Fundamentalne jednadžbe plohe

Jelena Sedlar

Fakultet građevinarstva, arhitekture i geodezije

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Dakle, svi vektori, pa tako i vektori $\mathbf{r}''_{uu}$, $\mathbf{r}''_{uv}$, $\mathbf{r}''_{vv}$, se mogu prikazati u toj bazi. Vrijedi

$$\mathbf{r}''_{uu} =$$

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$$\mathbf{r}''_{uu} = \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N},$$

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Dakle, svi vektori, pa tako i vektori $\mathbf{r}''_{uu}$, $\mathbf{r}''_{uv}$, $\mathbf{r}''_{vv}$, se mogu prikazati u toj bazi. Vrijedi

$$
\begin{aligned}
\mathbf{r}''_{uu} &= \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N}, \\
\mathbf{r}''_{uv} &=
\end{aligned}
$$

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Dakle, svi vektori, pa tako i vektori $\mathbf{r}''_{uu}$, $\mathbf{r}''_{uv}$, $\mathbf{r}''_{vv}$, se mogu prikazati u toj bazi. Vrijedi

$$\begin{aligned}\mathbf{r}''_{uu} &= \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N},\\ \mathbf{r}''_{uv} &= \Gamma^u_{uv}\mathbf{r}'_u + \Gamma^v_{uv}\mathbf{r}'_v + M\mathbf{N},\end{aligned}$$

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Dakle, svi vektori, pa tako i vektori $\mathbf{r}''_{uu}$, $\mathbf{r}''_{uv}$, $\mathbf{r}''_{vv}$, se mogu prikazati u toj bazi. Vrijedi

$$\begin{aligned} \mathbf{r}''_{uu} &= \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N}, \\ \mathbf{r}''_{uv} &= \Gamma^u_{uv}\mathbf{r}'_u + \Gamma^v_{uv}\mathbf{r}'_v + M\mathbf{N}, \\ \mathbf{r}''_{vv} &= \end{aligned}$$

Neka je $\mathbf{r}:U\rightarrow\mathbb{R}^3$ regularna parametrizirana ploha. Za svaki $(u,v)\in U$ vektori $\mathbf{r}_u'$, $\mathbf{r}_v'$ i $\mathbf{N}$ tvore bazu prostora.

Dakle, svi vektori, pa tako i vektori $\mathbf{r}_{uu}''$, $\mathbf{r}_{uv}''$, $\mathbf{r}_{vv}''$, se mogu prikazati u toj bazi. Vrijedi

$$\begin{aligned}
\mathbf{r}_{uu}'' &= \Gamma_{uu}^{u}\mathbf{r}_u'+\Gamma_{uu}^{v}\mathbf{r}_v'+L\mathbf{N},\\
\mathbf{r}_{uv}'' &= \Gamma_{uv}^{u}\mathbf{r}_u'+\Gamma_{uv}^{v}\mathbf{r}_v'+M\mathbf{N},\\
\mathbf{r}_{vv}'' &= \Gamma_{vv}^{u}\mathbf{r}_u'+\Gamma_{vv}^{v}\mathbf{r}_v'+N\mathbf{N}.
\end{aligned}$$

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Dakle, svi vektori, pa tako i vektori $\mathbf{r}''_{uu}$, $\mathbf{r}''_{uv}$, $\mathbf{r}''_{vv}$, se mogu prikazati u toj bazi. Vrijedi

$$\begin{aligned}
\mathbf{r}''_{uu} &= \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N}, \\
\mathbf{r}''_{uv} &= \Gamma^u_{uv}\mathbf{r}'_u + \Gamma^v_{uv}\mathbf{r}'_v + M\mathbf{N}, \\
\mathbf{r}''_{vv} &= \Gamma^u_{vv}\mathbf{r}'_u + \Gamma^v_{vv}\mathbf{r}'_v + N\mathbf{N}.
\end{aligned}$$

pri čemu se simboli Γ^k_{ij} nazivaju se *Christoffelovi simboli druge vrste*.

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Dakle, svi vektori, pa tako i vektori $\mathbf{r}''_{uu}$, $\mathbf{r}''_{uv}$, $\mathbf{r}''_{vv}$, se mogu prikazati u toj bazi. Vrijedi

$$\begin{aligned}
\mathbf{r}''_{uu} &= \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N}, \\
\mathbf{r}''_{uv} &= \Gamma^u_{uv}\mathbf{r}'_u + \Gamma^v_{uv}\mathbf{r}'_v + M\mathbf{N}, \\
\mathbf{r}''_{vv} &= \Gamma^u_{vv}\mathbf{r}'_u + \Gamma^v_{vv}\mathbf{r}'_v + N\mathbf{N}.
\end{aligned}$$

pri čemu se simboli Γ^k_{ij} nazivaju se *Christoffelovi simboli druge vrste*.

Ove formule nazivaju se *Gaussove derivacijske formule*.

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Dakle, svi vektori, pa tako i vektori $\mathbf{r}''_{uu}$, $\mathbf{r}''_{uv}$, $\mathbf{r}''_{vv}$, se mogu prikazati u toj bazi. Vrijedi

$$\begin{aligned}
\mathbf{r}''_{uu} &= \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N}, \\
\mathbf{r}''_{uv} &= \Gamma^u_{uv}\mathbf{r}'_u + \Gamma^v_{uv}\mathbf{r}'_v + M\mathbf{N}, \\
\mathbf{r}''_{vv} &= \Gamma^u_{vv}\mathbf{r}'_u + \Gamma^v_{vv}\mathbf{r}'_v + N\mathbf{N}.
\end{aligned}$$

pri čemu se simboli Γ^k_{ij} nazivaju se *Christoffelovi simboli druge vrste*.

Napomena.

Neka je $\mathbf{r} : U \to \mathbb{R}^3$ regularna parametrizirana ploha. Za svaki $(u, v) \in U$ vektori $\mathbf{r}_u'$, $\mathbf{r}_v'$ i $\mathbf{N}$ tvore bazu prostora.

Dakle, svi vektori, pa tako i vektori $\mathbf{r}_{uu}''$, $\mathbf{r}_{uv}''$, $\mathbf{r}_{vv}''$, se mogu prikazati u toj bazi. Vrijedi

$$\begin{aligned} \mathbf{r}_{uu}'' &= \Gamma_{uu}^u \mathbf{r}_u' + \Gamma_{uu}^v \mathbf{r}_v' + L\mathbf{N}, \\ \mathbf{r}_{uv}'' &= \Gamma_{uv}^u \mathbf{r}_u' + \Gamma_{uv}^v \mathbf{r}_v' + M\mathbf{N}, \\ \mathbf{r}_{vv}'' &= \Gamma_{vv}^u \mathbf{r}_u' + \Gamma_{vv}^v \mathbf{r}_v' + N\mathbf{N}. \end{aligned}$$

pri čemu se simboli Γ_{ij}^k nazivaju se *Christoffelovi simboli druge vrste*.

Napomena. Dakle, Christoffelovi simboli druge vrste su **definirani** upravo kao koeficijenti prikaza vektora $\mathbf{r}_{uu}''$, $\mathbf{r}_{uv}''$, $\mathbf{r}_{vv}''$ u bazi $\{\mathbf{r}_u', \mathbf{r}_v', \mathbf{N}\}$.

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$$\begin{aligned}
\mathbf{r}''_{uu} &= \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N}, \\
\mathbf{r}''_{uv} &= \Gamma^u_{uv}\mathbf{r}'_u + \Gamma^v_{uv}\mathbf{r}'_v + M\mathbf{N}, \\
\mathbf{r}''_{vv} &= \Gamma^u_{vv}\mathbf{r}'_u + \Gamma^v_{vv}\mathbf{r}'_v + N\mathbf{N}.
\end{aligned}$$

pri čemu se simboli Γ^k_{ij} nazivaju se *Christoffelovi simboli druge vrste*.

Napomena. Dakle, Christoffelovi simboli druge vrste su **definirani** upravo kao koeficijenti prikaza vektora $\mathbf{r}''_{uu}$, $\mathbf{r}''_{uv}$, $\mathbf{r}''_{vv}$ u bazi $\{\mathbf{r}'_u, \mathbf{r}'_v, \mathbf{N}\}$. Sada ih želimo **izraziti** pomoću koeficijenata prve i druge fundamentalne forme, te njihovih derivacija.

Fundamentalne jednadžbe plohe

$$\left(\mathbf{r}'_u\right)^2 = E$$

Fundamentale jednadžbe plohe

$$\left(\mathbf{r}'_u\right)^2 = E \quad /'_u \quad \Rightarrow$$

Fundamentalne jednadžbe plohe

$$\left(\mathbf{r}'_u\right)^2 = E \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uu} = E'_u \Rightarrow$$

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$$/'_v \quad \Rightarrow$$

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$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uv} = E'_v \Rightarrow$$

Fundamentalne jednadžbe plohe

$$\left(\mathbf{r}_u'\right)^2 = E \quad /_u' \quad \Rightarrow \quad 2\mathbf{r}_u' \cdot \mathbf{r}_{uu}'' = E_u' \Rightarrow \mathbf{r}_{uu}'' \cdot \mathbf{r}_u' = \frac{1}{2}E_u'$$

$$/_v' \quad \Rightarrow \quad 2\mathbf{r}_u' \cdot \mathbf{r}_{uv}'' = E_v' \Rightarrow \mathbf{r}_{uv}'' \cdot \mathbf{r}_u' = \frac{1}{2}E_v'$$

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$$\left(\mathbf{r}'_u\right)^2 = E \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uu} = E'_u \Rightarrow \mathbf{r}''_{uu} \cdot \mathbf{r}'_u = \frac{1}{2}E'_u$$

$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uv} = E'_v \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_u = \frac{1}{2}E'_v$$

$$\left(\mathbf{r}'_v\right)^2 = G$$

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$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uv} = E'_v \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_u = \frac{1}{2}E'_v$$

$$\left(\mathbf{r}'_v\right)^2 = G \quad /'_u \quad \Rightarrow$$

Fundamentalne jednadžbe plohe

$$\left(\mathbf{r}'_u\right)^2 = E \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uu} = E'_u \Rightarrow \mathbf{r}''_{uu} \cdot \mathbf{r}'_u = \frac{1}{2}E'_u$$

$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uv} = E'_v \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_u = \frac{1}{2}E'_v$$

$$\left(\mathbf{r}'_v\right)^2 = G \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{uv} = G'_u \Rightarrow$$

Fundamentalne jednadžbe plohe

$$\left(\mathbf{r}'_u\right)^2 = E \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uu} = E'_u \Rightarrow \mathbf{r}''_{uu} \cdot \mathbf{r}'_u = \frac{1}{2}E'_u$$

$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uv} = E'_v \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_u = \frac{1}{2}E'_v$$

$$\left(\mathbf{r}'_v\right)^2 = G \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{uv} = G'_u \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_u$$

Fundamentalne jednadžbe plohe

$$
\begin{aligned}
(\mathbf{r}_u')^2 = E \quad /_u' \quad &\Rightarrow \quad 2\mathbf{r}_u' \cdot \mathbf{r}_{uu}'' = E_u' \Rightarrow \mathbf{r}_{uu}'' \cdot \mathbf{r}_u' = \frac{1}{2}E_u' \\
/_v' \quad &\Rightarrow \quad 2\mathbf{r}_u' \cdot \mathbf{r}_{uv}'' = E_v' \Rightarrow \mathbf{r}_{uv}'' \cdot \mathbf{r}_u' = \frac{1}{2}E_v' \\
(\mathbf{r}_v')^2 = G \quad /_u' \quad &\Rightarrow \quad 2\mathbf{r}_v' \cdot \mathbf{r}_{uv}'' = G_u' \Rightarrow \mathbf{r}_{uv}'' \cdot \mathbf{r}_v' = \frac{1}{2}G_u' \\
/_v' \quad &\Rightarrow
\end{aligned}
$$

Fundamentalne jednadžbe plohe

$$\left(\mathbf{r}'_u\right)^2 = E \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uu} = E'_u \Rightarrow \mathbf{r}''_{uu} \cdot \mathbf{r}'_u = \frac{1}{2} E'_u$$

$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uv} = E'_v \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_u = \frac{1}{2} E'_v$$

$$\left(\mathbf{r}'_v\right)^2 = G \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{uv} = G'_u \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_v = \frac{1}{2} G'_u$$

$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{vv} = G'_v \Rightarrow$$

Fundamentalne jednadžbe plohe

$$\left(\mathbf{r}'_u\right)^2 = E \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uu} = E'_u \Rightarrow \mathbf{r}''_{uu} \cdot \mathbf{r}'_u = \frac{1}{2}E'_u$$

$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uv} = E'_v \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_u = \frac{1}{2}E'_v$$

$$\left(\mathbf{r}'_v\right)^2 = G \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{uv} = G'_u \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_u$$

$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{vv} = G'_v \Rightarrow \mathbf{r}''_{vv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_v$$

Fundamentalne jednadžbe plohe

$$\left(\mathbf{r}'_u\right)^2 = E \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uu} = E'_u \Rightarrow \mathbf{r}''_{uu} \cdot \mathbf{r}'_u = \frac{1}{2}E'_u$$

$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uv} = E'_v \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_u = \frac{1}{2}E'_v$$

$$\left(\mathbf{r}'_v\right)^2 = G \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{uv} = G'_u \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_u$$

$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{vv} = G'_v \Rightarrow \mathbf{r}''_{vv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_v$$

$$\mathbf{r}'_u \cdot \mathbf{r}'_v = F$$

Fundamentalne jednadžbe plohe

$$\left(\mathbf{r}'_u\right)^2 = E \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uu} = E'_u \Rightarrow \mathbf{r}''_{uu} \cdot \mathbf{r}'_u = \frac{1}{2}E'_u$$

$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uv} = E'_v \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_u = \frac{1}{2}E'_v$$

$$\left(\mathbf{r}'_v\right)^2 = G \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{uv} = G'_u \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_u$$

$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{vv} = G'_v \Rightarrow \mathbf{r}''_{vv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_v$$

$$\mathbf{r}'_u \cdot \mathbf{r}'_v = F \quad /'_u \quad \Rightarrow$$

Fundamentalne jednadžbe plohe

$$(\mathbf{r}'_u)^2 = E \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uu} = E'_u \Rightarrow \mathbf{r}''_{uu} \cdot \mathbf{r}'_u = \frac{1}{2}E'_u$$

$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uv} = E'_v \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_u = \frac{1}{2}E'_v$$

$$(\mathbf{r}'_v)^2 = G \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{uv} = G'_u \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_u$$

$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{vv} = G'_v \Rightarrow \mathbf{r}''_{vv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_v$$

$$\mathbf{r}'_u \cdot \mathbf{r}'_v = F \quad /'_u \quad \Rightarrow \quad \mathbf{r}''_{uu} \cdot \mathbf{r}'_v + \mathbf{r}'_u \cdot \mathbf{r}''_{uv} = F'_u \Rightarrow$$

Fundamentalne jednadžbe plohe

$$\left(\mathbf{r}_u'\right)^2 = E \quad /_u' \quad \Rightarrow \quad 2\mathbf{r}_u' \cdot \mathbf{r}_{uu}'' = E_u' \Rightarrow \mathbf{r}_{uu}'' \cdot \mathbf{r}_u' = \frac{1}{2}E_u'$$

$$/_v' \quad \Rightarrow \quad 2\mathbf{r}_u' \cdot \mathbf{r}_{uv}'' = E_v' \Rightarrow \mathbf{r}_{uv}'' \cdot \mathbf{r}_u' = \frac{1}{2}E_v'$$

$$\left(\mathbf{r}_v'\right)^2 = G \quad /_u' \quad \Rightarrow \quad 2\mathbf{r}_v' \cdot \mathbf{r}_{uv}'' = G_u' \Rightarrow \mathbf{r}_{uv}'' \cdot \mathbf{r}_v' = \frac{1}{2}G_u'$$

$$/_v' \quad \Rightarrow \quad 2\mathbf{r}_v' \cdot \mathbf{r}_{vv}'' = G_v' \Rightarrow \mathbf{r}_{vv}'' \cdot \mathbf{r}_v' = \frac{1}{2}G_v'$$

$$\mathbf{r}_u' \cdot \mathbf{r}_v' = F \quad /_u' \quad \Rightarrow \quad \mathbf{r}_{uu}'' \cdot \mathbf{r}_v' + \mathbf{r}_u' \cdot \mathbf{r}_{uv}'' = F_u' \Rightarrow \mathbf{r}_{uu}'' \cdot \mathbf{r}_v' + \frac{1}{2}E_v' = F_u'$$

Fundamentalne jednadžbe plohe

$$\left(\mathbf{r}_u'\right)^2 = E \quad /_u' \quad \Rightarrow \quad 2\mathbf{r}_u' \cdot \mathbf{r}_{uu}'' = E_u' \Rightarrow \mathbf{r}_{uu}'' \cdot \mathbf{r}_u' = \frac{1}{2}E_u'$$

$$/_v' \quad \Rightarrow \quad 2\mathbf{r}_u' \cdot \mathbf{r}_{uv}'' = E_v' \Rightarrow \mathbf{r}_{uv}'' \cdot \mathbf{r}_u' = \frac{1}{2}E_v'$$

$$\left(\mathbf{r}_v'\right)^2 = G \quad /_u' \quad \Rightarrow \quad 2\mathbf{r}_v' \cdot \mathbf{r}_{uv}'' = G_u' \Rightarrow \mathbf{r}_{uv}'' \cdot \mathbf{r}_v' = \frac{1}{2}G_u'$$

$$/_v' \quad \Rightarrow \quad 2\mathbf{r}_v' \cdot \mathbf{r}_{vv}'' = G_v' \Rightarrow \mathbf{r}_{vv}'' \cdot \mathbf{r}_v' = \frac{1}{2}G_v'$$

$$\mathbf{r}_u' \cdot \mathbf{r}_v' = F \quad /_u' \quad \Rightarrow \quad \mathbf{r}_{uu}'' \cdot \mathbf{r}_v' + \mathbf{r}_u' \cdot \mathbf{r}_{uv}'' = F_u' \Rightarrow \mathbf{r}_{uu}'' \cdot \mathbf{r}_v' + \frac{1}{2}E_v' = F_u'$$

$$\Rightarrow \quad \mathbf{r}_{uu}'' \cdot \mathbf{r}_v' = F_u' - \frac{1}{2}E_v'$$

Fundamentalne jednadžbe plohe

$$
\begin{aligned}
(\mathbf{r}'_u)^2 = E \quad /'_u \quad &\Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uu} = E'_u \Rightarrow \mathbf{r}''_{uu} \cdot \mathbf{r}'_u = \frac{1}{2}E'_u \\
/'_v \quad &\Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uv} = E'_v \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_u = \frac{1}{2}E'_v \\
(\mathbf{r}'_v)^2 = G \quad /'_u \quad &\Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{uv} = G'_u \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_u \\
/'_v \quad &\Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{vv} = G'_v \Rightarrow \mathbf{r}''_{vv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_v \\
\mathbf{r}'_u \cdot \mathbf{r}'_v = F \quad /'_u \quad &\Rightarrow \quad \mathbf{r}''_{uu} \cdot \mathbf{r}'_v + \mathbf{r}'_u \cdot \mathbf{r}''_{uv} = F'_u \Rightarrow \mathbf{r}''_{uu} \cdot \mathbf{r}'_v + \frac{1}{2}E'_v = F'_u \\
&\Rightarrow \quad \mathbf{r}''_{uu} \cdot \mathbf{r}'_v = F'_u - \frac{1}{2}E'_v \\
/'_v \quad &\Rightarrow
\end{aligned}
$$

Fundamentalne jednadžbe plohe

$$\left(\mathbf{r}_u'\right)^2 = E \quad /_u' \quad \Rightarrow \quad 2\mathbf{r}_u' \cdot \mathbf{r}_{uu}'' = E_u' \Rightarrow \mathbf{r}_{uu}'' \cdot \mathbf{r}_u' = \frac{1}{2}E_u'$$

$$/_v' \quad \Rightarrow \quad 2\mathbf{r}_u' \cdot \mathbf{r}_{uv}'' = E_v' \Rightarrow \mathbf{r}_{uv}'' \cdot \mathbf{r}_u' = \frac{1}{2}E_v'$$

$$\left(\mathbf{r}_v'\right)^2 = G \quad /_u' \quad \Rightarrow \quad 2\mathbf{r}_v' \cdot \mathbf{r}_{uv}'' = G_u' \Rightarrow \mathbf{r}_{uv}'' \cdot \mathbf{r}_v' = \frac{1}{2}G_u'$$

$$/_v' \quad \Rightarrow \quad 2\mathbf{r}_v' \cdot \mathbf{r}_{vv}'' = G_v' \Rightarrow \mathbf{r}_{vv}'' \cdot \mathbf{r}_v' = \frac{1}{2}G_v'$$

$$\mathbf{r}_u' \cdot \mathbf{r}_v' = F \quad /_u' \quad \Rightarrow \quad \mathbf{r}_{uu}'' \cdot \mathbf{r}_v' + \mathbf{r}_u' \cdot \mathbf{r}_{uv}'' = F_u' \Rightarrow \mathbf{r}_{uu}'' \cdot \mathbf{r}_v' + \frac{1}{2}E_v' = F_u'$$

$$\Rightarrow \quad \mathbf{r}_{uu}'' \cdot \mathbf{r}_v' = F_u' - \frac{1}{2}E_v'$$

$$/_v' \quad \Rightarrow \quad \mathbf{r}_{uv}'' \cdot \mathbf{r}_v' + \mathbf{r}_u' \cdot \mathbf{r}_{vv}'' = F_v'$$

Fundamentalne jednadžbe plohe

$$
\begin{aligned}
(\mathbf{r}'_u)^2 = E \quad /'_u \quad &\Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uu} = E'_u \Rightarrow \mathbf{r}''_{uu} \cdot \mathbf{r}'_u = \frac{1}{2}E'_u \\
/'_v \quad &\Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uv} = E'_v \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_u = \frac{1}{2}E'_v \\
(\mathbf{r}'_v)^2 = G \quad /'_u \quad &\Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{uv} = G'_u \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_u \\
/'_v \quad &\Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{vv} = G'_v \Rightarrow \mathbf{r}''_{vv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_v \\
\mathbf{r}'_u \cdot \mathbf{r}'_v = F \quad /'_u \quad &\Rightarrow \quad \mathbf{r}''_{uu} \cdot \mathbf{r}'_v + \mathbf{r}'_u \cdot \mathbf{r}''_{uv} = F'_u \Rightarrow \mathbf{r}''_{uu} \cdot \mathbf{r}'_v + \frac{1}{2}E'_v = F'_u \\
&\Rightarrow \quad \mathbf{r}''_{uu} \cdot \mathbf{r}'_v = F'_u - \frac{1}{2}E'_v \\
/'_v \quad &\Rightarrow \quad \mathbf{r}''_{uv} \cdot \mathbf{r}'_v + \mathbf{r}'_u \cdot \mathbf{r}''_{vv} = F'_v \Rightarrow \frac{1}{2}G'_u + \mathbf{r}'_u \cdot \mathbf{r}''_{vv} = F'_v
\end{aligned}
$$

Fundamentalne jednadžbe plohe

$$
\begin{aligned}
\left(\mathbf{r}'_u\right)^2 = E \quad /'_u \quad &\Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uu} = E'_u \Rightarrow \mathbf{r}''_{uu} \cdot \mathbf{r}'_u = \frac{1}{2}E'_u \\
/'_v \quad &\Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uv} = E'_v \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_u = \frac{1}{2}E'_v \\
\left(\mathbf{r}'_v\right)^2 = G \quad /'_u \quad &\Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{uv} = G'_u \Rightarrow \mathbf{r}''_{uv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_u \\
/'_v \quad &\Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{vv} = G'_v \Rightarrow \mathbf{r}''_{vv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_v \\
\mathbf{r}'_u \cdot \mathbf{r}'_v = F \quad /'_u \quad &\Rightarrow \quad \mathbf{r}''_{uu} \cdot \mathbf{r}'_v + \mathbf{r}'_u \cdot \mathbf{r}''_{uv} = F'_u \Rightarrow \mathbf{r}''_{uu} \cdot \mathbf{r}'_v + \frac{1}{2}E'_v = F'_u \\
&\Rightarrow \quad \mathbf{r}''_{uu} \cdot \mathbf{r}'_v = F'_u - \frac{1}{2}E'_v \\
/'_v \quad &\Rightarrow \quad \mathbf{r}''_{uv} \cdot \mathbf{r}'_v + \mathbf{r}'_u \cdot \mathbf{r}''_{vv} = F'_v \Rightarrow \frac{1}{2}G'_u + \mathbf{r}'_u \cdot \mathbf{r}''_{vv} = F'_v \\
&\Rightarrow \quad \mathbf{r}''_{vv} \cdot \mathbf{r}'_u = F'_v - \frac{1}{2}G'_u
\end{aligned}
$$

Fundamentalne jednadžbe plohe

$$\left(\mathbf{r}'_u\right)^2 = E \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uu} = E'_u \Rightarrow \boxed{\mathbf{r}''_{uu} \cdot \mathbf{r}'_u = \frac{1}{2}E'_u}$$

$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_u \cdot \mathbf{r}''_{uv} = E'_v \Rightarrow \boxed{\mathbf{r}''_{uv} \cdot \mathbf{r}'_u = \frac{1}{2}E'_v}$$

$$\left(\mathbf{r}'_v\right)^2 = G \quad /'_u \quad \Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{uv} = G'_u \Rightarrow \boxed{\mathbf{r}''_{uv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_u}$$

$$/'_v \quad \Rightarrow \quad 2\mathbf{r}'_v \cdot \mathbf{r}''_{vv} = G'_v \Rightarrow \boxed{\mathbf{r}''_{vv} \cdot \mathbf{r}'_v = \frac{1}{2}G'_v}$$

$$\mathbf{r}'_u \cdot \mathbf{r}'_v = F \quad /'_u \quad \Rightarrow \quad \mathbf{r}''_{uu} \cdot \mathbf{r}'_v + \mathbf{r}'_u \cdot \mathbf{r}''_{uv} = F'_u \Rightarrow \mathbf{r}''_{uu} \cdot \mathbf{r}'_v + \frac{1}{2}E'_v = F'_u$$

$$\Rightarrow \quad \boxed{\mathbf{r}''_{uu} \cdot \mathbf{r}'_v = F'_u - \frac{1}{2}E'_v}$$

$$/'_v \quad \Rightarrow \quad \mathbf{r}''_{uv} \cdot \mathbf{r}'_v + \mathbf{r}'_u \cdot \mathbf{r}''_{vv} = F'_v \Rightarrow \frac{1}{2}G'_u + \mathbf{r}'_u \cdot \mathbf{r}''_{vv} = F'_v$$

$$\Rightarrow \quad \boxed{\mathbf{r}''_{vv} \cdot \mathbf{r}'_u = F'_v - \frac{1}{2}G'_u}$$

Fundamentalne jednadžbe plohe

Sada imamo:

Fundamentalne jednadžbe plohe

Sada imamo:

$$\mathbf{r}''_{uu} = \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N}$$

Fundamentalne jednadžbe plohe

Sada imamo:

$$\mathbf{r}''_{uu} = \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N} \ \ / \cdot \mathbf{r}'_u \Rightarrow$$

Fundamentalne jednadžbe plohe

Sada imamo:

$$\mathbf{r}''_{uu} = \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N} \ / \cdot \mathbf{r}'_u \Rightarrow \quad \mathbf{r}''_{uu} \cdot \mathbf{r}'_u = \Gamma^u_{uu}\mathbf{r}'_u \cdot \mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v \cdot \mathbf{r}'_u + L\mathbf{N} \cdot \mathbf{r}'_u$$

Fundamentalne jednadžbe plohe

Sada imamo:

$$\mathbf{r}''_{uu} = \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N} \;\; / \cdot \mathbf{r}'_u \Rightarrow \quad \mathbf{r}''_{uu} \cdot \mathbf{r}'_u = \Gamma^u_{uu}\mathbf{r}'_u \cdot \mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v \cdot \mathbf{r}'_u + L\mathbf{N} \cdot \mathbf{r}'_u$$

$$/ \cdot \mathbf{r}'_v \Rightarrow$$

Fundamentalne jednadžbe plohe

Sada imamo:

$$\begin{aligned}\mathbf{r}''_{uu} = \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N} \ / \cdot \mathbf{r}'_u &\Rightarrow \quad \mathbf{r}''_{uu}\cdot\mathbf{r}'_u = \Gamma^u_{uu}\mathbf{r}'_u\cdot\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v\cdot\mathbf{r}'_u + L\mathbf{N}\cdot\mathbf{r}'_u \\ / \cdot \mathbf{r}'_v &\Rightarrow \quad \mathbf{r}''_{uu}\cdot\mathbf{r}'_v = \Gamma^u_{uu}\mathbf{r}'_u\cdot\mathbf{r}'_v + \Gamma^v_{uu}\mathbf{r}'_v\cdot\mathbf{r}'_v + L\mathbf{N}\cdot\mathbf{r}'_v\end{aligned}$$

Fundamentalne jednadžbe plohe

Sada imamo:

$$\mathbf{r}''_{uu} = \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N} \ \ / \cdot \mathbf{r}'_u \Rightarrow \ \ \mathbf{r}''_{uu}\cdot\mathbf{r}'_u = \Gamma^u_{uu}\mathbf{r}'_u\cdot\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v\cdot\mathbf{r}'_u + L\mathbf{N}\cdot\mathbf{r}'_u$$

$$/ \cdot \mathbf{r}'_v \Rightarrow \ \ \mathbf{r}''_{uu}\cdot\mathbf{r}'_v = \Gamma^u_{uu}\mathbf{r}'_u\cdot\mathbf{r}'_v + \Gamma^v_{uu}\mathbf{r}'_v\cdot\mathbf{r}'_v + L\mathbf{N}\cdot\mathbf{r}'_v$$

$$\frac{1}{2}E'_u = E\Gamma^u_{uu} + F\Gamma^v_{uu}$$

$$F'_u - \frac{1}{2}E'_v = F\Gamma^u_{uu} + G\Gamma^v_{uu}$$

Fundamentalne jednadžbe plohe

Sada imamo:

$$\mathbf{r}''_{uu} = \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N} \;\; / \cdot \mathbf{r}'_u \Rightarrow \;\; \mathbf{r}''_{uu}\cdot\mathbf{r}'_u = \Gamma^u_{uu}\mathbf{r}'_u\cdot\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v\cdot\mathbf{r}'_u + L\mathbf{N}\cdot\mathbf{r}'_u$$

$$/ \cdot \mathbf{r}'_v \Rightarrow \;\; \mathbf{r}''_{uu}\cdot\mathbf{r}'_v = \Gamma^u_{uu}\mathbf{r}'_u\cdot\mathbf{r}'_v + \Gamma^v_{uu}\mathbf{r}'_v\cdot\mathbf{r}'_v + L\mathbf{N}\cdot\mathbf{r}'_v$$

$$\frac{1}{2}E'_u = E\Gamma^u_{uu} + F\Gamma^v_{uu}$$

$$F'_u - \frac{1}{2}E'_v = F\Gamma^u_{uu} + G\Gamma^v_{uu}$$

Ovaj sustav se lako riješi, pa imamo

Fundamentalne jednadžbe plohe

Sada imamo:

$$\mathbf{r}''_{uu} = \Gamma^u_{uu}\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v + L\mathbf{N} \ / \cdot \mathbf{r}'_u \Rightarrow \ \mathbf{r}''_{uu}\cdot\mathbf{r}'_u = \Gamma^u_{uu}\mathbf{r}'_u\cdot\mathbf{r}'_u + \Gamma^v_{uu}\mathbf{r}'_v\cdot\mathbf{r}'_u + L\mathbf{N}\cdot\mathbf{r}'_u$$

$$/ \cdot \mathbf{r}'_v \Rightarrow \ \mathbf{r}''_{uu}\cdot\mathbf{r}'_v = \Gamma^u_{uu}\mathbf{r}'_u\cdot\mathbf{r}'_v + \Gamma^v_{uu}\mathbf{r}'_v\cdot\mathbf{r}'_v + L\mathbf{N}\cdot\mathbf{r}'_v$$

$$\frac{1}{2}E'_u = E\Gamma^u_{uu} + F\Gamma^v_{uu}$$

$$F'_u - \frac{1}{2}E'_v = F\Gamma^u_{uu} + G\Gamma^v_{uu}$$

Ovaj sustav se lako riješi, pa imamo

$$\Gamma^u_{uu} = \frac{GE'_u - 2FF'_u + FE'_v}{2(EG - F^2)}$$

$$\Gamma^v_{uu} = \frac{-FE'_u + 2EF'_u - EE'_v}{2(EG - F^2)}$$

Fundamentalne jednadžbe plohe

Sada imamo:

$$\mathbf{r}''_{uv} = \Gamma^u_{uv}\mathbf{r}'_u + \Gamma^v_{uv}\mathbf{r}'_v + M\mathbf{N} \quad / \cdot \mathbf{r}'_u \Rightarrow \quad \dots$$

$$/ \cdot \mathbf{r}'_v \Rightarrow \quad \dots$$

Fundamentalne jednadžbe plohe

Sada imamo:

$$\mathbf{r}''_{uv} = \Gamma^u_{uv}\mathbf{r}'_u + \Gamma^v_{uv}\mathbf{r}'_v + M\mathbf{N} \ / \cdot \mathbf{r}'_u \Rightarrow \quad \ldots$$
$$/ \cdot \mathbf{r}'_v \Rightarrow \quad \ldots$$

$$\mathbf{r}''_{vv} = \Gamma^u_{vv}\mathbf{r}'_u + \Gamma^v_{vv}\mathbf{r}'_v + N\mathbf{N} \ / \cdot \mathbf{r}'_u \Rightarrow \quad \ldots$$
$$/ \cdot \mathbf{r}'_v \Rightarrow \quad \ldots$$

Fundamentalne jednadžbe plohe

Sada imamo:

$$\mathbf{r}''_{uv} = \Gamma^u_{uv}\mathbf{r}'_u + \Gamma^v_{uv}\mathbf{r}'_v + M\mathbf{N} \;\; / \cdot \mathbf{r}'_u \Rightarrow \quad \ldots$$
$$/ \cdot \mathbf{r}'_v \Rightarrow \quad \ldots$$

$$\mathbf{r}''_{vv} = \Gamma^u_{vv}\mathbf{r}'_u + \Gamma^v_{vv}\mathbf{r}'_v + N\mathbf{N} \;\; / \cdot \mathbf{r}'_u \Rightarrow \quad \ldots$$
$$/ \cdot \mathbf{r}'_v \Rightarrow \quad \ldots$$

Rješavanjem ovih sustava dobivamo preostale Christoffelove simbole, tj.

$$\Gamma^u_{uv} = \frac{GE'_v - FG'_u}{2(EG - F^2)}$$
$$\Gamma^v_{uv} = \frac{EG'_u - FE'_v}{2(EG - F^2)}$$

Sada imamo:

$$\mathbf{r}''_{uv} = \Gamma^u_{uv}\mathbf{r}'_u + \Gamma^v_{uv}\mathbf{r}'_v + M\mathbf{N} \ / \cdot \mathbf{r}'_u \Rightarrow \quad \ldots$$
$$/ \cdot \mathbf{r}'_v \Rightarrow \quad \ldots$$

$$\mathbf{r}''_{vv} = \Gamma^u_{vv}\mathbf{r}'_u + \Gamma^v_{vv}\mathbf{r}'_v + N\mathbf{N} \ / \cdot \mathbf{r}'_u \Rightarrow \quad \ldots$$
$$/ \cdot \mathbf{r}'_v \Rightarrow \quad \ldots$$

Rješavanjem ovih sustava dobivamo preostale Christoffelove simbole, tj.

$$\begin{aligned}
\Gamma^u_{uv} &= \frac{GE'_v - FG'_u}{2(EG - F^2)} \\
\Gamma^v_{uv} &= \frac{EG'_u - FE'_v}{2(EG - F^2)} \\
\Gamma^u_{vv} &= \frac{-FG'_v + 2GF'_v - GG'_u}{2(EG - F^2)} \\
\Gamma^v_{vv} &= \frac{EG'_v - 2FF'_v + FG'_u}{2(EG - F^2)}.
\end{aligned}$$

Fundamentalne jednadžbe plohe

Što se tiče prikaza derivacija vektora normale $\mathbf{N}$ u bazi $\{\mathbf{r}'_u, \mathbf{r}'_v, \mathbf{N}\}$, vrijedi

Fundamentalne jednadžbe plohe

Što se tiče prikaza derivacija vektora normale $\mathbf{N}$ u bazi $\{\mathbf{r}'_u, \mathbf{r}'_v, \mathbf{N}\}$, vrijedi

$$\mathbf{N}'_u = \frac{FM - GL}{EG - F^2}\mathbf{r}'_u + \frac{FL - EM}{EG - F^2}\mathbf{r}'_v$$

Fundamentalne jednadžbe plohe

Što se tiče prikaza derivacija vektora normale $\mathbf{N}$ u bazi $\{\mathbf{r}'_u, \mathbf{r}'_v, \mathbf{N}\}$, vrijedi

$$
\begin{aligned}
\mathbf{N}'_u &= \frac{FM - GL}{EG - F^2}\mathbf{r}'_u + \frac{FL - EM}{EG - F^2}\mathbf{r}'_v \\
\mathbf{N}'_v &= \frac{FN - GM}{EG - F^2}\mathbf{r}'_u + \frac{FM - EN}{EG - F^2}\mathbf{r}'_v
\end{aligned}
$$

Fundamentalne jednadžbe plohe

Što se tiče prikaza derivacija vektora normale $\mathbf{N}$ u bazi $\{\mathbf{r}'_u, \mathbf{r}'_v, \mathbf{N}\}$, vrijedi

$$\begin{aligned} \mathbf{N}'_u &= \frac{FM - GL}{EG - F^2}\mathbf{r}'_u + \frac{FL - EM}{EG - F^2}\mathbf{r}'_v \\ \mathbf{N}'_v &= \frac{FN - GM}{EG - F^2}\mathbf{r}'_u + \frac{FM - EN}{EG - F^2}\mathbf{r}'_v \end{aligned}$$

pri čemu se ove formule nazivaju *Weingartenove derivacijske formule*.

Što se tiče prikaza derivacija vektora normale $\mathbf{N}$ u bazi $\{\mathbf{r}'_u, \mathbf{r}'_v, \mathbf{N}\}$, vrijedi

$$\begin{aligned}
\mathbf{N}'_u &= \frac{FM - GL}{EG - F^2}\mathbf{r}'_u + \frac{FL - EM}{EG - F^2}\mathbf{r}'_v \\
\mathbf{N}'_v &= \frac{FN - GM}{EG - F^2}\mathbf{r}'_u + \frac{FM - EN}{EG - F^2}\mathbf{r}'_v
\end{aligned}$$

pri čemu se ove formule nazivaju *Weingartenove derivacijske formule*.

U slučaju kad su krivulje zakrivljenosti ujedno koordinatne krivulje

Fundamentalne jednadžbe plohe

Što se tiče prikaza derivacija vektora normale $\mathbf{N}$ u bazi $\{\mathbf{r}'_u, \mathbf{r}'_v, \mathbf{N}\}$, vrijedi

$$\begin{aligned} \mathbf{N}'_u &= \frac{FM - GL}{EG - F^2}\mathbf{r}'_u + \frac{FL - EM}{EG - F^2}\mathbf{r}'_v \\ \mathbf{N}'_v &= \frac{FN - GM}{EG - F^2}\mathbf{r}'_u + \frac{FM - EN}{EG - F^2}\mathbf{r}'_v \end{aligned}$$

pri čemu se ove formule nazivaju *Weingartenove derivacijske formule*.

U slučaju kad su krivulje zakrivljenosti ujedno koordinatne krivulje (tj. $F = 0$ i $M = 0$),

Fundamentalne jednadžbe plohe

Što se tiče prikaza derivacija vektora normale $\mathbf{N}$ u bazi $\{\mathbf{r}'_u, \mathbf{r}'_v, \mathbf{N}\}$, vrijedi

$$\begin{aligned} \mathbf{N}'_u &= \frac{FM - GL}{EG - F^2}\mathbf{r}'_u + \frac{FL - EM}{EG - F^2}\mathbf{r}'_v \\ \mathbf{N}'_v &= \frac{FN - GM}{EG - F^2}\mathbf{r}'_u + \frac{FM - EN}{EG - F^2}\mathbf{r}'_v \end{aligned}$$

pri čemu se ove formule nazivaju *Weingartenove derivacijske formule*.

U slučaju kad su krivulje zakrivljenosti ujedno koordinatne krivulje (tj. $F = 0$ i $M = 0$), tada Weingartenove derivacijske formule poprimaju oblik

$$\begin{aligned} \mathbf{N}'_u &= -\frac{L}{E}\mathbf{r}'_u, \\ \mathbf{N}'_v &= -\frac{N}{G}\mathbf{r}'_v \end{aligned}$$

Fundamentalne jednadžbe plohe

Što se tiče prikaza derivacija vektora normale $\mathbf{N}$ u bazi $\{\mathbf{r}'_u, \mathbf{r}'_v, \mathbf{N}\}$, vrijedi

$$\begin{aligned}\mathbf{N}'_u &= \frac{FM-GL}{EG-F^2}\mathbf{r}'_u + \frac{FL-EM}{EG-F^2}\mathbf{r}'_v \\ \mathbf{N}'_v &= \frac{FN-GM}{EG-F^2}\mathbf{r}'_u + \frac{FM-EN}{EG-F^2}\mathbf{r}'_v\end{aligned}$$

pri čemu se ove formule nazivaju *Weingartenove derivacijske formule*.

U slučaju kad su krivulje zakrivljenosti ujedno koordinatne krivulje (tj. $F = 0$ i $M = 0$), tada Weingartenove derivacijske formule poprimaju oblik

$$\begin{aligned}\mathbf{N}'_u &= -\frac{L}{E}\mathbf{r}'_u, \\ \mathbf{N}'_v &= -\frac{N}{G}\mathbf{r}'_v\end{aligned}$$

i zovu se *Rodriguesove formule*.

Teorem (Gauss, Codazzi, Mainardi).

Teorem (Gauss, Codazzi, Mainardi). Za bilo koju regularnu plohu $\mathbf{r} : U \to \mathbb{R}^3$ vrijede jednadžbe

$$\begin{aligned}
EK &= \frac{\partial \Gamma^v_{uu}}{\partial v} - \frac{\partial \Gamma^v_{uv}}{\partial u} + \Gamma^u_{uu}\Gamma^v_{uu} + \Gamma^v_{uu}\Gamma^v_{uv} - \Gamma^u_{uv}\Gamma^v_{uu} - \Gamma^v_{uv}\Gamma^v_{uv} \\
FK &= \frac{\partial \Gamma^u_{uv}}{\partial u} - \frac{\partial \Gamma^u_{uu}}{\partial v} + \Gamma^u_{uv}\Gamma^v_{uv} - \Gamma^v_{uu}\Gamma^u_{vv} \\
FK &= \frac{\partial \Gamma^v_{uv}}{\partial v} - \frac{\partial \Gamma^v_{vv}}{\partial u} + \Gamma^u_{uv}\Gamma^v_{uv} - \Gamma^v_{uu}\Gamma^u_{vv} \\
GK &= \frac{\partial \Gamma^u_{vv}}{\partial u} - \frac{\partial \Gamma^u_{uv}}{\partial v} + \Gamma^u_{uu}\Gamma^u_{vv} + \Gamma^u_{uv}\Gamma^v_{vv} - \Gamma^u_{uv}\Gamma^u_{uv} - \Gamma^v_{uv}\Gamma^u_{vv}
\end{aligned}$$

koje se zovu Gaussove jednadžbe,

Fundamentalne jednadžbe plohe

Teorem (Gauss, Codazzi, Mainardi). Za bilo koju regularnu plohu $\mathbf{r} : U \to \mathbb{R}^3$ vrijede jednadžbe

$$
\begin{aligned}
EK &= \frac{\partial \Gamma^v_{uu}}{\partial v} - \frac{\partial \Gamma^v_{uv}}{\partial u} + \Gamma^u_{uu}\Gamma^v_{uu} + \Gamma^v_{uu}\Gamma^v_{uv} - \Gamma^u_{uv}\Gamma^v_{uu} - \Gamma^v_{uv}\Gamma^v_{uv} \\
FK &= \frac{\partial \Gamma^u_{uv}}{\partial u} - \frac{\partial \Gamma^u_{uu}}{\partial v} + \Gamma^u_{uv}\Gamma^v_{uv} - \Gamma^v_{uu}\Gamma^u_{vv} \\
FK &= \frac{\partial \Gamma^v_{uv}}{\partial v} - \frac{\partial \Gamma^v_{vv}}{\partial u} + \Gamma^u_{uv}\Gamma^v_{uv} - \Gamma^v_{uu}\Gamma^u_{vv} \\
GK &= \frac{\partial \Gamma^u_{vv}}{\partial u} - \frac{\partial \Gamma^u_{uv}}{\partial v} + \Gamma^u_{uu}\Gamma^u_{vv} + \Gamma^u_{uv}\Gamma^v_{vv} - \Gamma^u_{uv}\Gamma^u_{uv} - \Gamma^v_{uv}\Gamma^u_{vv}
\end{aligned}
$$

koje se zovu Gaussove jednadžbe, te jednadžbe

$$
\begin{aligned}
L'_v - M'_u &= L\Gamma^u_{uv} + M(\Gamma^v_{uv} - \Gamma^u_{uu}) + N\Gamma^v_{uu}, \\
M'_v - N'_u &= L\Gamma^u_{vv} + M(\Gamma^v_{vv} - \Gamma^u_{uv}) + N\Gamma^v_{uv}.
\end{aligned}
$$

koje se zovu Codazzi-Mainardi jednadžbe.

Teorem (Gaussov theorema egregium).

Fundamentalne jednadžbe plohe

Teorem (Gaussov theorema egregium). Gaussova zakrivljenost regularne plohe ovisi samo o koeficijentima prve fundamentalne forme i njihovim derivacijama prvog i drugog reda.

Teorem (Gaussov theorema egregium). Gaussova zakrivljenost regularne plohe ovisi samo o koeficijentima prve fundamentalne forme i njihovim derivacijama prvog i drugog reda.

Dokaz.

Fundamentalne jednadžbe plohe

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Dokaz. Pokaže se da vrijedi formula

$$K = \frac{1}{(EG-F^2)^2}\left(\begin{vmatrix} E & F & F'_v - \frac{1}{2}G'_u \\ F & G & \frac{1}{2}G'_v \\ \frac{1}{2}E'_u & F'_u - \frac{1}{2}E'_v & F''_{uv} - \frac{1}{2}E''_{vv} - \frac{1}{2}G''_{uu} \end{vmatrix} - \begin{vmatrix} E & F & \frac{1}{2}E'_v \\ F & G & \frac{1}{2}G'_u \\ \frac{1}{2}E'_v & \frac{1}{2}G'_u & 0 \end{vmatrix} \right).$$

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QED.

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Fundamentalne jednadžbe plohe

Može se pokazati da vrijedi i sljedeća formula za Gaussovu zakrivljenost

$$
\begin{aligned}
K &= \frac{1}{4(EG-F^2)^2}\begin{vmatrix} E & E'_u & E'_v \\ F & F'_u & F'_v \\ G & G'_u & G'_v \end{vmatrix} + \\
&\quad + \frac{1}{2\sqrt{EG-F^2}}\left(\frac{\partial}{\partial u}\left(\frac{F'_v - G'_u}{\sqrt{EG-F^2}}\right) + \frac{\partial}{\partial v}\left(\frac{F'_u - E'_v}{\sqrt{EG-F^2}}\right)\right)
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koja se naziva Frobeniusova, a koja u slučaju ortogonalnih koordinatnih krivulja ($F = 0$) postaje

$$K = \frac{1}{2\sqrt{EG}}\left(\frac{\partial}{\partial u}\left(\frac{-G'_u}{\sqrt{EG}}\right) + \frac{\partial}{\partial v}\left(\frac{-E'_v}{\sqrt{EG}}\right)\right).$$

Teorem (Bonnet).

Fundamentalne jednadžbe plohe

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Dokaz.

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Dokaz. Ispuštamo zbog složenosti.