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Reexamining a single-producer multi-retailer integrated inventory model with rework using algebraic method

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SUMMARY

In this study, a single-producer multi-retailer integrated inventory model with rework is reexamined using mathematical modeling and an algebraic method. It is assumed that a product is manufactured through an imperfect production process, and the reworking of random defective items is done right after the regular process in each cycle. After the entire lot is quality assured, multiple shipments will be delivered synchronously to m different retailers in each production cycle. The objective is to find the optimal production lot size and optimal number of shipments that minimizes total expected costs for such a specific supply chains system. The conventional approach uses differential calculus on system cost function to derive the optimal production- shipment policy (Chiu et al. [1]); in contrast, the proposed algebraic approach is a straightforward method that enables practitioners who may not have sufficient knowledge of calculus to understand and manage real-world systems more effectively.

Key words: *supply chains, algebraic approach, lot size, multiple retailers, rework, multiple shipments.*

1. INTRODUCTION

An inventory model that employs mathematical techniques to determine the most economical production lot was first proposed by Taft [2]. This is also known as the economic production quantity (EPQ) model [3]. The EPQ model assumes a continuous inventory issuing policy to satisfy product demand. However, in real-life vendor-buyer integrated systems, multiple or periodic deliveries of end items are often used. Therefore, determination of the optimal number of delivery that minimizes the production-delivery cost becomes a critical management decision for such a system. Schwarz [4], first studied a one-warehouse N-retailer inventory system with the objective of determining the stock refilling policy that minimizes the expected total system cost per unit time. Studies related to different aspects of the supply chain optimization have since been extensively carried out [5-11].

Another special focus of the present study is the product quality obtained from the producer. The classic economic production quantity (EPQ) model assumes a perfect production, however, in a real life production environment, make it likely various unpredictable factors random defective items are produced. We consider that all nonconforming items are reworked and repaired in order to assure that the entire lot has the expected quality. In past decades, many studies have attempted to address the issues of defective products and quality assurance in production systems [12-18].

Recently, Grubbström and Erdem [19], proposed an algebraic derivation for solving the economic order quantity (*EOQ*) model with backlogging. Their method does not reference to first- or second-order derivatives. Similar approaches were applied to solve various aspects of production and supply chain optimizatio [20-23]. This paper extends such an algebraic approach in order to reexamine a single-producer multi-retailer integrated inventory model with a rework process [1].

2. MODEL DESCRIPTION AND MODELLING

In this study, we present an algebraic approach to reexamine a single-producer multi-retailer integrated inventory model with a rework process [1]. Such a specific supply chains model is described as follows. Consider that a product can be made at an annual production rate P, and the process may randomly generate an x portion of nonconforming items at a rate of d. All items produced are screened and the inspection expense is included in the unit production cost C. All defective items are assumed to be re-workable at a rate of P_1, and a rework process starts right after the end of regular production in each cycle (see Figure 1).

Under the normal operation, to prevent shortages from occurring, the constant production rate P must satisfy $(P\text{-}d\text{-}\lambda)>0$, where λ is the sum of annual demands of retailers and $d=Px$. This study considers that a multi-shipment policy and the finished items can only be delivered to the retailers when the entire lot is quality assured in the end of rework. Each retailer has its own annual demand rate λ_i. Fixed quantity n installments of the finished batch are delivered to multiple retailers synchronously at a fixed interval of time during the downtime t_3 (Figure 1).

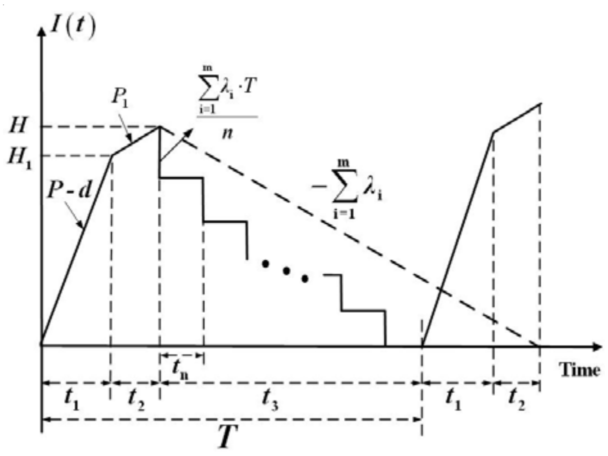

Fig. 1 On-hand inventory of perfect quality items in producer side

All cost-related parameters used in this study are as follows: the unit production cost C, production setup cost K, unit holding cost h, unit cost C_R and unit holding cost h_1 for each reworked item, unit disposal cost C_S, the fixed delivery cost K_{1i} per shipment delivered to retailer i, unit holding cost h_{2i} for item kept by retailer i, and unit shipping cost C_i for item shipped to retailer i. Other notation includes:

- H_1 – level of on-hand inventory in units when regular production process ends,
- H – maximum level of on-hand inventory in units when the rework process ends,
- t_1 – the production uptime for the proposed system,
- t_2 – time required for reworking the nonconforming items produced in each cycle,
- t_3 – time required for delivering all quality assured finished products to retailers,
- Q – production lot size per cycle, a decision variable (to be determined),
- n – number of fixed quantity installments of the finished batch to be delivered to retailers for each cycle, a decision variable (to be determined),
- m – number of retailers,
- t_n – a fixed interval of time between each installment of finished products delivered during production downtime t_2,
- T – production cycle length,
- $I(t)$– on-hand inventory of perfect quality items at time t,
- $I_c(t)$ – on-hand inventory at the retailers at time t,
- $TC(Q,n)$ – total production-inventory-delivery costs per cycle for the proposed system,
- $E[TCU(Q,n)]$ – total expected production-inventory-delivery costs per unit time for the proposed system.

From Figure 1 and with reference to Ref. [1], note that the total production-inventory-delivery cost per cycle $TC(Q,n)$ consists of the following: the setup cost, variable production cost, the cost for the reworking, disposal cost, the fixed and variable delivery cost, holding cost during production uptime t_1 and reworking time t_2, and holding cost for finished goods kept by both the manufacturer and the customer during the delivery time t_3, i.e.:

$$TC(Q,n) = CQ + K + C_R[xQ] + n\sum_{i=1}^{m} K_{1i} + \sum_{i=1}^{m} C_i \lambda_i T +$$
$$+ h\left[\frac{H_1 + dt_1}{2}(t_1) + \frac{H_1 + H}{2}(t_2) + \left(\frac{n-1}{2n}\right)Ht_3\right] +$$
$$+ h_1 \frac{P_1 \cdot t_2}{2}(t_2) + \frac{1}{2}\sum_{i=1}^{m} h_{2i}\lambda_i\left[\frac{Tt_3}{n} + (t_1 + t_2)T\right] + \tag{1}$$

Taking the randomness of defective rate into account, the expected values of x are used in cost analysis of this study. Substituting all parameters [1] and with further derivations, the expected cost $E[TCU(Q,n)]$ can be obtained as follows:

$$E\left[TCU(Q,n)\right] = C\sum_{i=1}^{m}\lambda_i + \frac{1}{Q}\left(K + n\sum_{i=1}^{m}K_{1i}\right)\sum_{i=1}^{m}\lambda_i + C_R E[x]\sum_{i=1}^{m}\lambda_i + \sum_{i=1}^{m}C_i\lambda_i +$$
$$+\frac{h}{2}\left(Q\sum_{i=1}^{m}\lambda_i\right)\left[\frac{1}{P}+\frac{1}{P_1}\left(2E[x]-\left(E[x]\right)^2\right)\right]+\left(\frac{n-1}{2n}\right)\left(hQ\sum_{i=1}^{m}\lambda_i\right)\left[\frac{1}{\sum_{i=1}^{m}\lambda_i}-\frac{1}{P}-\frac{E[x]}{P_1}\right]+$$
$$+\frac{h_1}{2}\left(\frac{1}{P_1}\right)\left(E[x]\right)^2\cdot Q\sum_{i=1}^{m}\lambda_i+\left(\frac{n-1}{2n}\right)\left(\sum_{i=1}^{m}h_{2i}\lambda_i Q\right)\left[\frac{1}{P}+\frac{E[x]}{P_1}\right]+\left(\frac{1}{2n}\right)\frac{1}{\sum_{i=1}^{m}\lambda_i}\left(\sum_{i=1}^{m}h_{2i}\lambda_i Q\right) \tag{2}$$

3. THE PROPOSED ALGEBRAIC APPROACH

It should be noted that Eq. (2) contains two decision variables, namely Q and n. Moreover, there are several different forms of decision variables in the right-hand side of Eq. (2), such as Q, Q^{-1}, nQ^{-1}, and Qn^{-1}. Hence, Eq. (2) can be rearranged as:

$$E\left[TCU(Q,n)\right] = C\sum_{i=1}^{m}\lambda_i + C_R E[x]\sum_{i=1}^{m}\lambda_i + \sum_{i=1}^{m}C_i\lambda_i + K\sum_{i=1}^{m}\lambda_i(Q^{-1}) +$$
$$+\frac{1}{2}\left[\left(h\sum_{i=1}^{m}\lambda_i\right)\left[\frac{E[x]-\left(E[x]\right)^2}{P_1}+\frac{1}{\sum_{i=1}^{m}\lambda_i}\right]+\left(\frac{h_1}{P_1}\right)\left(E[x]\right)^2\sum_{i=1}^{m}\lambda_i+\sum_{i=1}^{m}h_{2i}\lambda_i\left[\frac{1}{P}+\frac{E[x]}{P_1}\right]\right](Q)+$$
$$+\left(\frac{1}{2}\right)\left[\frac{1}{\sum_{i=1}^{m}\lambda_i}-\frac{1}{P}-\frac{E[x]}{P_1}\right]\left(\sum_{i=1}^{m}h_{2i}\lambda_i - h\sum_{i=1}^{m}\lambda_i\right)\left(n^{-1}Q\right)+\left(\sum_{i=1}^{m}K_{1i}\right)\sum_{i=1}^{m}\lambda_i(nQ^{-1}) \tag{3}$$

Let:

$$\beta_1 = C\sum_{i=1}^{m}\lambda_i + C_R E[x]\sum_{i=1}^{m}\lambda_i + \sum_{i=1}^{m}C_i\lambda_i \tag{4}$$

$$\beta_2 = K\sum_{i=1}^{m}\lambda_i \tag{5}$$

$$\beta_3 = \frac{1}{2}\left[\left(h\sum_{i=1}^{m}\lambda_i\right)\left[\frac{E[x]-\left(E[x]\right)^2}{P_1}+\frac{1}{\sum_{i=1}^{m}\lambda_i}\right]+\left(\frac{h_1}{P_1}\right)\left(E[x]\right)^2\sum_{i=1}^{m}\lambda_i+\sum_{i=1}^{m}h_{2i}\lambda_i\left[\frac{1}{P}+\frac{E[x]}{P_1}\right]\right] \tag{6}$$

$$\beta_4 = \frac{1}{2}\left[\frac{1}{\sum_{i=1}^{m}\lambda_i} - \frac{1}{P} - \frac{E[x]}{P_1}\right]\left(\sum_{i=1}^{m} h_{2i}\lambda_i - h\sum_{i=1}^{m}\lambda_i\right) \tag{7}$$

$$\beta_5 = \left(\sum_{i=1}^{m} K_{1i}\right)\sum_{i=1}^{m}\lambda_i \tag{8}$$

Then Eq. (3) becomes:

$$E\left[TCU(Q,n)\right] = \beta_1 + \beta_2(Q^{-1}) + \beta_3(Q) + \beta_4\left(n^{-1}Q\right) + \beta_5(nQ^{-1}) \tag{9}$$

If Eq. (9) is rearranged, then:

$$E\left[TCU(Q,n)\right] = \beta_1 + (Q^{-1})\left[\sqrt{\beta_3}Q - \sqrt{\beta_2}\right]^2 + \left(n^{-1}Q\right)\left[\sqrt{\beta_5}\left(nQ^{-1}\right) - \sqrt{\beta_4}\right]^2 + \\ + 2\sqrt{\beta_3 \cdot \beta_2} + +2\sqrt{\beta_5 \cdot \beta_4} \tag{10}$$

It is noted that Eq. (10) will be minimized if its second and third terms equal zero. That is:

$$Q = \sqrt{\frac{\beta_2}{\beta_3}} \tag{11}$$

and:

$$n = \sqrt{\frac{\beta_4}{\beta_5}} \cdot Q \tag{12}$$

Substituting Eqs. (5) and (6) into Eq. (11), and substituting Eqs. (7), (8) and (11) into Eq. (12), the optimal number of shipments n^* is:

$$n^* = \sqrt{\frac{K\left(\sum_{i=1}^{m} h_{2i}\lambda_i - h\sum_{i=1}^{m}\lambda_i\right)\left[\left(\sum_{i=1}^{m}\lambda_i\right)^{-1} - \left(\frac{1}{P} + \frac{E[x]}{P_1}\right)\right]}{\sum_{i=1}^{m} K_{1i}\left\{\begin{array}{l} h\sum_{i=1}^{m}\lambda_i\left[\frac{1}{P} + \frac{1}{P_1}\left[2E[x] - \left(E[x]\right)^2\right]\right] + h + \\ + \left(\sum_{i=1}^{m} h_{2i}\lambda_i - h\sum_{i=1}^{m}\lambda_i\right)\left(\frac{1}{P} + \frac{E[x]}{P_1}\right) + h_1\left(E[x]\right)^2 \sum_{i=1}^{m}\lambda_i\left(\frac{1}{P_1}\right)\end{array}\right\}}} \tag{13}$$

Note that Eq. (13) is identical to the one obtained by using the conventional differential calculus method [1]. Now, in order to find the integer value of n^* that minimizes the expected system cost, the two adjacent integers to n must be examined respectively for cost minimization [23]. Let n^+ denote the smallest integer greater than or equal to n (derived from Eq. (13)) and n^- denote the largest integer less than or equal to n. Because n^* is either n^+ or n^-, we can first treat $E[TCU(Q,n)]$ (Eq. (2)) as a cost function with a single decision variable Q, and do the following rearrangements:

$$E\left[TCU(Q,n)\right] = \beta_1 + \left[\beta_2 + \beta_5(n)\right](Q^{-1}) + \left[\beta_3 + \beta_4\left(n^{-1}\right)\right](Q) \tag{14}$$

or:

$$E\left[TCU(Q,n)\right] = \beta_1 + (Q^{-1})\left[Q\sqrt{\beta_3 + \beta_4\left(n^{-1}\right)} - \sqrt{\beta_2 + \beta_5(n)}\right]^2 + \\ + 2\sqrt{\beta_3 + \beta_4\left(n^{-1}\right)} \cdot \sqrt{\beta_2 + \beta_5(n)} \tag{15}$$

Upon derivation of Eq. (15), note that $E[TCU(Q,n)]$ will be minimized if the second term of Eq. (15) equals zero. That is:

$$Q=\sqrt{\frac{\beta_2+\beta_5(n)}{\beta_3+\beta_4\left(n^{-1}\right)}} \tag{16}$$

Substituting Eqs. (5), (6), (7) and (8) into Eq. (16), the optimal production lot size is:

$$Q^*=\sqrt{\frac{2\left(K+n\sum_{i=1}^{m}K_{1i}\right)\sum_{i=1}^{m}\lambda_i}{\left\{\begin{array}{l}h\sum_{i=1}^{m}\lambda_i\left[\frac{1}{P}+\frac{1}{P_1}\left[\left(2E[x]-\left(E[x]\right)^2\right)\right]\right]+h_1\left(E[x]\right)^2\sum_{i=1}^{m}\lambda_i\frac{1}{P_1}+\\+\left(\frac{n-1}{n}\right)\left[h+\left(\sum_{i=1}^{m}h_{2i}\lambda_i-h\sum_{i=1}^{m}\lambda_i\right)\left(\frac{1}{P}+\frac{E[x]}{P_1}\right)\right]+\sum_{i=1}^{m}h_{2i}\lambda_i\left(n\sum_{i=1}^{m}\lambda_i\right)^{-1}\end{array}\right\}}} \tag{17}$$

It should be noted that Eq. (17) is identical to the one obtained by using the conventional differential calculus method [1]. Finally, to find the optimal replenishment-delivery (Q^*, n^*) policy, one can substitute all related system parameters, along with n^+ and n^-, in Eq. (17). Then, applying the resulting (Q, n^+) and (Q, n^-) in Eq. (3) respectively, the one that gives the minimum expected system cost is selected as the optimal (Q^*, n^*) policy.

4. NUMERICAL EXAMPLE

The aforementioned algebraic approach and its resulting Eqs. (13), (17), and (3) are verified in this section using the same numerical example as used in Ref. [1]. Consider in a single-producer multi-retailer integrated inventory model that a product can be made at a rate $P = 60{,}000$ units per year and its annual demands λ_i from 5 different retailers are *650*, *350*, *450*, *800*, and *750* respectively (total demand $\lambda = 3000$ per year). The random defective rate x follows a uniform distribution over the range of [*0*, *0.3*]. All nonconforming items are reworked and repaired at a reworking rate $P_1 = 3600$ per year.

Other values of parameters are $C = \$100$, $K = \$35000$, $h = \$25$, $h_1 = \$60$, $C_R = \$60$, K_{1i} for retailer i are *\$400*, *\$100*, *\$300*, *\$450*, and *\$250* respectively, h_{2i} for retailer i are *\$70*, *\$80*, *\$75*, *\$60*, and *\$65* respectively, C_i for retailer i are *\$0.5*, *\$0.4*, *\$0.3*, *\$0.2*, and *\$0.1* respectively.

First determine the optimal number of delivery by computing Eq. (13), one has $n = 4.51$. Then, examine the two adjacent integers to n and applying Eq. (17), one obtains (Q, n^+) = (*2310,5*) and (Q, n^-) = (*2228,4*). Finally, substitute these (Q, n^+) and (Q, n^-) in Eq. (3) respectively. Choosing the one that gives the minimum system cost as our optimal policy, one obtains (Q, n^+) = (*2310,5*), and total expected cost $E[TCU(Q^*,n^*)] = \$438{,}211$.

The research results were confirmed to be identical to those obtained by the traditional method presented in Ref. [1].

5. CONCLUDING REMARKS

This study proposes an algebraic approach for determining the optimal production-delivery policy for a single-producer multi-retailer integrated inventory model with a rework process. Unlike the conventional method which uses differential calculus on the system cost function to find the optimal policy [1], the proposed algebraic approach is a straightforward method that may enable practitioners with little knowledge of calculus to understand and manage such a real-life supply chains system more effectively.

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PREISPITIVANJE INTEGRIRANOG MODELA SKLADIŠTENJA KOJI UKLJUČUJE JEDNOG PROIZVOĐAČA, VIŠE TRGOVACA NA MALO I DORADU PROIZVODA KORISTEĆI ALGEBARSKU METODU

SAŽETAK

U ovom članku se preispituje integrirani model skladištenja koji obuhvaća jednog proizvođača, više trgovaca na malo i doradu proizvoda koristeći matematičko modeliranje i algebarsku metodu. Pretpostavlja se da je proizvod rezultat nesavršenog proizvodnog procesa, a dorada slučajno izabranih neispravnih proizvoda se obavlja odmah nakon redovitog proizvodnog procesa u svakom ciklusu. Nakon što je dokazana kvaliteta cjelokupno proizvedene količine, višestruke pošiljke će biti istovremeno isporučene na m različitih trgovaca na malo u svakom proizvodnom ciklusu. Cilj je pronaći optimalnu sveukupnu količinu proizvodnje i optimalni broj pošiljki koji minimaliziraju ukupne očekivane troškove za takve specifične sustave opskrbnih lanaca. Konvencionalni pristup koristi diferencijani račun baziran na funkciji troška sustava kako bi se dobila politika optimalne proizvodnje-isporuke (Chiu i sur. [1]), dok je predloženi algebarski pristup neposredna metoda koja omogućuje ljudima iz prakse, koji nemaju dovoljno znanje o diferencijalnom računu, bolje razumijevanje i učinkovitije upravljanje stvarnim sustavima.

Ključne riječi: *opskrbni lanac, algebarski pristup, količina robe, više trgovaca na malo, dorada proizvoda, višestruke isporuke.*