical analysis to be lower than the bulk fluid shear strength and the shear strength of the contact surface adhering layer. The contact-fluid interfacial shear strength seems to be the shear strength between the contact surface adhering layer and the bulk fluid. This shear strength is determined by the welding between the bulk fluid and the contact surface adhering layer. This welding is determined by the attraction forces between the bulk fluid molecules and the contact surface adhering layer molecules. It seems that this welding is determined by the temperature of the contact-fluid interface. For high contact surface temperatures, the attraction forces between these molecules are usually greatly reduced due to the chemical and physical reactions and fluid thermal desorption at the contact surface adhering layer-bulk fluid interface. The shear strength of this interface usually can only be low [17]. For this case, the bulk fluid pressure is not the factor that strongly influences this interfacial shear strength. On the other hand, when the bulk fluid pressure is lower than the solidification pressure of this interface, the physical and chemical characters of this interface are not considerably changed by the bulk fluid pressure and the influence of the bulk fluid pressure on this interfacial shear strength can be neglected. When the bulk fluid pressure is higher than the solidification pressure of this interface, the bulk fluid pressure considerably changes the physical and chemical characters of this interface and strongly influences the attraction forces and the shear strength between the bulk fluid molecules and the contact surface adhering layer molecules. Even for low contact surface temperatures, the above pressure influence on the shear strength of the contact surface adhering layer-bulk fluid interface may also be present.

Assume that:

$$\Delta\tau_l=\alpha_{\tau l}\Delta_{p,} \quad for\ p>p_s \qquad (5)$$

where p is the bulk fluid pressure, τ_l is the shear strength of the contact surface adhering layer-bulk fluid interface, p_s is the solidification pressure of this interface, $\alpha_{\tau l}$ is constant, $\Delta\tau_l$ and Δp are respectively the variations in τ_l and p. Therefore, according to the fore paragraph and based on Eq. (5), the shear strength of the contact surface adhering layer-bulk fluid interface also satisfies Eq. (4).

In the theoretical modelling of the EHL or mixed EHL in isothermal condition, the contact-fluid interfacial shear strength does not need to be critically distinguished as the fluid shear strength or the shear strength of the contact-fluid interface but is the minimum of them [23]. In EHL and mixed EHL, the contact-fluid interfacial shear strength can be assumed to follow Eq. (4), where the parameter p_s is not critically distinguished as the bulk fluid solidification pressure or the solidification pressure of the contact-fluid interface and is one of them.

5.2 Contact-fluid interfacial slippage in EHL and mixed EHL

In classical hydrodynamic lubrication theories including classical elastohydrodynamic lubrication theories, the condition of the no slip at the contact-fluid interface was assumed. The physical basis of this hypothesized non slip boundary condition is the strong interaction at the contact-fluid interface, with significant intermolecular forces being developed at the contact-fluid interface. In an actual hydrodynamic lubrication, the fluid film across the fluid film thickness is usually consisted of two kinds of fluid film layers, i.e. the highly solidified layer adhering and ordered to the contact surfaces and the disordered bulk fluid film layer between the contact surface adhering layers respectively at the two contact surfaces. Rozeanu and Tipei [51] proposed that in an actual hydrodynamic lubrication, when the fluid film is relatively thick, the real boundary condition across the fluid film thickness is at the contact surface adhering layer-bulk fluid interface. They showed that in an actual hydrodynamic lubrication, due to the entropy discontinuity occurring at the contact surface adhering layer-bulk fluid interface, the shear strength of the contact surface adhering layer-bulk fluid interface is rather limited. They proposed that in an actual hydrodynamic lubrication, the bulk fluid film can slip at the contact surface adhering layer-bulk fluid interface due to the limited shear strength of this interface and the velocity of the hydrodynamic fluid film at the boundary across the fluid film thickness i.e. at this interface is thus different from the speed of the contact surface at this interface due to this fluid film slippage. The experiment carried out by them showed the occurrence of the fluid film slippage at the contact-fluid interface and its effect on the fluid film pressure distribution in a partial sliding hydrodynamic journal bearing. They showed by both theory and experiments that in an actual hydrodynamic lubrication, the fluid film slippage at the contact-fluid interface can occur and thus invalidates the assumption of the no slip boundary condition at the contact-fluid interface based on by classical hydrodynamic lubrication theories. They theoretically showed that in a hydrodynamic lubrication, the fluid film slippage at the contact-fluid interface causes the change of the distribution of the fluid film velocity within the bulk fluid film and the significant reduction of the fluid film pressure. Their experiment verified this conclusion.

In an actual elastohydrodynamic lubrication or mixed elastohydrodynamic lubrication, the fluid film slippage at the contact-fluid interface and its effect on the fluid film pressure described in the fore paragraph also occur due to the limited shear strength of the contact-fluid interface as described in the fore section. In an actual EHL or mixed EHL, the fluid film slippage can also occur within the fluid film as a result of the film shear stress reaching the film shear strength within the fluid film due to the fluid film thermal effect. A fluid response indicating the loss of fluid film adherence to the contact-fluid interface, i.e. the fluid film slippage at the contact-fluid interface was observed by Kaneta et al. [52] with a special optical interferometry technique. However, in an EHL or mixed EHL, the fluid film slippage both at the contact-fluid interface and within the fluid film is very difficult to directly detect in general operating conditions due to the small contact width, the extremely thin fluid film and the very short transit time of the whole EHL or mixed EHL contact. It is extremely difficult to directly observe by experiment any slippage phenomena of an EHL film element such as the slipping velocity magnitude, its dependence on operating parameters, and the extent of the domains in the whole EHL or mixed EHL contact where fluid film slippage occurs and to make a direct correlation between fluid film slippage and fluid film thickness. Kaneta et al. [53] observed by experiment the phenomena of the dimples occurring in sliding point contact EHL under medium loads. As commented by Zhang [25], the combined effect of the contact-fluid interfacial slippage and the fluid film viscous heating may be responsible for their observed dimple phenomena.

The experiments of EHL or mixed EHL and of even any hydrodynamic lubrication directly indicating the occurrence of the fluid film slippage either at the contact-fluid interface or within the fluid film in these lubrications are presently still few. However, theoretical modelling of these hydrodynamic lubrications based on the assumption of the contact-fluid interfacial shear strength and the contact-fluid interfacial slippage confirms the occurrence of the fluid film slippage in these hydrodynamic lubrications due to well fitting experimental results as described in the subsequent section. Nevertheless, experiments of EHL or mixed EHL need to be more intensively carried out in the future to better substantiate the occurrence of the fluid film slippage and its effect in EHL or mixed EHL.

5.3 Theoretical modelling of EHL and mixed EHL based on the assumption of the contact-fluid interfacial shear strength and the contact-fluid interfacial slippage

Based on the fact of the contact-fluid interfacial shear strength and the contact-fluid interfacial slippage in a hydrodynamic lubrication and in EHL found by experiments, the theoretical modelling of EHL and mixed EHL based on the assumption of the contact-fluid interfacial shear strength and the contact-fluid interfacial slippage were also developed in the past time.

Ehret et al. [54] developed the model for isothermal EHL of ideally smooth point contacts for large slide-roll ratios based on the non-Newtonian fluid model considering the contact-fluid interfacial shear strength and the contact-fluid interfacial slippage. The results obtained from his model had the capability of explaining the phenomena of the dimples in EHL contacts experimentally observed by Kaneta et al. [53] due to qualitatively matching their experimental results. However, his model was not completely able to explain the dimples in EHL observed by Kaneta et al.[53] due to not quite fitting the experimental results obtained by Kaneta et al. [53] and due to several questionable results obtained from his model. It was commented by Zhang et al. [25] that this may be caused by the over assumptions taken in his model.

Zhang [23] developed the model for isothermal EHL of ideally smooth line contacts for small, medium and large slide-roll ratios based on the non-Newtonian fluid model considering the contact-fluid interfacial shear strength and the contact-fluid interfacial slippage. He showed by his model that the EHL film pressure and the EHL film load-carrying capacity both are significantly reduced especially for medium and large slide-roll ratios due to the contact-fluid interfacial shear strength and the contact-fluid interfacial slippage effect. His result of the contact-fluid interfacial slippage effect on EHL film pressure and load-carrying capacity is qualitatively agreeable with the conclusion on the contact-fluid interfacial slippage effect on the fluid film pressure and load-carrying capacity of a hydrodynamic journal bearing drawn by Rozeanu and Snarsky [49]. He applied his model to isothermal pure rolling EHL of ideally smooth line contacts for the condition of heavy loads, high rolling speeds and high bulk fluid temperatures. The results obtained from his model for this condition well fits the experimental results of the EHL film thickness and its variations with loads and rolling speeds obtained by Kannel and Bell [36] for the same operating condition, which had been difficult to explain from any EHL theories before. He [21] later applied his model to this EHL for wider operating conditions and for much more common fluids and showed that his model essentially fits the experimental results obtained by Kannel and Bell [36] for the same wide operating conditions. Based on his model, Zhang [21] proposed the theory of mixed rheologies in EHL as described in the above sections. He [21] showed that the mixed rheologies regime in EHL is a new mode of mixed lubrication in concentrated contacts due to the contact-fluid interfacial shear strength and the contact-fluid interfacial slippage effect, as also described in the above sections. Zhang [12] also developed the model for isothermal simple sliding mixed EHL of line contacts for medium loads and rolling speeds based on the non-Newtonian fluid model considering the contact-fluid interfacial shear strength and the contact-fluid interfacial slippage. He showed by this model that his obtained mixed EHL results were qualitatively different from the mixed EHL results before due to the contact-fluid interfacial shear strength and the contact-fluid interfacial slippage effect. He [12] showed that due to this effect, the EHL film pressure was usually instead not rippled at all and the manufactured roughness of the contact surfaces of the mixed EHL contact usually totally persisted in the condition of medium/heavy loads and large slide-roll ratios, as reviewed in the above sections. He [12] showed that as a result, the overall EHL film and the local EHL film both were severely thinning in the mixed EHL due to the contact-fluid interfacial shear strength and contact-fluid interfacial slippage effect and due to the persisting contact surface roughness locally penetrating into the EHL film. These mixed EHL results were qualitatively new and had not been obtained before by any other mixed EHL models which had ignored the contact-fluid interfacial shear strength and contact-fluid interfacial slippage effect in a mixed EHL. Therefore, the contact-fluid interfacial shear strength and contact-fluid interfacial slippage effect has important impacts on both EHL and mixed EHL.

5.4 Commenting remarks

As reviewed in the fore section, the contact-fluid interfacial shear strength and contact-fluid interfacial slippage effect has so important impacts on EHL and mixed EHL that it would raise the revolutionary of the understanding of the EHL and mixed EHL regimes in concentrated contacts and thus the revolutionary of the theories of EHL and mixed EHL. The contact-fluid interfacial shear strength and contact-fluid interfacial slippage effect needs to be incorporated in the theoretical modelling of mixed EHL in the future. On the other hand, this effect in EHL and mixed EHL needs to be studied more intensively and extensively by experiments in the future. This will answer and justify the quarrel on the necessity of establishing new EHL and mixed EHL theories based on the contact-fluid interfacial shear strength and contact-fluid interfacial slippage effect. This will also prove the correctness of the developed models and theories of EHL and mixed EHL based on the condition of the contact-fluid interfacial shear strength and the contact-fluid interfacial slippage and will further prove the predictive and application values of these developed models and theories of EHL and mixed EHL to a practical EHL or mixed EHL contact. Advance on these theoretical and experimental research subjects is of significant interest and will be seen soon.

6. FUTURE MODE OF MIXED EHL

According to the reviews and descriptions in the above sections, this section proposes the future mode of mixed EHL, i.e. the more generalized mode of mixed EHL proposed in the present paper in the above sections which needs to be taken in the theoretical modelling of mixed EHL in the future. The future mode of mixed EHL is representatively pictured in Figure 8.

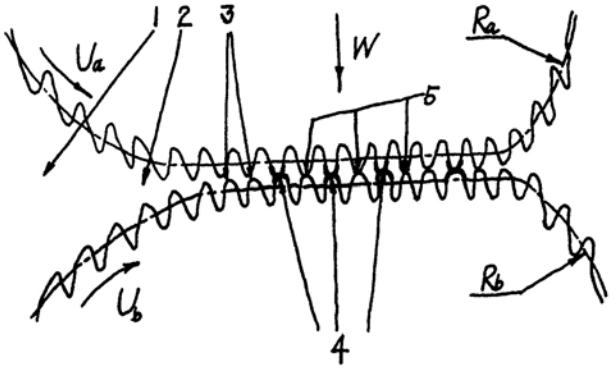

Fig. 8 Representative and general description of the future mode of mixed EHL: 1 - viscoelastic continuum (relatively thick) fluid film lubrication, 2 - viscoplastic continuum (relatively thick) fluid film lubrication, 3 - non-continuum fluid film lubrication, i.e. physical adsorbed layer boundary lubrication, 4 - oxidized chemical boundary layer lubrication, 5 - fresh metal to metal dry contact

In the future mode of mixed EHL, in a mixed EHL contact, the fluid film between the rough contact surfaces may be locally molecularly thin, i.e. non-continuum across the fluid film thickness on some separate locations of the whole mixed EHL contact. At the same time, the oxidized chemical boundary layer lubrication and the dry contact between the opposing asperities of the contact surfaces may occur respectively on some other separate locations of the whole mixed EHL contact. In the other zones of the whole mixed EHL contact, where the fluid film is relatively thick and thus continuum across the fluid film thickness, the fluid rheological behavior is shear-thinning viscoelastic or viscoplastic depending on the operating condition.

In the theoretical modelling of the future mode of mixed EHL, detailed, fine and time-dependent results for mixed EHL are of purpose. In this mixed EHL modelling, the fluid rheological model needs to be non-Newtonian and can be implemented to the fluid film in the whole fluid film thickness range, i.e. the molecularly thin and thus non-continuum fluid film and the relatively thick and thus continuum fluid film both respectively simultaneously locally occurring in the mixed EHL contact. On the other hand, the contact-fluid interfacial shear strength and contact-fluid interfacial slippage effect in a mixed EHL needs to be incorporated by this fluid rheological model. This fluid rheological model can be expressed by Eq. (1) as an example. In this mixed EHL modelling, the mixed EHL contact is more of engineering and real. The contact surface roughness is most frequently taken as engineering and real contact surface roughness. In this mixed EHL modelling, the frictional heating effect both within the fluid film and in the dry contact of asperity needs to be incorporated. The lubrication stage transition as occurs in a practical mixed EHL contact and experimentally found by Begelinger and Gee [18, 19] and by Tabor [20] needs to be studied with the operating condition by this mixed EHL modelling. As a further research, the friction, wear and contact surface destruction, i.e. contact surface failure as respectively occur in a practical mixed EHL contact and experimentally found by Begelinger and Gee [18, 19] and by Tabor [20] in dependence on the operating condition also needs to be modelled by this mixed EHL modelling.

The future mode of mixed EHL has the essential features of thinning, mixed and partial lubricating films respectively simultaneously locally occurring in a mixed EHL contact as typically occur in the mixed EHL in the industry and typically shown by the results of both experiments and theoretical modelling of mixed EHL obtained in the past time, as reviewed and described in the above sections.

The experiments of mixed EHL respectively carried out by Begelinger and Gee [18, 19] and by Tabor [20] showed that even on the mixed EHL stage of relatively thick overall mixed EHL film, usually, lubricating films with different rheological behaviors also respectively simultaneously locally occur in a mixed EHL contact and are thus mixed in the whole mixed EHL contact. These mixed lubricating films in a mixed EHL contact are respectively the relatively thick and thus continuum fluid film, the molecularly thin and thus non-continuum fluid film, the oxidized chemical boundary layer lubrication film in local small areas of the whole mixed EHL contact and the instant dry contact between the opposing asperities of the contact surfaces in local small areas of the whole mixed EHL contact. The experiments of mixed EHL of Begelinger and Gee [18, 19] and of Tabor [20] showed that these small areas of oxidized chemical boundary layer lubrication and dry contact of asperity have significant effects on the transition of the stage of mixed EHL in a practical mixed EHL contact. In a practical mixed EHL contact, the occurrence of these oxidized chemical boundary layer lubrication and dry contact of asperity usually quickly causes the mixed EHL stage transited from the mixed EHL stage of relatively thick overall mixed EHL film to the mixed EHL stage of oxidized chemical boundary layer lubrication mainly and widely occurring in the Hertzian zone, i.e. the first primary transition of the mixed EHL stage as reviewed and described in the above sections [18, 19]. These show that the future mode of mixed EHL better fits the mode of a real mixed EHL than the classical and modern modes of mixed EHL as described in the above sections.

7. CONCLUSIONS

The present paper reviews the developments of mixed elastohydrodynamic lubrication (EHL) in the industry and in the academic research in the past time. In the review of the academic research of mixed EHL in the past time, the experimental study of mixed EHL and the theoretical modelling of mixed EHL in the past time both are reviewed. These reviews show that the essential features of mixed EHL are thinning, mixed and partial lubricating films usually respectively simultaneously locally occurring in a mixed EHL contact both in the industry and in the academic research. In a mixed EHL contact, the thinning lubricating films includes the overall thinning EHL films and the local thinning EHL films. These thinning EHL films can be respectively caused by the effects of the fluid non-Newtonian rheological behavior, the contact surface roughness or the frictional heating of the whole mixed EHL contact or by the combined effect of these factors in a mixed EHL contact. The overall thinning EHL films and the local thinning EHL films both can simultaneously occur in a mixed EHL contact depending on the operating condition. These EHL film thinning both can be severe and even extreme in severe operating conditions in a mixed EHL contact. In a mixed EHL contact, the mixed lubricating films are defined as the lubricating films with different rheological behaviors respectively simultaneously locally occurring in a mixed EHL contact and thus mixed in the whole mixed EHL contact. These lubricating films with different rheological behaviors in a mixed EHL contact include the relatively thick and thus continuum fluid film, the molecularly thin and thus non-continuum fluid film, the oxidized chemical boundary layer lubrication film and the dry contact between the opposing asperities of the contact surfaces. According to the experimental study of mixed EHL respectively carried out by Begelinger and Gee [18, 19] and by Tabor [20] and the theoretical modelling of mixed EHL by Zhang and his colleagues [21, 29, 30, 31], the relatively thick and thus continuum fluid film, the molecularly thin and thus non-continuum fluid film, the oxidized chemical boundary layer lubrication and the dry contact between the opposing asperities of the contact surfaces usually respectively simultaneously locally occur in a mixed EHL contact and respectively have significant effects on the performance of a mixed EHL contact. In a practical mixed EHL contact, usually, the dry contact between the opposing asperities of the contact surfaces is instant and only occurs in local small areas of the whole mixed EHL contact due to the oxidization of the fresh metals of the contact surfaces in the newly formed asperity contact between the contact surfaces by the oxygen within the mixed EHL contact. In a mixed EHL contact, the partial lubricating film is here defined as the lubricating film with a certain rheological behavior in a mixed EHL which only occurs in part of the whole mixed EHL contact and is thus partial in the whole mixed EHL contact due to the lubricating films with different rheological behaviors respectively simultaneously locally occurring in the mixed EHL contact or/and the dry contact between the opposing asperities of the contact surfaces locally occurring in the whole mixed EHL contact, instead of defined as the Newtonian fluid film simultaneously locally occurring in a mixed EHL contact and thus mixed in the whole mixed EHL contact with the dry contact between the opposing asperities of the contact surfaces locally occurring in the contact as classically defined. In a practical mixed EHL contact, due to the mixed lubricating films and the dry contact between the opposing asperities of the contact surfaces respectively simultaneously locally occurring in the mixed EHL contact as described above, the lubricating film is actually usually complicated and very partial.

According to the developments and characteristics of the modes of mixed EHL studied in the theoretical modelling of mixed EHL in the past time, the modes of mixed EHL taken in the theoretical modelling of mixed EHL are here classified as three kinds, i.e. the classical mode of mixed EHL, the modern mode of mixed EHL and the future mode of mixed EHL. The classical mode of mixed EHL is defined as the mode of mixed EHL in which the hydrodynamic fluid film occurs in the entire area of the whole mixed EHL contact and is relatively thick and thus continuum across the fluid film thickness in the whole mixed EHL contact. The modes of mixed EHL taken in the theoretical modelling of mixed EHL before 1990s all belong to this mode of mixed EHL. The modern mode of mixed EHL is defined as the mode of mixed EHL in which the lubricating films with different rheological behaviors and the dry contact between the contact surfaces respectively simultaneously locally occur in different areas of the whole mixed EHL contact and are thus mixed in the whole mixed EHL contact depending on the operating condition due to the severe thinning of the overall and local lubricating films and the disappearance of the local lubricating film in a mixed EHL contact. The modern mode of mixed EHL is the modes of mixed EHL proposed and taken in the EHL modelling in the beginning of this century by Zhang and his colleagues [21, 29, 30, 31]. The future mode of mixed EHL is defined as the mode of mixed EHL in which the lubricating films with different rheological behaviors and the dry contact respectively simultaneously occur in different areas of the whole mixed EHL contact between the rough contact surfaces depending on the operating condition and may respectively locally occur in more irregular areas of the contact due to the local lubricating film thickening, thinning and disappearance in more irregular areas of the contact respectively due to the furrow denting out of and the ridge penetration into the fluid film of the contact surfaces. In the future mode of mixed EHL, in a mixed EHL contact, the fluid film between the rough contact surfaces may be locally molecularly thin, i.e. non-continuum across the fluid film thickness on some separate locations of the whole mixed EHL contact. At the same time, the oxidized chemical boundary layer lubrication and the dry contact between the opposing asperities of the contact surfaces may occur respectively on some other separate locations of the whole mixed EHL contact. In the other zones of the whole mixed EHL contact, where the fluid film is relatively thick and thus continuum across the fluid film thickness, the fluid rheological behavior is shear-thinning viscoelastic or viscoplastic depending on the operating condition.

As commented in section 3.1, the classical mode of mixed EHL does essentially not hold any of the essential features of a practical mixed EHL contact which are thinning, mixed and partial lubricating films in the whole mixed EHL contact. The classical mode of mixed EHL does therefore qualitatively not fit the mode of a practical mixed EHL. This mode of mixed EHL is oversimplified compared to a practical mixed EHL and should be rejected in the mixed EHL modelling in the future. As commented in sections 2.1 and 2.2.1.2, the configuration of the modern mode of mixed EHL well agrees with the mode of a practical mixed EHL. The modern mode of mixed EHL is thus much better than the classical mode of mixed EHL. As commented in section 2.2.2.1, the modern mode of mixed EHL also has its limitations of applicability to a practical mixed EHL contact due to its shortcomings. The future mode of mixed EHL has the essential features of thinning, mixed and partial lubricating films respectively simultaneously locally occurring in a mixed EHL contact as typically occur in the mixed EHL in the industry and typically shown by the results of both experiments and theoretical modelling of mixed EHL obtained in the past time. The future mode of mixed EHL better fits the mode of a real mixed EHL than the classical and modern modes of mixed EHL, according to the experimental results of mixed EHL respectively obtained by Begelinger and Gee [18, 19] and by Tabor [20]. The future mode of mixed EHL therefore needs to be taken in the theoretical modelling of mixed EHL in the future.

In the theoretical modelling of the future mode of mixed EHL, detailed, fine and time-dependent results for mixed EHL are of purpose. In this mixed EHL modelling, the fluid rheological model needs to be non-Newtonian and can be implemented to the fluid film in the whole fluid film thickness range, i.e. the molecularly thin and thus non-continuum fluid film and the relatively thick and thus continuum fluid film both respectively simultaneously locally occurring in the mixed EHL contact. On the other hand, the contact-fluid interfacial shear strength and contact-fluid interfacial slippage effect in a mixed EHL needs to be incorporated by this fluid rheological model. This fluid rheological model can be expressed by Eq. (1) as an example. In this mixed EHL modelling, the mixed EHL contact is more of engineering and real. The contact surface roughness is most frequently taken as engineering and real contact surface roughness. In this mixed EHL modelling, the frictional heating effect both within the fluid film and in the dry contact of asperity needs to be incorporated. The lubrication stage transition as occurs in a practical mixed EHL contact and experimentally found by Begelinger and Gee [18, 19] and by Tabor [20] needs to be studied with the operating condition by this mixed EHL modelling. As a further research, the friction, wear and contact surface destruction, i.e. contact surface failure as respectively occur in a practical mixed EHL contact and experimentally found by Begelinger and Gee [18, 19] and by Tabor [20] in dependence on the operating condition also needs to be modelled by this mixed EHL modelling.

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NOMENCLATURE

C = $\tau_{l0} / (E`G^{0.4})$

C_0 = $\tau_{l0} / E`$

C_p = dimensionless shear strength of the contact-fluid interface in the EHL inlet zone, $\tau_{l0} / (E`G^{0.4})$

E' = compound Young's modulus of the contact surfaces

G = dimensionless material parameter of the contact, $\alpha E'$

G_{ncf}^{eff}, η_{ncf}^{eff} and $\tau_{l,ncf}$ = respectively shear modulus of elasticity, viscosity and shear strength of the fluid which is either non-continuum or continuum across the fluid film thickness

h = fluid film thickness

h_c = EHL film thickness at the Hertzian contact center

$h_{cr,\ ncf}$ = critical thickness of the non-continuum fluid film

$h_{ctv\text{-}p}$ = central EHL film thickness in thermal condition based on the assumption of the contact-fluid interfacial shear strength

h_{ciN} = central EHL film thickness in isothermal condition based on the Newtonian fluid model

$H_{c,pr}^{v-p}$ = fluid film thickness at the Hertzian contact center in isothermal pure rolling EHL of ideally smooth line contacts due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect

H_c = dimensionless EHL film thickness at the Hertzian contact center, h_c/R

k = constant

p = fluid film pressure

p_h = maximum Hertzian contact pressure

p_s = fluid solidification pressure or solidification pressure of the contact-fluid interface

R = compound curvature radius of the contact surfaces of the EHL contact

R_{cr0} = critical compound curvature radius of the contact surfaces of the EHL contact in isothermal pure rolling EHL of ideally smooth line contacts for the occurrence of the non-continuum fluid film in the Hertzian contact zone due to the contact-fluid interfacial shear strength and contact-fluid interfacial slip effect.

S = slide-roll ratio, $2(U_a-U_b)/(U_a+U_b)$

t = time

u_a, u_b = circumferential speeds of the upper and lower contact surfaces of the EHL contact respectively

U_{ch} = dimensionless characteristic rolling speed, $0.0372W^{1.50}/G$

U_a, U_b = dimensionless circumferential speeds of the upper and lower contact surfaces of the EHL contact respectively, $\eta_0 u_a/E`$, $\eta_0 u_b/E`R$,

U_{b1}, K_1 and n_1 = respectively functions of the dimensionless shear strength C_p of the contact-fluid interface in the inlet zone ($C_p=\tau_{l0}/(kE`G^{0.4})$)

U = dimensionless rolling speed of the EHL contact, $(U_a+U_b)/2$

w = carried load of the whole contact per unit axial length

W = dimensionless load, $w/E`R$

α = constant depending on the fluid chemical composition or fluid viscosity-pressure index

$\alpha_{\tau l}$ = bulk fluid shear strength-pressure proportionality

β = constant

η_0, η = bulk fluid viscosities at ambient pressure and film pressure respectively

$\dot{\gamma}$ = fluid film shear strain rate

τ = fluid film shear stress

$\tau_0(p,h) = \tau_{l,\,ncf}(p,h)/2.8$

$\tau^{i}_{l,ncf}$ = shear strength of the contact-fluid interface

τ_{l0} = shear strength of the contact-fluid interface in the inlet zone (at ambient pressure) or bulk fluid shear strength at ambient pressure

τ_l = shear strength of the contact surface adhering layer-bulk fluid interface or fluid shear strength

$\Delta\tau_l$ and Δp = respectively variations in τ_l and p

PREGLED MIJEŠANOG PODMAZIVANJA U KONCENTRIRANIM DODIRIMA: PRORJEĐIVANJE, MIJEŠANI I PARCIJALNI SLOJEVI, KLASIČNI, MODERNI I BUDUĆI OBLICI

SAŽETAK

Ovaj rad opisuje miješano podmazivanje u koncentriranim dodirima, tj. miješano elastohidrodinamičko podmazivanje (miješano EHL) kako linijski tako i u dodirnim točkama. Zbog tangencijalnog ne-Newton-ovskog prorijeđivanja fluida, hrapavosti dodirnih površina i toplinskih djelovanja trenja, naročito u grubim radnim uvjetima miješanog EHL, sloj fluida može biti jako prorijeđen, a debljina fluidnog sloja u Hertzian-ovoj zoni može se reducirati do veličine molekule i čak postupnog nestajanja. Posljedica ovoga je pojava oksidiranog kemijskog graničnog sloja podmazivanja ili suhog dodira između suprotnih hrapavih dodirnih površina gdje nestaje sloj fluida. U stvari, miješani EHL u koncentriranim dodirima mogu se postupno prenositi od miješanog EHL od relativno debelog sloja fluida do kemijskog graničnog sloja podmazivanja, koji se uglavnom pojavljuju u Hertzian-ovoj zoni te dalje do razaranja dodirnih površina, tj. odnošenje oksidiranog kemijskog graničnog sloja sa dodirnih površina ili suhih dodira između hrapavih površina, koje se uglavnom pojavljuju u Hertzian-ovoj zoni u kojoj nestaje sloj fluida, kad se poveća oporost radnih uvjeta, tj. nosivo opterećenje dodira, brzina klizanja dodira (kontakta), sveukupna temperatura fluida postupno se povećava. Čak u miješanom EHL, koji ima debeli sloj fluida, male površine oksidiranog kemijski graničnog sloja podmazivanja, može nastati neposredan suhi dodir na mjestu između suprotnih hrapavih dodirnih površina u uobičajenim radnim uvjetima.Ovo granično podmazivanje kao i suhi dodir imaju važan utjecaj na djelovanje i prijelaz ovog miješanog EHL relativno debelog sloja fliuda rezultirajući brzim prijelazom ovog miješanog EHL u kemijski granični sloj podmazivanja, koji se uglavnom pojavljuje u Hertzian-ovoj zoni. Sve to pokazuje da su sloj podmazivanja i suhi dodir obično izmiješani zbog različitog reološkog ponašanja također izmiješani zbog različitog reološkog ponašanja filmova za podmazivanje, koji se simultano pojavljuju u različitim područjima izmiješanog EHL dodira. Izmiješani slojevi podmazivanja, koji se simultano pojavljuju u dotičnom izmiješanom EHL, sadrže viskoelastičan kontinuum, relativno debeli, sloj fluida, viskoplastičan kontinuum, relativno debeli, sloj fluida, nekontinuum sloj fluida, tj. fizički apsorbiran granični sloj filma za podmazivanje, kemijski granični sloj kao i iščezavajući sloj podmazivanja, tj. suhi dodir. Svaki od ovih izmiješanih filmova za podmazivanje mogu samo lokalno biti prisutni, stoga samo djelomično prisutni u dotičnom izmiješanom EHL dodiru. S obzirom na razvoj i buduće tendencije, oblik izmiješanog EHL u zgusnutim dodirima dijeli se na tri vrste: klasičan, moderan i budući oblik izmiješanog EHL u koncentriranim dodirima. Klasičan oblik miješanog EHL u koncentriranim dodirima odnosi se na oblik izmiješanog EHL u zgusnutim dodirima u kojima je sloj fluida između dodirnih površina sveukupno relativno debeo. Tako se kontinuum dodiruje kroz debljinu sloja fluida u čitavom izmiješanom EHL. Moderan oblik izmiješanog EHL u koncentriranim dodirima odnosi se na oblik miješanog EHL u kojima je sloj fluida između dodirnih površina molekularno tanak. Tako se nekontinuum kroz debljinu sloja fluida na nekim odvojenim mjestima *dodira ili suhog kontakta hrapavosti pojavljuje na nekim odvojenim mjestima dodira, dok je u drugim zonama dodir sloja fluida relativno debeo, tako da je kontinuum kroz debljinu filma fluida i ponašanje ne-Newton-ovskog fluida, tangencijalno stanjivanje, viskoelastično ili viskoplastično, ovisno o radnim uvjetima. Budući oblik izmiješanog EHL u koncentriranim dodirima odnosi se na oblik izmiješanog EHL u koncentriranim 2dodirima u kojima sloj fluida između dodirnih površina može biti molekularno tanak, tj. nekontinuum kroz debljinu sloja fluida na nekim odvojenim mjestima dodira. Istovremeno kemijski granični sloj podmazivanja i suhi dodir mogu se pojaviti na nekim donjim odvojenim mjestima dodira, dok u drugim zonama dodira sloj (film) fluida je relativno debeo, te je kontinuum kroz debljinu sloja fluida i ponašanje fluida viskoelastično tangencijalno stanjivanje ili viskoplastično ovisno o radnim uvjetima. Zaključuje se da budući oblik izmiješanog EHL u koncentriranim dodirima najbolje očituje oblik izmiješanog EHL u stvarnim koncentriranim dodirima između ova tri oblika izmiješanog EHL u koncentriranim dodirima. To je smjernica budućeg istraživanja miješanog EHL. Također se može reći da proučavanje budućeg oblika miješanog EHL u koncentriranim dodirima, kao i detaljni te pouzdani rezultati ovisni o vremenu su svrsishodni te da miješani EHL dodir pripada tehnici i stvarnosti. U ovoj studiji o miješanom EHL model fluida mora biti ne-Newton-ovski, hrapavost dodirne površine treba shvatiti da pripada tehnici i stvarnosti, a djelovanje zagrijavanja trenja i u sloju tekućine i u suhom dodiru hrapavosti treba biti uključeno. U proučavanju budućeg oblika miješanog EHL, prijelaznu fazu podmazivanja, koja se pojavljuje u stvarnim dodirima miješanog EHL, treba proučavati u radnim uvjetima. Također treba proučavati trenje, trošenje te trošenje dodirnih površina koji se pojavljuju u stvarnim EHL dodirima u smislu njihove ovisnosti o radnim uvjetima.*

Ključne riječi: *elastohidrodinamičko podmazivanje, miješano podmazivanje, koncentrirani dodiri, oblici podmazivanja, stanjivanje, miješani slojevi, parcijalni slojevi.*