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Optimal production cycle time for multi-item FPR model with rework and multi-shipment policy

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SUMMARY

This paper determines the optimal common production cycle time for a multi-item finite production rate (FPR) model with rework and multi-shipment policy. The classic FPR model considers production planning for a single product with perfect quality production and a continuous issuing policy. However, in real life production environments, vendors often plan to produce multiple products in turn on a single machine in order to maximize the machine utilization. Also, due to various uncontrollable factors, generation of nonconforming items in any given production run is inevitable. It is also common for vendors to adopt multiple/periodic delivery policy for distributing their finished goods to customers. In this study, it is assumed that all nonconforming items can be reworked and repaired in the same cycle when regular production ends at additional cost per each reworked item. Our objective is to determine the optimal common production cycle time that minimizes the long-run average cost per unit time and to study the effect of rework on the optimal common cycle time for such a specific multi-item FPR model with rework and multi-shipment policy. Mathematical modeling is used, and the expected system cost for the proposed model is derived and proved to be convex. Finally, a closed-form optimal cycle time is obtained. A numerical example and sensitivity analysis is provided to show the practical use of our obtained results.

Key words: *multi-item production, finite production rate, common cycle time, rework, optimization, multi-shipment.*

1. INTRODUCTION

The classic finite production rate (FPR) model [1-3] considers production planning for a single product with perfect quality production and a continuous issuing policy. However, in real life production environments, vendors often plan to produce multiple products in turn on a single machine in order to maximize the machine utilization. Bergstrom and Smith [4] used the linear decision rules to a multi-item formulation which solves directly the optimum sales, production, and inventory levels for individual items in future periods. They showed that their proposed formulation can seek a solution to maximize the firm's profit over the time horizon by applying it to a firm producing a line of electric motors. Rosenblatt and Finger [5] considered a problem of multi-item production in a single facility. The proposed facility was an electrochemical machining system and the products were impact sockets of various sizes for power wrenches. A grouping procedure of the various items was adopted. A modified version of an existing algorithm was applied to insure production cycle times which are multiples of the shortest production cycle time. Tamura [6] presented an approximation procedure used to solve a production planning problem for a multistage production system which produces many different components and assembles them into finished products under capacity limitations. A generalized production planning model was built using the mixed-integer programming. The solution procedure was approximated by a linear programming method. Different algorithms were developed in detail for a two-stage production problem. Numerical example was provided to examine the validity and efficiency of the proposed algorithms. Studies related to various aspects of multi-item production planning and optimization issues have since been extensively conducted [7-12].

Also, in real life production environments, generation of defective items in any given production run is inevitable due to various uncontrollable factors. Mak [13] developed a mathematical model for an inventory system in which the number of units of acceptable quality in a replenishment lot is uncertain and the demand is partially captive. It was assumed that the fraction of the demand during the stock-out period, which can be backordered, is a random variable whose probability distribution is known. The optimal replenishment policy was synthesized for such a system. A numerical example was used to illustrate the theory. The results indicated that the optimal replenishment policy is sensitive to the nature of the demand during the stock-out period. Hariga and Ben-Daya [14] considered the economic production quantity problem in the presence of imperfect processes. The time to shift from the in-control state to the out-of-control state was assumed to be flexible, and they provided distribution-based and distribution-free bounds on the optimal cost. For the exponential case, they compared the optimal solutions to approximate solutions proposed in the literature. Many studies have since been conducted to address different aspects of imperfect production systems as well as quality assurance issues in production [15-21]. Another unrealistic assumption in the classic FPR model is the continuous inventory issuing policy. In real life; however, it is common for vendors to adopt multiple or periodic delivery policy for shipping their finished goods to customers. Schwarz et al. [22] examined the fill-rate of one-warehouse N-identical retailer distribution system. An approximation model was adopted from a prior study to maximize the system fill-rate subject to a constraint on the system safety stock. As a result, properties of the fill-rate policy were suggested to provide management when looking into system optimization. Hill [23] studied a model in which a manufacturing company purchases a raw material, manufactures a product (at a finite rate) and ships a fixed quantity of the product to a single customer at fixed and regular intervals of time, as specified by the customers. The objective was to determine a purchasing and production schedule which minimizes the total cost of purchasing, manufacturing and stockholding. Additional studies have also been extensively carried out to address the various aspects of periodic or multiple deliveries issues [24-31].

The purpose of this study is to determine the optimal common production cycle time for a multi-item finite production rate (FPR) model with rework and multi-shipment policy, and to study the effect of rework on the optimal common cycle time. For little attention has been paid to this area, this paper is intended to bridge the gap.

2. PROBLEM DESCRIPTION AND MATHEMATICAL MODELLING

This study examines the optimal common production cycle time for a multi-item finite production rate model with rework and multi-shipment policy. Consider there are L products to be made in turn on a single machine. All items made are screened and the unit inspection cost is included in the unit production cost C_i. During the manufacturing process, for each product i (where $i = 1, 2, \ldots, L$), there is an x_i portion of nonconforming items being produced randomly at a rate d_i. All nonconforming items can be reworked and repaired at a rate of P_{2i} right after the end of regular production process in each cycle at an additional cost C_{Ri}. Under the normal operation, the constant production rate for product i, P_{1i} must satisfy statement $(P_{1i} - d_i - \lambda_i) > 0$, where λ_i is the annual demand rate for product i, where d_i can be expressed as $d_i = x_i P_{1i}$. Unlike the classic FPR model assumes a continuous issuing policy to meet the product demand, this research adopts a multi-shipment policy. We assume that the finished items for each product i can only be delivered to customers if the whole production lot is quality assured at the end of rework process of each product i. Fixed quantity n installments of the finished batch are delivered at a fixed interval of time during delivery time t_{3i} (refer to Figure 1).

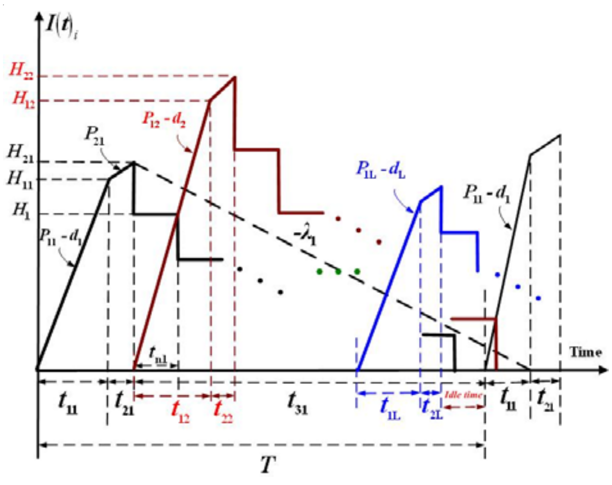

Fig. 1 On-hand inventory of perfect quality items for product i in a common production cycle

Other cost-related parameters used in this study include: unit holding cost h_i, production setup cost K_i, unit holding cost h_{1i} for each reworked item, the fixed delivery cost K_{1i} per shipment for product i, and unit shipping cost C_{Ti} for product i. Additional notation is listed as follows:

- T – common production cycle length, a decision variable (to be determined),
- H_{1i} – maximum level of on-hand inventory for product i when regular production ends,
- H_{2i} – maximum level of on-hand inventory in units for product i when rework process ends,
- t_{1i} – the production uptime for product i in the proposed system,
- t_{2i} – the rework time for product i in the proposed system,
- Q_i – production lot size per cycle for product i,
- n – number of fixed quantity installments of the finished batch to be delivered to customers in each cycle, it is assumed to be a constant for all products,
- t_{ni} – a fixed interval of time between each installment of finished products delivered during t_{2i}, for product i.
- $I(t)_i$ – on-hand inventory of perfect quality items for product i at time t,
- $ID(t)_I$ – on-hand inventory of defective items for product i at time t,
- $TC(Q_i)$ – total production-inventory-delivery costs per cycle for product i,
- $E[TCU(Q)]$ – total expected production-inventory-delivery costs per unit time for L products in the proposed system.
- $E[TCU(T)]$ – total expected production-inventory-delivery costs per unit time for L products in the proposed system using common production cycle time T as the decision variable.

The following equations can obtain directly from Figure 1:

$$T = t_{1i} + t_{2i} + t_{3i} = \frac{Q_i}{\lambda_i} \tag{1}$$

$$t_{1i} = \frac{Q_i}{P_{1i}} = \frac{H_{1i}}{P_{1i} - d_i} \tag{2}$$

$$t_{3i} = nt_{ni} = T - \left(t_{1i} + t_{2i}\right) \tag{3}$$

$$H_{1i} = \left(P_{1i} - d_i\right)t_{1i} \tag{4}$$

$$H_{2i} = H_{1i} + P_{2i}t_{2i} \tag{5}$$

The on-hand inventory of defective items during production uptime t_1 (see Figure 2) and the time required for reworking the defective items are:

$$d_i t_{1i} = x_i Q_i \tag{6}$$

$$t_{2i} = \frac{x_i Q_i}{P_{2i}} \tag{7}$$

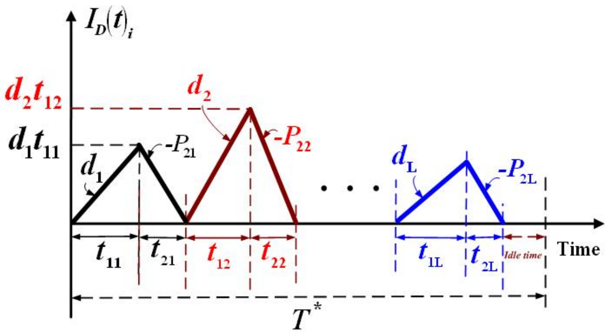

Fig. 2 On-hand inventory of defective items for product i in a common production cycle

Total delivery costs for product i (n shipments) in a cycle are:

$$nK_{1i} + C_{Ti}Q_i \tag{8}$$

The variable holding costs for finished products kept by the manufacturer, during the delivery time t_3 where n fixed-quantity installments of the finished batch are delivered to customers at a fixed interval of time, are as follows (see Appendix A in Ref. [29]):

$$h_i\left(\frac{n-1}{2n}\right)H_{2i}\,t_{3i} \tag{9}$$

Total production-inventory-delivery cost per cycle $TC(Q_i)$ for L products, consists of the variable production cost, setup cost, rework cost, fixed and variable delivery cost, holding cost during production uptime t_{1i} and rework time t_{2i}, and holding cost for finished goods kept during the delivery time t_3. Therefore, total $TC(Q_i)$ for L products is:

$$\sum_{i=1}^{L} TC\left(Q_i\right) = \sum_{i=1}^{L} \left\{ \begin{array}{l} C_iQ_i + K_i + C_{Ri}x_iQ_i + nK_{1i} + C_{Ti}Q_i + h_{1i}\left[\dfrac{P_{1i}t_{2i}}{2}\left(t_{2i}\right)\right] + \\ + h_i\left[\dfrac{H_{1i} + d_it_1}{2}\left(t_{1i}\right) + \dfrac{H_{1i} + H_{2i}}{2}\left(t_{2i}\right) + \dfrac{n-1}{2n}\left(H_{2i}t_{3i}\right)\right] \end{array} \right\} \tag{10}$$

Defective rate x is assumed to be a random variable with a known probability density function. In order to take the randomness of x into account, the expected values of x can be used in the cost analysis. Substituting all parameters from equations (1) to (9) into Eq. (10), and with further derivations, the expected $E[TCU(Q)]$ can be obtained as follows:

$$E\left[TCU(Q)\right]=E\left[\sum_{i=1}^{L}TC(Q_i)\right]\frac{1}{E\left[T\right]}=$$

$$=\sum_{i=1}^{L}\left\{\begin{array}{l}C_i\lambda_i+C_{\mathrm{R}i}\lambda_i E(x_i)+C_{\mathrm{T}i}\lambda_i+\dfrac{K_i\lambda_i}{Q_i}+\dfrac{nK_{1i}\lambda_i}{Q_i}+\dfrac{h_{1i}Q_i}{2}\left(\dfrac{\lambda_i E(x_i)^2}{P_{2i}}\right)+\\+\dfrac{h_iQ_i\lambda_i}{2}\left[\dfrac{E(x_i)}{P_{2i}}-\dfrac{E(x_i)^2}{P_{2i}}+\dfrac{1}{\lambda_i}-\left(\dfrac{1}{n}\right)(\dfrac{1}{\lambda_i}-\dfrac{1}{P_{1i}}-\dfrac{E(x_i)}{P_{2i}})\right]\end{array}\right\} \tag{11}$$

where $E\left[T\right]=Q_i/\lambda_i$.

By applying Eq. (1), Eq. (11) can be converted into $E[TCU(T)]$ as follows:

$$E\left[TCU(T)\right]=\sum_{i=1}^{L}\left\{\begin{array}{l}C_i\lambda_i+C_{\mathrm{R}i}\lambda_i E(x_i)+C_{\mathrm{T}i}\lambda_i+\dfrac{K_i}{T}+\dfrac{nK_{1i}}{T}+\dfrac{h_{1i}T\lambda_i}{2}\left(\dfrac{\lambda_i E(x_i)^2}{P_{2i}}\right)+\\+\dfrac{h_iT\lambda_i^2}{2}\left[\dfrac{E(x_i)}{P_{2i}}-\dfrac{E(x_i)^2}{P_{2i}}+\dfrac{1}{\lambda_i}-\left(\dfrac{1}{n}\right)(\dfrac{1}{\lambda_i}-\dfrac{1}{P_{1i}}-\dfrac{E(x_i)}{P_{2i}})\right]\end{array}\right\} \tag{12}$$

3. DERIVING THE OPTIMAL PRODUCTION CYCLE TIME

The optimal common production cycle time can be obtained by minimizing the expected cost function $E[TCU(T)]$. Differentiating $E[TCU(T)]$ with respect to T gives first and second derivative as:

$$\frac{\partial E\left[TCU(T)\right]}{\partial T}=\sum_{i=1}^{L}\left\{\begin{array}{l}-\dfrac{K_i}{T^2}-\dfrac{nK_{1i}}{T^2}+\dfrac{h_{1i}\lambda_i}{2}\left(\dfrac{\lambda_i E(x_i)^2}{P_{2i}}\right)+\\+\dfrac{h_i\lambda_i^2}{2}\left[\dfrac{E(x_i)}{P_{2i}}-\dfrac{E(x_i)^2}{P_{2i}}+\dfrac{1}{\lambda_i}-\left(\dfrac{1}{n}\right)\left(\dfrac{1}{\lambda_i}-\dfrac{1}{P_{1i}}-\dfrac{E(x_i)}{P_{2i}}\right)\right]\end{array}\right\} \tag{13}$$

$$\frac{\partial^2 E\left[TCU(T)\right]}{\partial T^2}=\sum_{i=1}^{L}\left\{\frac{2\left(K_i+nK_{1i}\right)}{T^3}\right\} \tag{14}$$

Equation (14) is resulting positive because K_i, n, K_{1i}, and T are all positive. Second derivative of $E[TCU(T)]$ with respect to T is greater than zero, hence $E[TCU(T)]$ is a convex function for all T different from zero.

3.1 Derivation of T*

The optimal common production cycle time T^* can be obtained by setting first derivative of $E[TCU(T)]$ equal to zero:

$$\frac{\partial E\left[TCU(T)\right]}{\partial T}=\sum_{i=1}^{L}\left\{\begin{array}{l}-\dfrac{K_i}{T^2}-\dfrac{nK_{1i}}{T^2}+\dfrac{h_{1i}\lambda_i}{2}\left(\dfrac{\lambda_i E(x_i)^2}{P_{2i}}\right)+\\+\dfrac{h_i\lambda_i^2}{2}\left[\dfrac{E(x_i)}{P_{2i}}-\dfrac{E(x_i)^2}{P_{2i}}+\dfrac{1}{\lambda_i}-\left(\dfrac{1}{n}\right)\left(\dfrac{1}{\lambda_i}-\dfrac{1}{P_{1i}}-\dfrac{E(x_i)}{P_{2i}}\right)\right]\end{array}\right\}=0 \tag{15}$$

With further derivations one obtains:

$$T^{*}=\sqrt{\frac{2\sum_{i=1}^{L}\left(K_i+nK_{1i}\right)}{\sum_{i=1}^{L}\left\{\begin{array}{l}h_i\lambda_i^2\left[\frac{E(x_i)}{P_{2i}}-\frac{E(x_i)^2}{P_{2i}}+\frac{1}{\lambda_i}-\left(\frac{1}{n}\right)\left(\frac{1}{\lambda_i}-\frac{1}{P_{1i}}-\frac{E(x_i)}{P_{2i}}\right)\right]+\\+h_{1i}\lambda_i\left(\frac{\lambda_i E(x_i)^2}{P_{2i}}\right)\end{array}\right\}}} \tag{16}$$

4. NUMERICAL EXAMPLE

Consider that a vendor plans a routine production schedule to produce five products in turn on a single machine using a common production cycle policy. Annual demands λ_i for five different products are *3000*, *3200*, *3400*, *3600*, and *3800* units respectively. Production rate P_{1i} for each product is *58000*, *59000*, *60000*, *61000*, and *62000* units respectively. Random defective rates x_i during production uptime for each product follow the uniform distribution over the intervals of [*0*, *0.05*], [*0*, *010*], [*0*, *0.15*], [*0*, *020*], and [*0*, *0.25*] respectively. All defective items are assumed to be repairable at the rates P_{2i} of *1800*, *2000*, *2200*, *2400*, and *2600* respectively, at additional reworking costs of *$50*, *$55*, *$60*, *$65*, and *$70* per item. Values of other parameters are:

K_i = production set up costs are *$3800*, *$3900*, *$4000*, *$4100*, and *$4200*, respectively,

C_i = unit manufacturing costs are *$80*, *$90*, *$100*, *$110*, and *$120* respectively,

h_i = unit holding costs are *$10*, *$15*, *$20*, *$25*, and *$30* respectively,

h_{1i} = unit holding costs per reworked are *$30*, *$35*, *$40*, *$45*, and *$50* respectively,

K_{1i} = the fixed delivery costs per shipment are *$1800*, *$1900*, *$2000*, *$2100*, and *$2200*,

C_{Ti} = unit transportation costs are *$0.1*, *$0.2*, *$0.3*, *$0.4*, and *$0.5* respectively,

n = number of shipments per cycle, in this study it is assumed to be a constant 4.

Applying Eqs. (16) and (12) one obtains optimal common production cycle time $T^* = 0.6026$ (years) and total expected production-inventory-delivery costs per unit time for L products in the proposed system $E[TCU(T^*=0.6026)] = \$2,008,926$. Variation of the common production cycle time T effects on the system cost $E[TCU(T)]$ are illustrated in Figure 3. Variation of average random defective rate effects on the optimal cycle time T^* and on the expected system cost $E[TCU(T^*)]$ are depicted in Figure 4. It should be noted that as the average random defective rate $E[x_i]$ increases, optimal common production cycle time T^* decreases, while the expected system cost $E[TCU(T^*)]$ increases significantly.

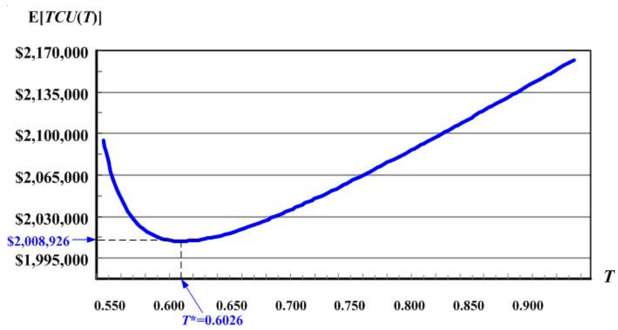

Fig. 3 Variation of the common production cycle time T effects on the system cost E[TCU(T)]

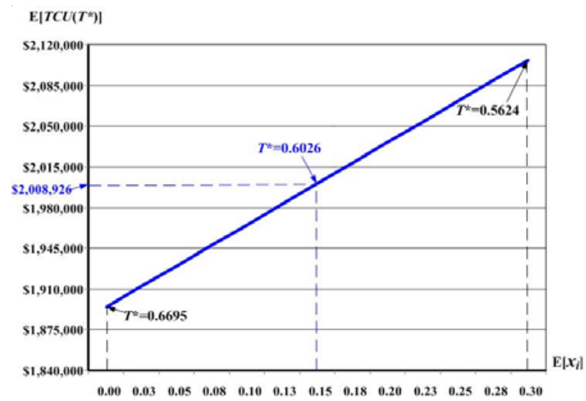

Fig. 4 Variation of average random defective rate effects on the optimal cycle time T and on the expected system cost E[TCU(T*)]*

5. CONCLUDING REMARKS

The classic FPR model considers production lot sizing for a single product with perfect production and a continuous inventory issuing policy. However, in real life manufacturing environment, vendors often plan to produce multiple products in turn on a single machine in order to maximize the machine utilization. During the production process, various uncontrollable factors make it likely that some nonconforming items are produced. Also, the delivery of finished products to outside clients is commonly under a practical periodic multiple shipments plan. Therefore, it is important for management to be able to know the effects of the reworking of defective items and multi-shipment policy on the multi-item FPR system.

This paper determines the optimal common production cycle time for the aforementioned FPR system and studies effects of rework on the optimal cycle time and on the expected system cost. The obtained results are intended to assist management in practice to better plan and control such a realistic multi-item production system. In future research, one interesting topic would be to consider imperfect rework effects on the common production cycle time for the same model.

6. ACKNOWLEDGEMENTS

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OPTIMALNO TRAJANJE PROIZVODNOG CIKLUSA ZA MODEL OGRANIČENE PROIZVODNJE VIŠESTRUKIH PROIZVODA S DORADOM I POLITIKOM VIŠEKRATNE ISPORUKE

SAŽETAK

Ovaj rad definira optimalno trajanje zajedničkog proizvodnog ciklusa za model ograničene proizvodnje (FPR) višestrukih proizvoda s doradom i politikom višekratne isporuke. Klasičan FPR model pretpostavlja planiranje proizvodnje jednog proizvoda uz savršenu kvalitetu proizvodnje i politiku kontinuirane isporuke proizvoda. Međutim, u stvarnoj proizvodnji, prodavači često planiraju naizmjeničnu proizvodnju višestrukih proizvoda na istom stroju kako bi maksimizirali iskorištenje stroja. Osim toga, pojava oštećenih proizvoda za vrijeme bilo kojeg proizvodnog ciklusa je neizbježna uslijed različitih čimbenika koji se ne mogu kontrolirati; stoga je uobičajeno da prodavač usvoji politiku višekratne/periodične isporuke za distribuiranje dovršenih proizvoda kupcima. U ovoj studiji se pretpostavlja da će se svi oštećeni proizvodi doraditi i popraviti nakon završetka redovne proizvodnja u istom proizvodnom ciklusu, po dodatnoj cijeni za svaki popravljeni proizvod. Cilj je bio definirati optimalno trajanje zajedničkog proizvodnog ciklusa koje minimizira dugoročni prosječni trošak po jedinici vremena te analizirati učinak dorade proizvoda na optimalno trajanje zajedničkog proizvodnog ciklusa za model ograničene proizvodnje (FPR) višestrukih proizvoda s popravcima i politikom višekratne isporuke. Korišteno je matematičko modeliranje, a dobiven je očekivani trošak sustava te je dokazana njegova konveksnost. Konačno, postignut je zatvoreni oblik optimalnog trajanja ciklusa. Prikazan je i numerički primjer te analiza osjetljivosti kako bi se demonstrirala praktična primjena dobivenih rezultata.

Ključne riječi: *proizvodnja višestrukih proizvoda, ograničena proizvodnja (FPR), trajanje zajedničkog ciklusa, dorada proizvoda, optimizacija, višekratna isporuka.*