1.2. Za zadani nosač analitičkom metodom odrediti progib na mjestu djelovanja sile P i kut zaokreta uz ležajeve A i B.

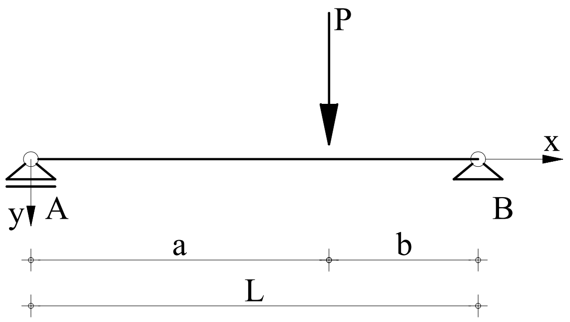
$a = 4\text{m}$

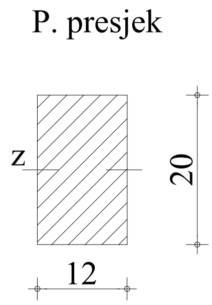
$b = 2\text{m}$

$L = 6\text{m}$

$P = 20\text{kN}$

$E = 20 \cdot 10^3 \ \dfrac{\text{kN}}{\text{cm}^2}$





$$I_z = \frac{b \cdot h^3}{12} = \frac{12 \cdot 20^3}{12} = 8 \cdot 10^3 \text{cm}^4$$

Reakcije

$\sum M_A = 0$

$R_B \cdot L - P \cdot a = 0$

$R_B = \dfrac{P \cdot a}{L} = \dfrac{20 \cdot 4}{6} = 13.33\text{kN}$

$\sum M_B = 0$

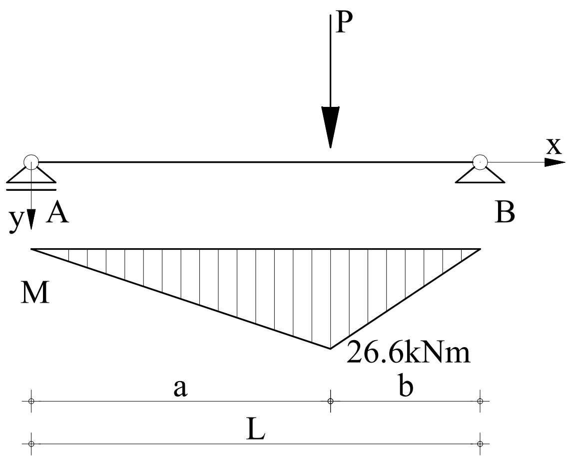
$R_A \cdot L - P \cdot b = 0$

$R_A = \dfrac{P \cdot b}{L} = \dfrac{20 \cdot 2}{6} = 6.67\text{kN}$

$\sum V = 0$

$R_a + R_B = P$

$6.67\text{kN} + 13.33\text{kN} = 20\text{kN}$

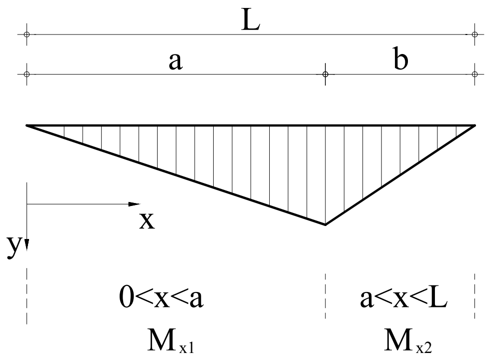


Jednadžba elastične linije

Jednadžbe za elastičnu liniju; kut zaokreta i progib, se mogu dobiti iz neposrednog integriranja jednadžbe:

$$\frac{d^2 w}{dx^2} = -\frac{M_z}{E \cdot I_z} \tag{1}$$

Moment je potrebno zapisati kao funkciju od položaja po gredi (x) i opterećenja, ali ga moramo podijeliti po dijelovima 0-a, i a-L:



$0 \le x < a \qquad R_A = \frac{P \cdot b}{L} \qquad M_{x1} = R_A \cdot x = \frac{P \cdot b}{L} \cdot x \qquad (2)$

$a \le x \le L \qquad R_A = \frac{P \cdot b}{L} \qquad M_{x2} = R_A \cdot x - P(x-a) = = \frac{P \cdot b}{L} \cdot x - P(x-a) \qquad (3)$

Uvrštavajući izraze za moment (2) i (3) u jednadžbu (1) dobivamo jednadžbe (4) i (5)

$$M_z \rightarrow \frac{d^2w}{dx^2} = -\frac{M_z}{E \cdot I_z}$$

Dobivamo diferencijalne jednadžbu elastične linije po dijelovima.

Lijevi dio

$0 \le x \le a \qquad M_{x1} = \frac{P \cdot b}{L} \cdot x \qquad \frac{d^2w}{dx^2} = -\frac{\frac{P \cdot b}{L} \cdot x}{E \cdot I_z} \qquad (4)$

$$E \cdot I_z \frac{d^2w}{dx^2} = -\frac{P \cdot b}{L} \cdot x$$

$$E \cdot I_z \frac{d^2w}{dx^2} = -\frac{P \cdot b}{L} \cdot x \Big|^{\int}$$

Integracijom dobivamo i konstante, koje ćemo označiti s indeksom prema području u kojem se nalaze.

$$E \cdot I_z \frac{dw}{dx} = E \cdot I_z \cdot \varphi = -\frac{P \cdot b}{L} \cdot \frac{x^2}{2} + C_1 \Big|^{\int}$$

$$E \cdot I_z \cdot w = -\frac{P \cdot b}{L} \cdot \frac{x^3}{6} + C_1 \cdot x + D_1$$

Desni dio

$a < x \leq L \qquad M_{x2} = \frac{P \cdot b}{L} \cdot x - P(x-a) \qquad \frac{d^2w}{dx^2} = -\frac{\frac{P \cdot b}{L} \cdot x - P(x-a)}{E \cdot I_z}$ (5)

$$E \cdot I_z \frac{d^2w}{dx^2} = -(\frac{P \cdot b}{L} \cdot x - P(x-a))$$

$$E \cdot I_z \frac{d^2w}{dx^2} = -(\frac{P \cdot b}{L} \cdot x - P(x-a))\Big|\int$$

$$E \cdot I_z \frac{dw}{dx} = E \cdot I_z \cdot \varphi = -\frac{P \cdot b}{L} \cdot \frac{x^2}{2} + \frac{P \cdot (x-a)^2}{2} + C_2\Big|\int$$

$$E \cdot I_z \cdot w = -\frac{P \cdot b}{L} \cdot \frac{x^3}{6} + \frac{P \cdot (x-a)^3}{6} + C_2 \cdot x + D_2$$

Određivanje nepoznatih konstanti

Kako bi odredili četiri nepoznate konstante potrebno je postaviti uvjete kontinuiteta, te time dobiti dodatne potrebne jednadžbe. U točki x=a progib s lijeve i desne strane mora biti jednak. Isto vrijedi i za kut zaokreta.

$x = a \qquad \varphi_L = \varphi_D \qquad w_L = w_D$

$$\varphi_{Lijevo} = \varphi_{Desno}$$

$$-\frac{Pbx^2}{2L} + C_1 = -\frac{Pbx^2}{2L} + \frac{P(x-a)^2}{2} + C_2$$

$$-\frac{Pba^2}{2L} + C_1 = -\frac{Pba^2}{2L} + \frac{P(a-a)^2}{2} + C_2$$

$$C_1 = C_2 = C$$

$$w_{Lijevo} = w_{Desno}$$

$$-\frac{Pbx^3}{6L} + C_1x + D_1 = -\frac{Pbx^3}{6L} + \frac{P(x-a)^3}{6} + C_2x + D_2$$

$$-\frac{Pba^3}{6L} + Ca + D_1 = -\frac{Pba^3}{6L} + \frac{P(a-a)^3}{6} + Ca + D_2$$

$$D_1 = D_2 = D$$

Uvrstimo li rubne uvjete možemo odrediti konstante C i D.

$x = 0 \qquad w_0 = 0$

S obzirom da se točka $x = 0$ nalazi u području $0 \leq x < a$ izraz je:

$$E \cdot I_z \cdot w = -\frac{P \cdot b}{L} \cdot \frac{x^3}{6} + C \cdot x + D$$

$$0 = D$$

S obzirom da se točka $x = L$ nalazi u području $a \leq x \leq L$ izraz je:

$x = L \qquad w_L = 0$

$$E \cdot I_z \cdot w = -\frac{P \cdot b}{L} \cdot \frac{x^3}{6} + \frac{P \cdot (x-a)^3}{6} + C \cdot x$$

$$0=-\frac{P\cdot b}{L}\cdot\frac{L^3}{6}+\frac{P\cdot(L-a)^3}{6}+C\cdot L$$

$$0=-\frac{P\cdot b\cdot L^2}{6}+\frac{P\cdot b^3}{6}+C\cdot L$$

$$C=\frac{P\cdot b\cdot(L^2-b^2)}{6\cdot L}$$

Kada uvrstimo dobivene vrijednosti za konstante dobivamo analitičko rješenje za elastičnu liniju.

Lijevi dio

$0\le x<a$

$$w=\frac{1}{E\cdot I_z}\left(-\frac{P\cdot b}{L}\cdot\frac{x^3}{6}+\frac{P\cdot b\cdot(L^2-b^2)}{6\cdot L}\cdot x\right) \quad (6)$$

Desni dio

$a\le x\le L$

$$w=\frac{1}{E\cdot I_z}\left(-\frac{P\cdot b}{L}\cdot\frac{x^3}{6}+\frac{P\cdot(x-a)^3}{6}+\frac{P\cdot b\cdot(L^2-b^2)}{6\cdot L}\cdot x\right) \quad (7)$$

Progib grede ispod hvatišta sile

Potrebno je uvrstiti vrijednost x=a ili u jednadžbu (6) ili u (7) kako bi izračunali traženi progib.

$x=a$

$$w_P=\frac{1}{E\cdot I_z}\left(-\frac{P\cdot b}{L}\cdot\frac{x^3}{6}+\frac{P\cdot b\cdot(L^2-b^2)}{6\cdot L}\cdot x\right)\Bigg|^{x=a}$$

$$w_P=\frac{1}{E\cdot I_z}\left(-\frac{P\cdot b}{L}\cdot\frac{a^3}{6}+\frac{P\cdot b\cdot(L^2-b^2)}{6\cdot L}\cdot a\right)$$

$$w_P=\frac{1}{E\cdot I_z}\frac{P\cdot a\cdot b\cdot(L^2-a^2-b^2)}{6\cdot L}$$

$$w_P=\frac{1}{20\cdot10^3\frac{kN}{cm^2}\cdot8\cdot10^3cm^4}\frac{P\cdot400cm\cdot200cm\cdot((600cm)^2-(400cm)^2-(200cm)^2)}{6\cdot600cm}$$

$$w_P=0.444cm$$

Kutevi zaokreta na ležajevima

Potrebno je uvrstiti vrijednost x u jednadžbu za kut zaokreta

$x_A=0$

$x_B=6m$

Kuta zaokreta uz ležaj "A" - koristimo jednadžbu za lijevi dio nosača

$$\varphi_A=\frac{1}{E\cdot I_z}\left(-\frac{P\cdot b}{L}\cdot\frac{x^2}{2}+\frac{P\cdot b\cdot(L^2-b^2)}{6\cdot L}\right)\Bigg|^{x=0}$$

$$\varphi_A=\frac{1}{E\cdot I_z}\left(\frac{P\cdot b\cdot(L^2-b^2)}{6\cdot L}\right)$$

$$\varphi_A=\frac{1}{20\cdot10^3\frac{kN}{cm^2}\cdot8\cdot10^3cm^4}\left(\frac{20kN\cdot2m\cdot((6m)^2-(2m)^2)}{6\cdot6m}\right)$$

$$\varphi_A=0.002\dot{2}rad=0.1273^\circ$$

Kuta zaokreta uz ležaj "B" - koristimo jednadžbu za desni dio nosača

$$\varphi_A = \frac{1}{E \cdot I_z}\left(-\frac{P \cdot b}{L} \cdot \frac{x^2}{2} + \frac{P \cdot (x-a)^2}{2} + \frac{P \cdot b \cdot (L^2 - b^2)}{6 \cdot L}\right)\Bigg|^{x=L}$$

$$\varphi_A = \frac{1}{E \cdot I_z}\left(-\frac{P \cdot b}{L} \cdot \frac{L^2}{2} + \frac{P \cdot (L-a)^2}{2} + \frac{P \cdot b \cdot (L^2 - b^2)}{6 \cdot L}\right)$$

$$\varphi_B = \frac{1}{20 \cdot 10^3 \frac{kN}{cm^2} \cdot 8 \cdot 10^3 cm^4}\left(-\frac{P \cdot 2m}{6m} \cdot \frac{(6m)^2}{2} + \frac{P \cdot (6m - 4m)^2}{2} + \frac{P \cdot 2m \cdot ((6m)^2 - (2m)^2)}{6 \cdot 6m}\right)$$

$$\varphi_B = -0.00278 rad = -0.1592^\circ$$