3.5. Na skici je prikazana konstrukcija dizalice (apsolutno kruta greda i vitki čelični štapovi). Zadana je nosivost dizalice P.
Potrebno je dimenzionirati štapove AD i CE, ako je poznato da je štap AD punog kružnog poprečnog presjeka promjera d, a štap CE šupljeg kružnog poprečnog presjeka unutrašnjeg promjera Du i vanjskog promjera Dv.
Za odabrane poprečne presjeke štapova izračunati naprezanja u njima, pri djelovanju sile P, te pomak točke F.

Poznato je:

$$\sigma_R = 24.0 \frac{kN}{cm^2}$$

$$\sigma_P = 21.0 \frac{kN}{cm^2}$$

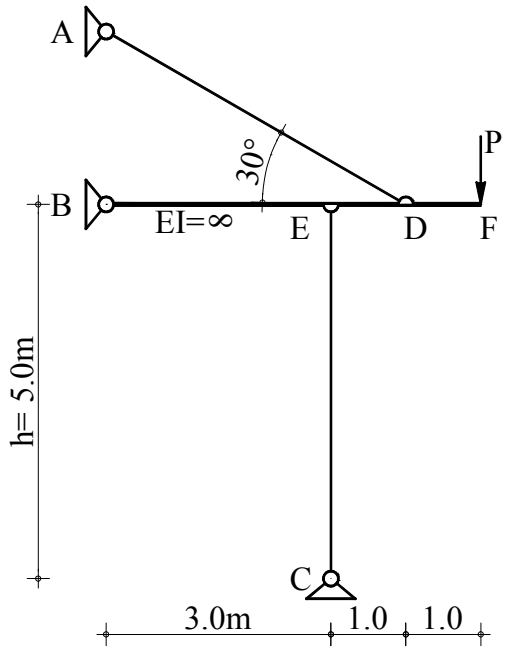
$$\nu = 1.5$$

$$\nu_i = 2.5$$

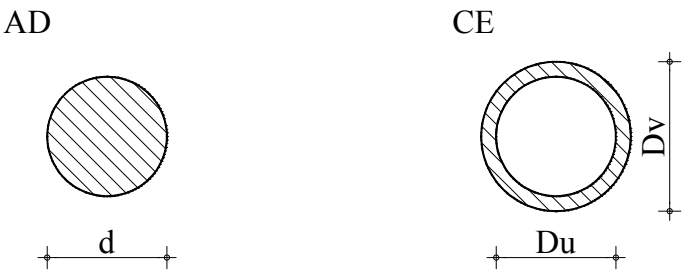
$$E = 2 \cdot 10^4 \frac{kN}{cm^2}$$

$$\frac{A_{AD}}{A_{CE}} = \frac{1}{3.65}$$

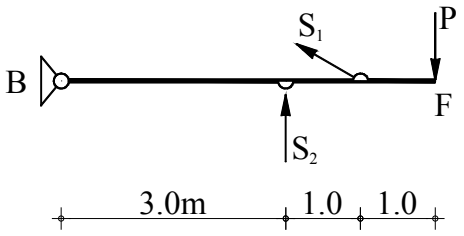
$$\frac{D_u}{D_v} = \frac{4}{5}$$



Karakteristike poprečnog presjeka



Uvjet ravnoteže:

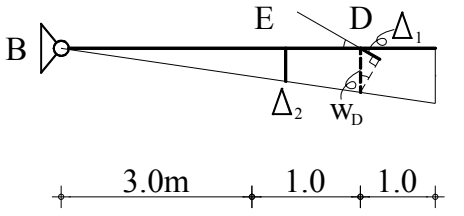


$$\sum M_B = 0$$

$$S_1 \cdot \sin 30° \cdot 4m + S_2 \cdot 3m - P \cdot 5m = 0$$

Uvjet deformacija:

Potrebno je uspostaviti vezu između produljenja štapova i sila koje se javljaju u štapovima. Podatak koji nam je potreban da bi pravilno uspostavili vezu je omjer površina poprečnih presjeka štapova, kao i njihove duljine, te nacrtati plan pomaka.



Možemo postaviti odnose:

$$\left.\begin{aligned} \frac{\Delta_2}{3} &= \frac{w_D}{4} \\ w_D &= \frac{\Delta_1}{\sin 30°} \end{aligned}\right\} \rightarrow \frac{\Delta_2}{3} = \frac{\Delta_1}{4 \cdot \sin 30°} \Rightarrow \Delta_1 = \frac{4 \cdot \sin 30°}{3} \Delta_2$$

$$\left.\begin{aligned} \Delta_1 &= \frac{S_1 \cdot L_1}{E \cdot A_1} \\ \Delta_2 &= \frac{S_2 \cdot L_2}{E \cdot A_2} \end{aligned}\right\} \rightarrow \frac{S_1 \cdot L_1}{E \cdot A_1} = \frac{4 \cdot \sin 30°}{3} \frac{S_2 \cdot L_2}{E \cdot A_2}$$

$$S_1 = S_2 \frac{4 \cdot \sin 30°}{3} \frac{A_1 \cdot L_2}{A_2 \cdot L_1}$$

Uvrstimo u

$$S_2 \frac{4 \cdot \sin 30°}{3} \frac{A_1 \cdot L_2}{A_2 \cdot L_1} \cdot \sin 30° \cdot 4m + S_2 \cdot 3m - P \cdot 5m = 0$$

$$S_2 \left(\frac{4 \cdot \sin^2 30°}{3} \frac{1 \cdot 5m}{3.65 \cdot \frac{4}{\cos 30°}} \cdot 4m + 3m\right) = P \cdot 5m$$

$$S_2 \cdot 3.39 = 60kN \cdot 5$$

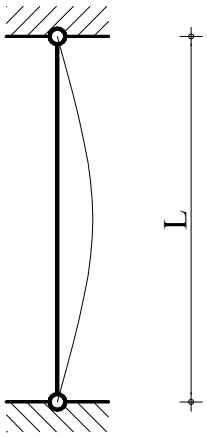
$$S_2 = \frac{300kN}{3.39} = 88.35kN$$

$$S_1 = S_2 \frac{4 \cdot \sin 30°}{3} \frac{A_1 \cdot L_2}{A_2 \cdot L_1} = 17.47\,\text{kN}$$

Kada smo odredili sile u štapovima, potrebno ih je dimenzionirati. Tlačni štap je potrebno provjeriti na izvijanje, ali i na dopuštena naprezanja.

Duljina izvijanja (slučaj zglobnog pridržanja na krajevima)

$$L_i = L_{CE} = 5\text{m}$$



Kritična sila za izvijanje

$$P_{kr} = \frac{\pi^2 \cdot E \cdot I_{min}}{L_i^{\,2}}$$

$$\frac{P_{kr}}{\nu_i} = P_{dop} \geq S_2$$

$$\frac{\pi^2 \cdot E \cdot I_{min}}{L_i^{\,2} \cdot \nu_i} \geq S_2 \Rightarrow I_{min} \geq \frac{L_i^{\,2} \cdot \nu_i \cdot S_2}{\pi^2 \cdot E}$$

$$I_{min} = \frac{\pi \cdot D_V^{\,4}}{64}\left[1 - (\frac{D_U}{D_V})^4\right] = \frac{\pi \cdot D_V^{\,4}}{64}(1 - (0.8)^4)$$

$$\frac{\pi \cdot D_V^{\,4}}{64}(1 - (0.8)^4) \geq \frac{L_i^{\,2} \cdot \nu_i \cdot S_2}{\pi^2 \cdot E}$$

$$D_V^{\,4} \geq \frac{64}{\pi \cdot (1 - (0.8)^4)} \cdot \frac{L_i^{\,2} \cdot \nu_i \cdot S_2}{\pi^2 \cdot E}$$

$$D_V^{\,4} \geq \frac{64}{\pi \cdot (1 - 0.4096)} \cdot \frac{(500\text{cm})^2 \cdot 2.5 \cdot 88.35\text{kN}}{\pi^2 \cdot 2.1 \cdot 10^4 \frac{\text{kN}}{\text{cm}^2}}$$

$$D_V^{\,4} \geq 34.505 \cdot 266.42\text{cm}^4$$

$$D_V^{\,4} \geq 9192.259\text{cm}^4$$

$$D_V \geq \sqrt[4]{9192.259\text{cm}^4} = 9.79\text{cm}$$

Odabrano:

$$D_V = 10.0\,\text{cm}$$

$$D_U = 8.0\,\text{cm}$$

Kontrola naprezanja u štapu CE

$$D_V = 10.0\,\text{cm}$$

$$D_U = 8.0\,\text{cm}$$

$$A_{CE} = \frac{D_V^{\,2} \cdot \pi}{4} - \frac{D_U^{\,2} \cdot \pi}{4} = 28.27\,\text{cm}^2$$

$$\sigma_{dop} = \frac{\sigma_R}{\nu} = \frac{24.0}{1.5}\frac{\text{kN}}{\text{cm}^2} = 16.0\frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_{CE} = \frac{S_2}{A_{CE}} = \frac{88.35\text{kN}}{28.27\text{cm}^2} = 3.12\frac{\text{kN}}{\text{cm}^2} < \sigma_{dop}$$

Kontrola naprezanja u štapu AD

$$\frac{A_{AD}}{A_{CE}} = \frac{1}{3.65}$$

$$A_{AD} = A_{CE}\frac{1}{3.65} = 28.27\text{cm}^2\frac{1}{3.65} = 7.75\text{cm}^2$$

$$\sigma_{dop} = \frac{\sigma_R}{\nu} = \frac{24.0}{1.5}\frac{\text{kN}}{\text{cm}^2} = 16.0\frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_{CE} = \frac{S_1}{A_{AD}} = \frac{17.47\text{kN}}{7.75\text{cm}^2} = 2.25\frac{\text{kN}}{\text{cm}^2} < \sigma_{dop}$$

Promjer štapa AD je:

$$A_{AD} = 7.75\text{cm}^2 = \frac{d^2 \cdot \pi}{4} \Rightarrow d = \sqrt{\frac{4 \cdot 7.75\text{cm}^2}{\pi}}$$

$$d = 3.14\,\text{cm}$$

Pomak hvatišta sile – točka F

Odrediti ćemo iz sličnosti trokuta

$$\frac{\Delta_2}{3} = \frac{w_F}{5}$$

$$w_F = \frac{3}{5}\Delta_2 = \frac{3}{5} \cdot \frac{S_2 \cdot L_{CD}}{E \cdot A_{CD}} = \frac{3}{5} \cdot \frac{88.35\text{kN} \cdot 500\text{cm}}{2.1 \cdot 10^4 \frac{\text{kN}}{\text{cm}^2} \cdot 28.27\text{cm}^2}$$

$$w_F = 0.0446\text{cm}$$