# APPLICATION OF FINITE DISCRETE ELEMENT METHOD IN DYNAMIC ANALYSIS OF MASONRY STRUCTURES

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**Abstract.** This paper presents the application of the numerical models for dynamic analysis of dry stone masonry structures strengthened with clamps and bolts and confined masonry structures based on finite discrete element method (FDEM). The proposed models considers the effects of the behavior of masonry structures due to dynamic action, crack initiation and propagation, energy dissipation mechanisms due to nonlinear effects, inertial effects due to motion, contact impact and attaining state of the rest that is a consequence of energy dissipation mechanisms in the structure. The models also include all relevant effects related to behavior of reinforcement in concrete and clams and bolts in stone blocks. The application of the model was performed on examples related to behavior of dry stone masonry structures strengthened with clamps and bolts as well as confined masonry wall.

## 1 INTRODUCTION

Building constructions using stone or clay bricks which are held together by mortar is one of the oldest building techniques which are still in use today. In spite of the simplicity which is manifested during the construction of masonry structures, understanding and describing mechanical behaviour of those structures, especially in conditions of seismic loading, represents a true challenge. The reason is the nature of masonry structures which might or might not be filled in with mortar in joints among the blocks. Such structures show a complex and particular nonlinear behaviour.

Many masonry structures are located in seismically active zones in which earthquakes exposed their vulnerability. With the aim of increasing their resistance, many of dry stone historical structures were further strengthened by steel clamps and bolts, while newly build masonry structures with mortar joints are usually confined with reinforced concrete members.

Numerical investigation of the behaviour of masonry structures under seismic loading is very complex and demands precise modelling of the stone blocks, steel clamps and bolts in dry stone structures as well as masonry infill and reinforced concrete elements in confined masonry structures. One of the dominant causes of nonlinearity in these structures is crack opening which leads to localized failure and stands out as a serious challenge in numerical modelling. Therefore, a realistic modelling of crack initiation and propagation is one of the key factors that affect the reliability of the model for analysis of the structures, especially

those subjected to earthquakes.

The most commonly used numerical tool for the analysis of masonry structures is the finite element method where the material is regarded as a fictitious homogeneous orthotropic continuum. These models encounter a significant limitation to simulate strong discontinuities between different blocks of the masonry. For overcoming these limitations the joint interface elements were developed to model the discontinuities. Another approach for modelling of masonry structures are based on a discrete element method. The common idea in different applications of the discrete element method to masonry structures is the idealization of the material as a discontinuum where joints are modelled as contact surfaces between different blocks. In recent times an increasing number of models attempted to combine the advantages of finite and discrete element methods. One of them is finite discrete element method (FDEM) which is subject of this paper. The FDEM was firstly developed for the simulation of fracturing problems considering deformable particles that may split and separate during the analysis.

In this work, the application of the numerical model for dynamic analysis of masonry structures based on a finite-discrete element method [1] is presented. The proposed model [2-5] considers the effects of the behaviour of dry stone masonry structures with or without steel clamps and bolts [3], masonry structures with mortar among the blocks [4] and confined masonry structures [6-8] due to dynamic action. The model includes crack initiation and propagation, energy dissipation mechanisms due to nonlinear effects, inertial effects due to motion, contact impact and attaining state of the rest that is a consequence of energy dissipation mechanisms in the structure. The application of the model was performed on several examples.

## 2 NUMERICAL MODEL

In this numerical model masonry structure is considered as assemblage of discrete elements. Each discrete element, which can split and separate during the analysis, is discretized by constant triangular finite elements. Material non-linearity, fracture and fragmentation are considered through the contact elements which are implemented within the finite element mesh of each block. Contact interaction between discrete elements is considered through the contact interaction algorithm based on the principle of potential contact forces [9] which include the Coulomb-type law for friction [10].

### 2.1 Numerical model for dry stone masonry strengthened with clamps and bolts

In this model each stone block is considered as one discrete element which is additionally discretized by constant strain triangular finite elements. The metal clamps which have only tension bearing capacity and metal bolts which have dominantly shear bearing capacity were modelled with 1D elements which can be placed in arbitrary positions inside the stone finite elements. Numerical model of clamps and bolts take into account cyclic behavior, yielding, stiffness degradation, failure and the influence of pulling out of the clamps and the bolts from the stone block [3, 5].

Discretization of dry stone masonry structure with embedded steel clamps and bolts is shown in Figure 1.
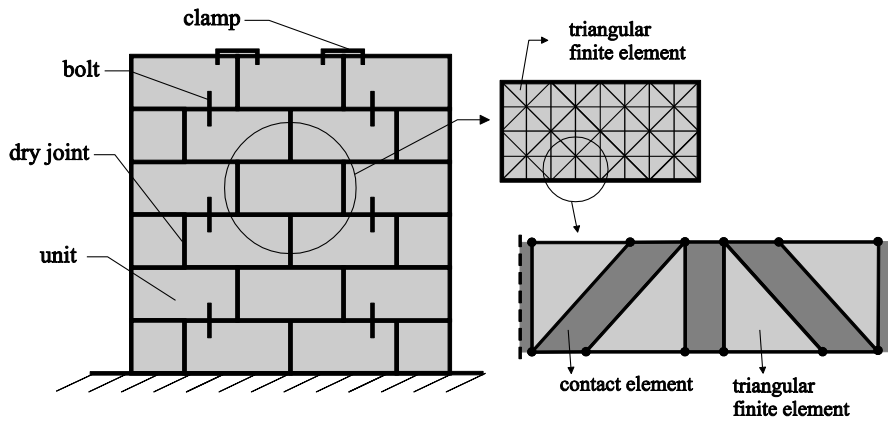
**Figure 1**: Discretization of dry stone masonry structure strengthen with steel clamps and bolts

## 2.2  Numerical model for confined masonry structures

In this model of confined masonry structure, masonry infill is considered as an assemblage of extended unit elements connected with zero thickness interface elements which simulate the behavior of the mortar joints and unit-mortar interface (Figure 2).
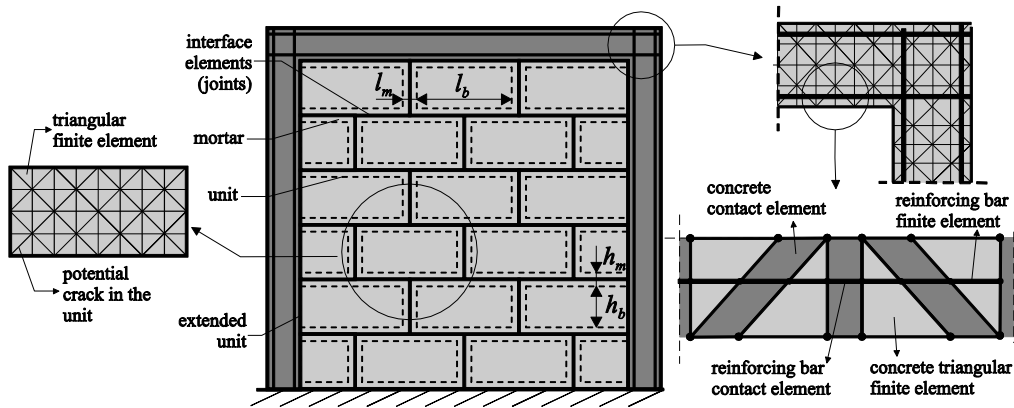


**Figure 2**: Discretization of confined masonry structure

The extended units are discretized by triangular finite elements which take into account the orthotropic and cyclic behavior, failure and softening in compression (Figure 3).

The numerical model of the interface element takes into account the possibility of failure and softening behavior in tension and shear, increasing of fracture energy in shear due to increasing pre-compression stress, decreasing of the friction coefficient due to the shear displacement increasing as well as cyclic behavior of the interface element [4, 5].

Reinforced concrete confined members are discretized with triangular finite elements for concrete, while the reinforcing bars are modeled with linear one-dimensional elements which can be placed in arbitrary position inside the concrete finite elements.
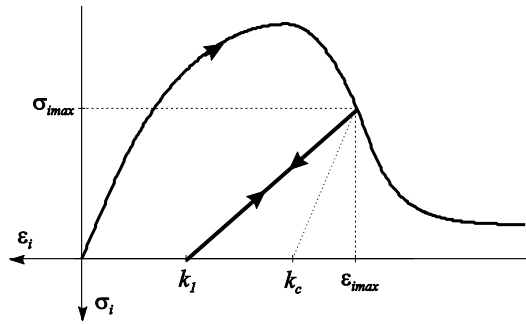
**Figure 3**: Failure, softening and cyclic behavior in compression

The nonlinear behavior of concrete in tension and shear is modeled in the plane joint element implemented between the triangular finite elements, while the non-linear behavior of the reinforcing bar is modeled in the one-dimensional joint element implemented between linear finite elements of reinforcing bar (Figure 2).
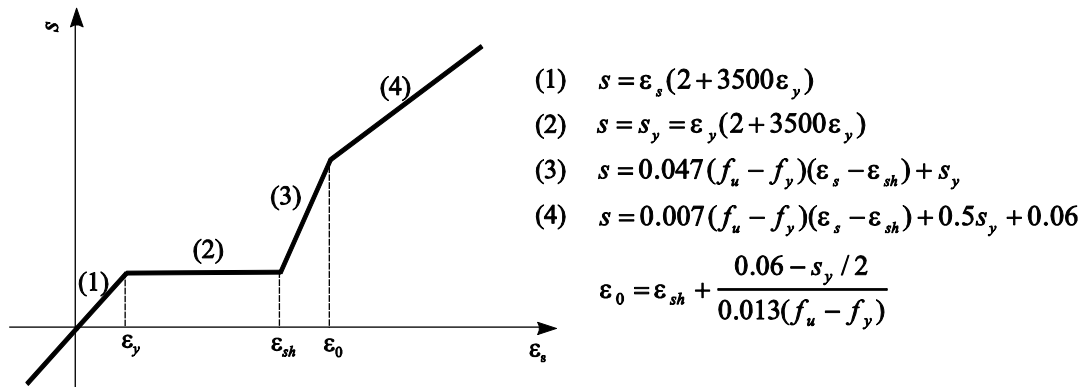


(1) $\quad s = \varepsilon_s(2 + 3500\varepsilon_y)$

(2) $\quad s = s_y = \varepsilon_y(2 + 3500\varepsilon_y)$

(3) $\quad s = 0.047(f_u - f_y)(\varepsilon_s - \varepsilon_{sh}) + s_y$

(4) $\quad s = 0.007(f_u - f_y)(\varepsilon_s - \varepsilon_{sh}) + 0.5s_y + 0.06$

$$\varepsilon_0 = \varepsilon_{sh} + \frac{0.06 - s_y/2}{0.013(f_u - f_y)}$$

**Figure 4:** Steel strain-slip relation under monotonic loading

The numerical model of confined reinforced concrete member take into account the interaction between the reinforcement and concrete which was taken into consideration by steel strain-slip relation (Figure 4), the influence of adjacent cracks to the slip of reinforcing bar, local slip of reinforcing bar near the crack plane when the bar undergoes high plastic deformation under reversed cyclic loading, the influence of the curvature of the reinforcing bar to yield stress reduction of the steel, the cyclic behavior of concrete and steel added to the existing combined single and smeared crack model, which is an essential mechanism for energy loss in post-cracking response of the structure [6-8].

## 3 NUMERICAL EXAMPLES

### 3.1 Seismic analysis of the Prothyron in Split - Croatia

The following example shows the application of the presented model (FDEM method) for the simulation of the dynamic response of the structure of the Prothyron in Split (Figure 5) to seismic load.

The Prothyron is located on the south side of the Peristyle. Four massive granite columns with Dorian capitals are located at the entrance of the Prothyron. The capitals support the broad gable with an arch in the middle. The structure was originally built of dry stone blocks with steel bolts embedded between columns, capitals and the upper beams. Throughout history deformations of the stone blocks which constitute the broad gable have occurred with the movement of the central columns. Due to the movement of the blocks, restoration of the structure was performed using steel clamps during the period of the Austro-Hungarian Empire.



**Figure 5**: The Prothyron entrance at the Peristyle in Split

In order to evaluate the dynamic response of the structure, the incremental dynamic analysis of the original geometry of the structure before the restoration was applied. The analyses were performed for two cases: structure without metal bolts and clamps and structure with steel bolts and clamps (after restoration). For both cases the structure was exposed to horizontal ground acceleration (Figure 6) which was recorded on 15.4.1979. in Dubrovnik on rock soil during an earthquake with the epicentre in Petrovac (Montenegro). The accelelogram was firstly scaled on peak ground acceleration $a_g$=0.22g which is valid for Split. After that the acceleration was gradually increased to the collapse of the structure.
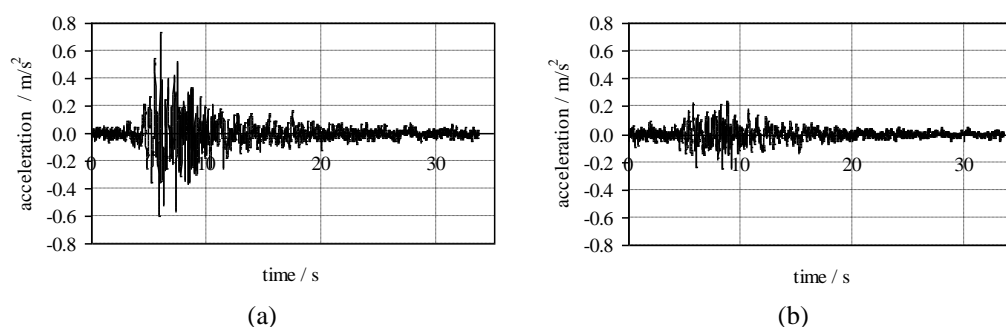


**Figure 6**: Ground acceleration recorded during the Petrovac earthquake (1979. Montenegro): (a) horizontal direction; (b) vertical direction

Figure 7 shows the geometry and the finite element mesh of the structure. Material characteristics used in the numerical analysis are shown in Table 1 and Table 2.
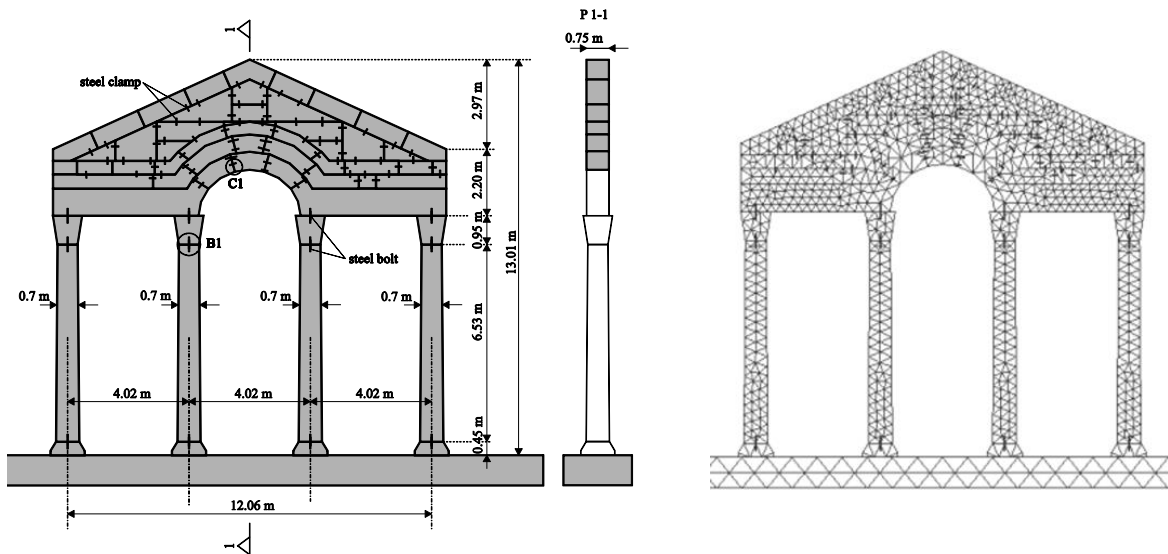
**Figure 7**: Geometry and finite element mesh of the Prothyron structure

**Table 1**: Material characteristics of the stone

| Young's Modulus $E$ (MPa) | Static sliding friction $\mu_{st}$ | Dynamic sliding friction $\mu_{din}$ |
|---|---|---|
| 48400 | 0.6 | 0.6 |

**Table 2**: Material characteristics of clamps and bolts

| Sample | Young's Modulus $E$ (MPa) | Tensile strength $f_{st}$ (MPa) | Shear strength $f_{su}$ (MPa) |
|---|---|---|---|
| Bolt | 181000 | - | 239 |
| Clamp | 65000 | 125 | - |

Dynamic analysis of the structure for the acceleration of $a_g$=0.22g showed that the movement of the blocks, i.e. opening of the joints appeared for a dry stone while for a dry stone structure with steel bolts and clamps there was no opening of the joints. The collapse of dry stone structure occurred for the acceleration of $a_g$=0.45g (Figure 8).

The behavior of the structure with steel bolts and cooper clamps for an acceleration $a_g$=1.2 g over time is shown in Figure 9. The figure shows that the broad gable with embedded cooper clamps behaves as one body.

The performed analysis shows that the original structure before its restoration with the cooper clamps had low seismic resistance because the significant displacement of the central block occurred at a design acceleration $a_g$=0.22 g. The influence of embedded cooper clamps for seismic resistance of structure is significant since the collapse of the structure appears for $a_g$=1.2 g.
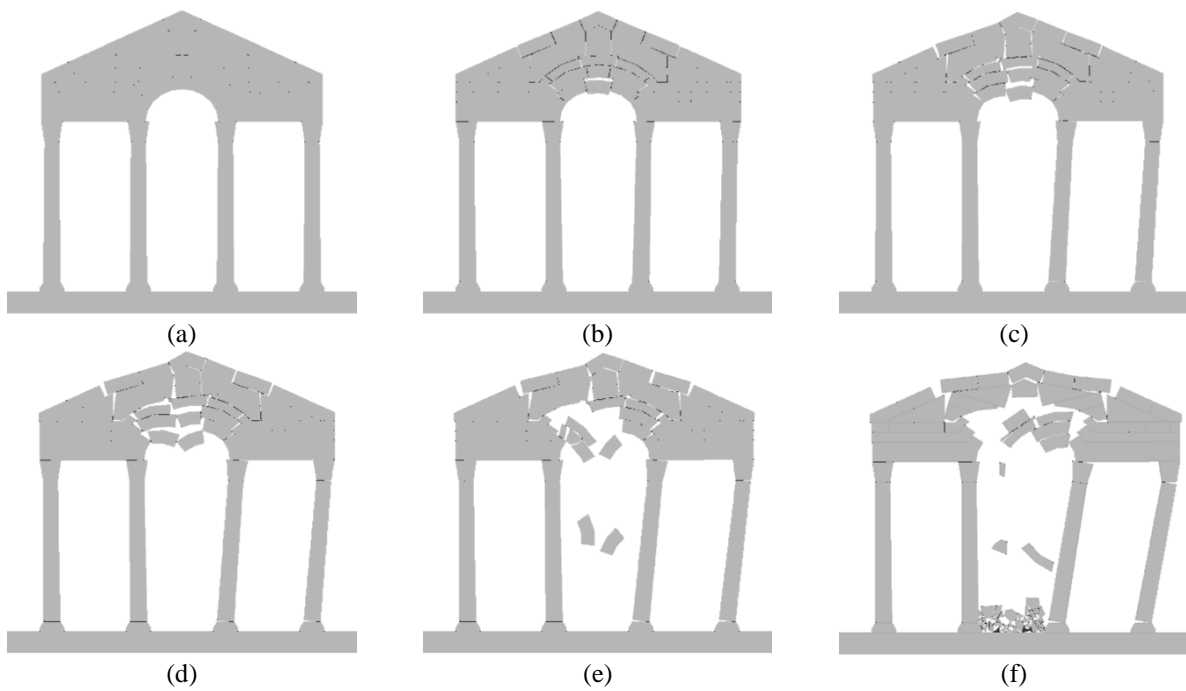
**Figure 8**: Collapse mechanism of dry stone structure with steel bolts in time: (a) *t*=0.0 s; (b) *t*=11.8 s; (c) *t*=17.2 s; (d) *t*=17.9 s; (e) *t*=18.7 s; (f) *t*=19.6 s.
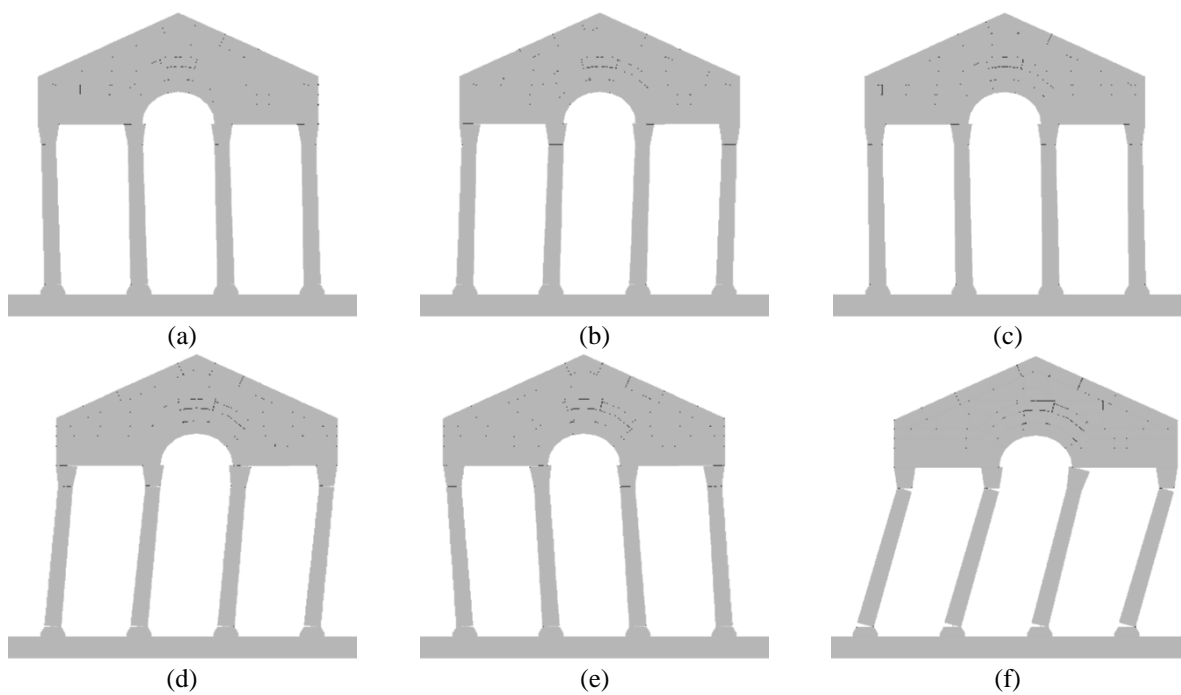


**Figure 9**: Dry stone structure with steel bolts and cooper clamps in time: (a) *t*=9.8 s; (b) *t*=12.0 s; (c) *t*=15.1 s; (d) *t*=23.5 s; (e) *t*=27.6 s; (f) *t*=32.2 s.

### 3.2 Seismic analysis of the confined masonry wall

The application of the FDEM for simulation of the dynamic response of the structure was performed on a confined masonry wall with geometry and discretization shown in Figure 10. Wall thickness was equal 0.25 m. Material characteristics of the wall are given in Table 3 and 4. Vertical load at each story was adopted in amount of 0.5 MPa.

The wall was exposed to horizontal ground acceleration (Figure 6a). The accelelogram was firstly scaled on peak ground acceleration $a_g$=0.22g which is valid for Split. After that the acceleration was gradually increased to the collapse of the structure.
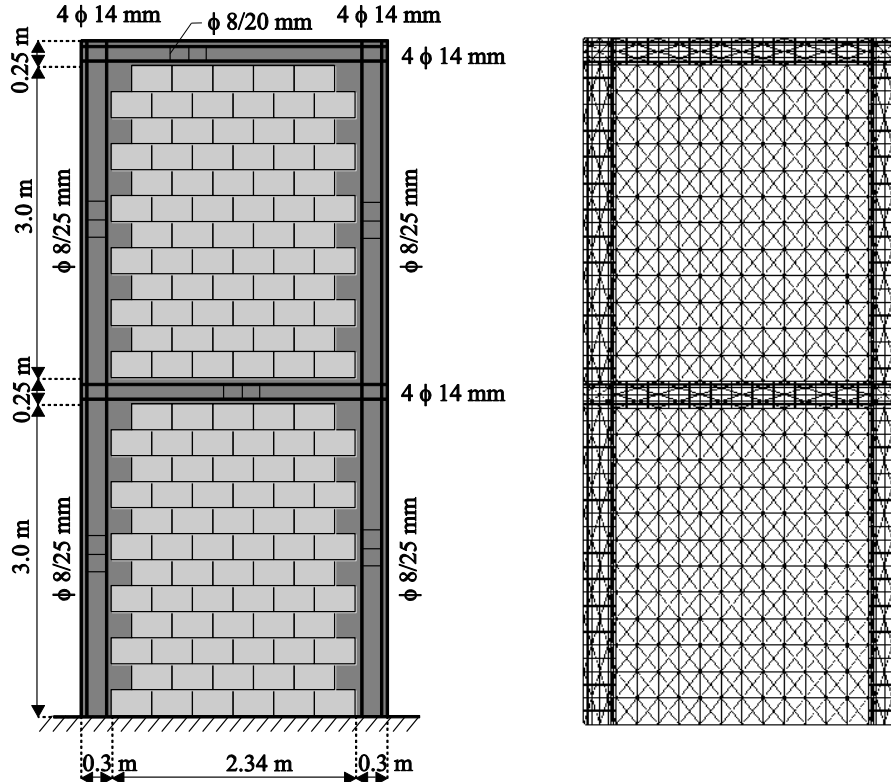
**Figure 10**: Geometry and discretization of confined masonry wall

**Table 3**: Material characteristics of reinforced concrete

| Concrete | | Steel | |
|---|---|---|---|
| Modulus of elasticity (MPa) | 30 500 | Modulus of elasticity (MPa) | 210 000 |
| Tensile strength (MPa) | 3.8 | Yield strength ( MPa) | 500 |
| Compressive strength (MPa) | 38.0 | Ultimate strength ( MPa) | 650 |
| Fracture energy (N/m) | 150 | | |

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**Table 4**: Material characteristics of unit and interface element

| Unit | | Interface element | |
|---|---|---|---|
| Modulus of elasticity (MPa) | 1033 | Tensile strength (MPa) | 0.16 |
| Poisson ratio | 0.141 | Shear strength (MPa) | 0.224 |
| Compressive strength $f_{cx}$ (MPa) | 2.7 | Fracture energy in tension (N/m) | 12 |
| Compressive strength $f_{cy}$ (MPa) | 10.33 | Fracture energy in shear (N/m) | 50 |

For pick ground acceleration $a_g$=4.0 m/s$^2$ crack initiation and propagation in the wall in different time steps are shown in Figure 11, while the displacements of the first and second story in time are shown in Figure 12.



(a)　　　　　　　　　　　(b)　　　　　　　　　　　(c)
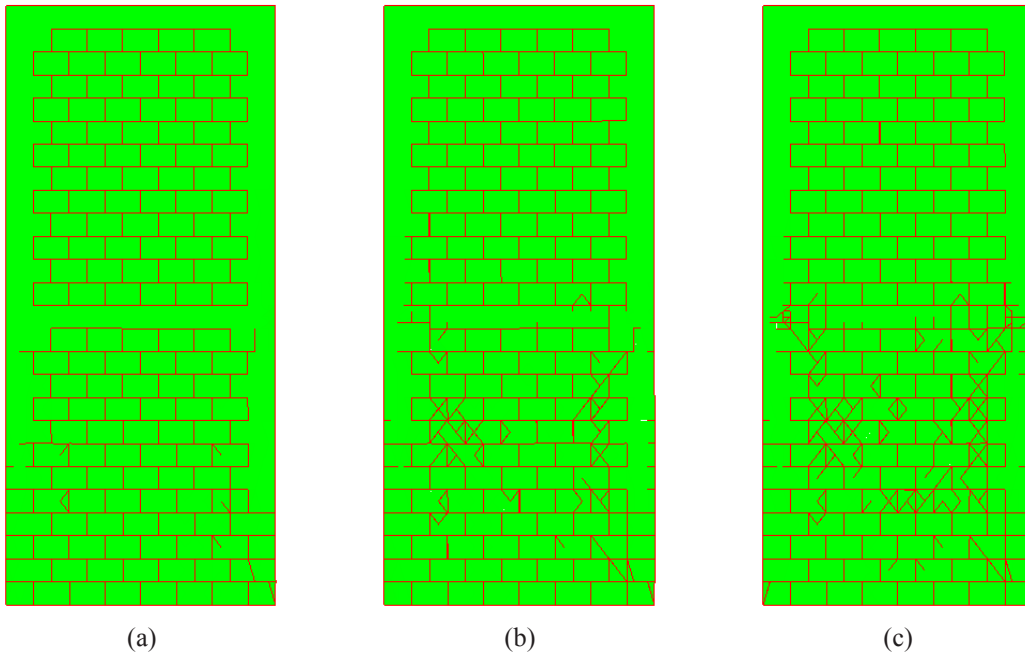
**Figure 11**: Crack initiation and propagation for $a_g$=4.0 m/s$^2$ in time: (a) $t$=4.85 s; (b) $t$=8.75 s; (c) $t$=12.0 s

It can be observed that for the peak ground acceleration of 4.0 m/s$^2$ significant cracks have been appeared in units and reinforced concrete confined member at the first floor. Maximal achieved displacement of the top of the wall was 40.0 mm, which is equal to $H$/150 where $H$ is the total height of the wall (Figure 12).

Figure 13 shows the maximal displacement of the top of the wall $u/H$ in relation on the peak ground acceleration obtained by incremental dynamic analysis. It can be seen that the collapse of the wall was achieved during the maximal acceleration of 4.0 m/$^2$ when the displacement of the top of the wall was $u/H = 0.67\%$.
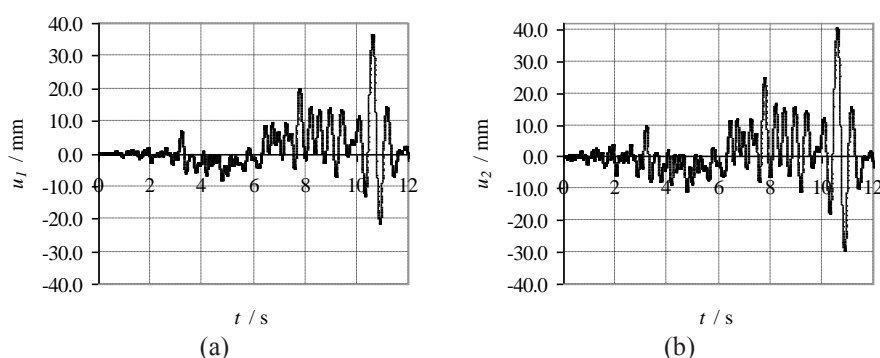
**Figure 12**: Displacements for $a_g$=4.0 m/s$^2$ in time: (a) first story (b) second story
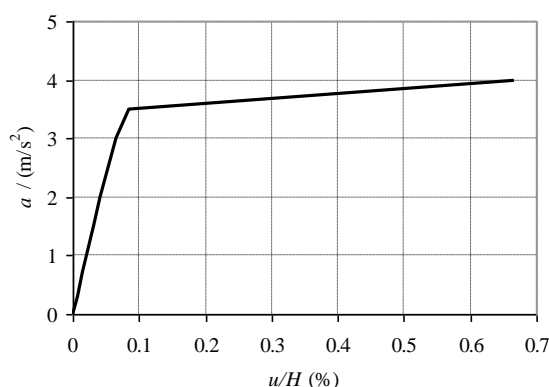


**Figure 13**: Maximal displacement of the top of the wall in relation on the peak ground acceleration

## 12 CONCLUSIONS

This paper presents the application of the numerical models which are based on finite discrete element method in seismic analysis of dry stone masonry structure strengthened with clamps and bolts and confined masonry structures.

Numerical model for dry stone masonry structures strengthened with clamps and bolts takes into account the contact interaction between stone blocks, energy dissipation during impact, block deformability, fracture and fragmentation of stone blocks, cyclic behavior, yielding and failure of the clamps and bolts, as well as the influence of pulling out of the clamps and bolts from the stone block. The application of this numerical model was shown in the incremental dynamic analysis of the structure of the Prothyron in Split.

The material model for confined masonry structures consists of numerical model for masonry infill and numerical model for reinforced concrete confined members.

Numerical model for masonry infill takes into account orthotropic and cyclic behavior in units, the possibility of failure and softening in compression, possibility of failure and softening behavior in mortar joints, increasing of fracture energy in mortar joints due to increasing pre compression stress, decreasing friction coefficient due to increasing shear displacement as well as cyclic behavior.

The numerical model of confined reinforced concrete member take into account embedded

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model of reinforcing bars, the interaction between the reinforcement and concrete, the influence of adjacent cracks to the slip of reinforcing bar, local slip of reinforcing bar near the crack plane when the bar undergoes high plastic deformation under reversed cyclic loading, the influence of the curvature of the reinforcing bar to yield stress reduction of the steel, the cyclic behavior of concrete and steel.

The performance of this numerical model was shown in incremental dynamic analysis of two story confined masonry wall.

### Acknowledgements

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