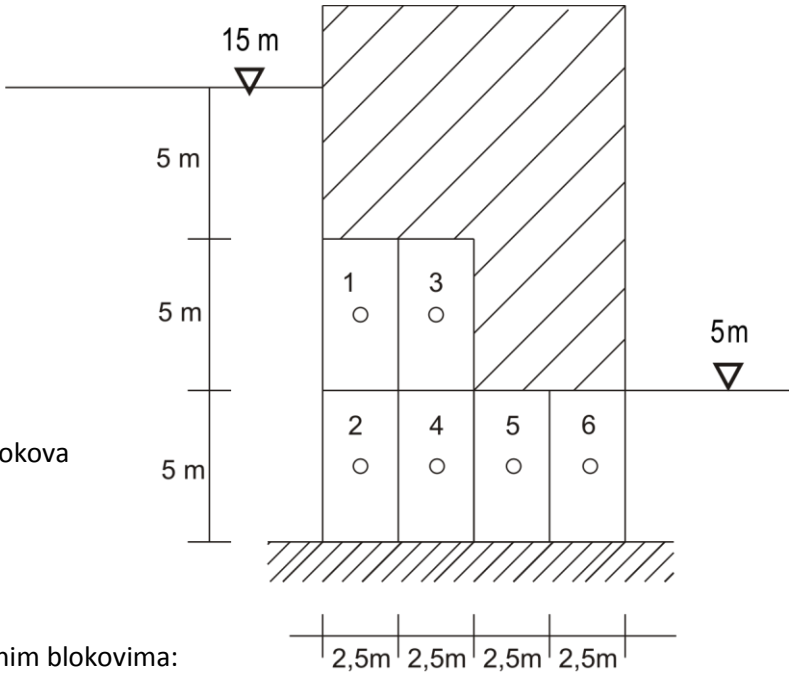
VJEŽBE 16/10/2012

METODA KONAČNIH RAZLIKA - MODFLOW

Riješiti problem tečenja (potencijale u blokovima -h; protoke na rubovima blokova -q) koristeći MODFLOW formulaciju:

1.PRIMJER:



$$K_1 = K_2 = 10^{-4} m/s \rightarrow K_{1,2}$$
$$K_3 = K_4 = 5 \cdot 10^{-5} m/s \rightarrow K_{3,4}$$
$$K_5 = K_6 = 10^{-5} m/s \rightarrow K_{5,6}$$
$$n = 0,2$$

Koeficijenti propusnosti između blokova (na granicama) se dobivaju kao harmonijske srednje vrijednosti koeficijenata koji pripadaju graničnim blokovima:

$$K_{1,2/3,4} = \frac{2 \cdot K_{1,2} \cdot K_{3,4}}{K_{1,2} + K_{3,4}} = \frac{10^{-8}}{1,5 \cdot 10^{-4}} = 6,67 \cdot 10^{-5} m/s \rightarrow K_A$$

$$K_{3,4/5,6} = \frac{2 \cdot K_{3,4} \cdot K_{5,6}}{K_{3,4} + K_{5,6}} = \frac{10^{-9}}{6 \cdot 10^{-5}} = 1,67 \cdot 10^{-5} m/s \rightarrow K_B$$

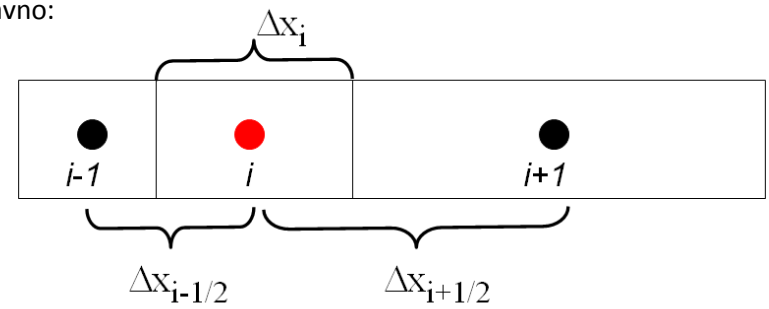
- Rubni uvjeti:

Lijeva granica i desna granica – kinematički(prisilni) rubni uvjet, zadani h!!! $h_1 = h_2 = 15m$; $h_6 = 5m$

Donja i gornja granica – nepropusne!

Primjena metode konačnih diferencija izravno:

$$\frac{\partial}{\partial x}(K_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(K_y \frac{\partial h}{\partial y}) = 0$$



$$\frac{1}{(\Delta x)_{i,j,k}} K_{x_{(i+1/2,j,k)}} \frac{h_{i+1,j,k} - h_{i,j,k}}{(\Delta x)_{i+1/2,j,k}} - K_{x_{(i-1/2,j,k)}} \frac{h_{i,j,k} - h_{i-1,j,k}}{(\Delta x)_{i-1/2,j,k}}$$

- Blokovi u području:

3.

$$\frac{1}{\Delta x}\left(0-K_A\cdot\frac{h_3-h_1}{\Delta x}\right)+\frac{1}{\Delta y}\left(0-K_{3,4}\cdot\frac{h_3-h_4}{\Delta y}\right)=0$$
$$-\left(4K_A+K_{3,4}\right)\cdot h_3+K_{3,4}\cdot h_4=-60K_A$$

4.

$$\frac{1}{\Delta x}\left(K_B\cdot\frac{h_5-h_4}{\Delta x}-K_A\cdot\frac{h_4-h_2}{\Delta x}\right)+\frac{1}{\Delta y}\left(K_{3,4}\cdot\frac{h_3-h_4}{\Delta y}-0\right)=0$$
$$K_{3,4}\cdot h_3-\left(K_{3,4}+4K_A+4K_B\right)\cdot h_4+4K_B\cdot h_5=-60K_A$$

5.

$$\frac{1}{\Delta x}\left(K_{5,6}\cdot\frac{h_6-h_5}{\Delta x}-K_B\cdot\frac{h_5-h_4}{\Delta x}\right)+\frac{1}{\Delta y}\cdot 0=0$$
$$K_B\cdot h_4-\left(K_{5,6}+K_B\right)\cdot h_5=-5K_{5,6}$$

- Rješenje sustava:

$$\begin{bmatrix}-31,67 & 5 & 0\\ 5 & -38,33 & 6,67\\ 0 & 1,67 & -2,67\end{bmatrix}\cdot\begin{bmatrix}h_3\\ h_4\\ h_5\end{bmatrix}=\begin{bmatrix}-400\\ -400\\ -5\end{bmatrix}$$

- Napomena: obe strane sustava su podijeljene sa 10^{-5} radi jednostavnijeg proračuna
- Sustav rješen u excelu/derive-u/matlabu...

  $h_3=14,88\,m$
- $h_4=14,25\,m$

  $h_5=10,78\,m$

- Izračun protoka u blokovima sa zadanim prisilnim rubnim uvjetom:

1.

$$q_1 = -K_A \cdot \frac{h_3 - h_1}{\Delta x}$$
$$q_1 = 3,2 \cdot 10^{-6} m/s \rightarrow Q_1 = q_1 \cdot \Delta y = 1,6 \cdot 10^{-5}\, m^3/s$$

2.

$$q_2 = -K_A \cdot \frac{h_4 - h_2}{\Delta x}$$
$$q_2 = 2 \cdot 10^{-5} m/s \rightarrow Q_2 = q_2 \cdot \Delta y = 10^{-4}\, m^3/s$$

6.

$$q_6 = -K_{5,6} \cdot \frac{h_6 - h_5}{\Delta x}$$
$$q_6 = 2,313 \cdot 10^{-5} m/s \rightarrow Q_6 = q_6 \cdot \Delta y = 1,16 \cdot 10^{-4}\, m^3/s$$

- Provjera ravnoteže cijelog sustava

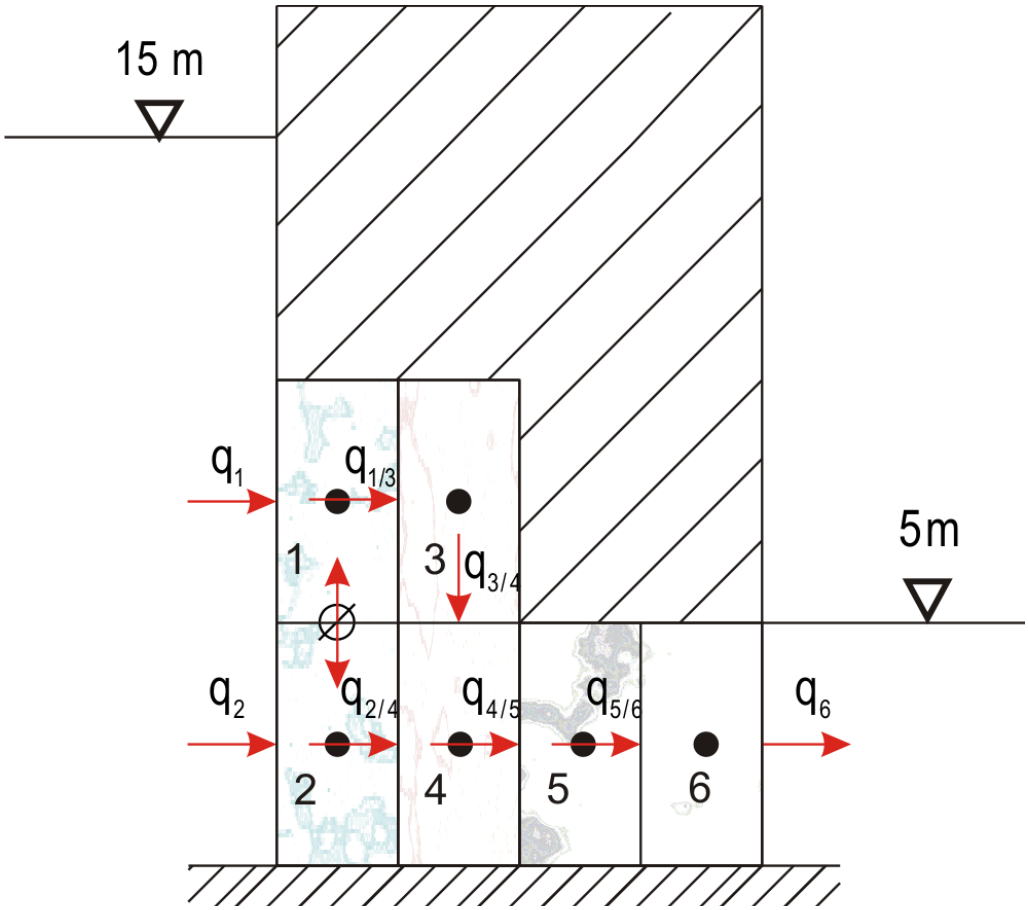
$$Q_1 + Q_2 = Q_6$$
$$1,6 \cdot 10^{-5} + 10^{-4} = 1,16 \cdot 10^{-4}$$
$$1,16 \cdot 10^{-4} = 1,16 \cdot 10^{-4}$$

- Proračun stvarnih brzina na rubovima blokova:

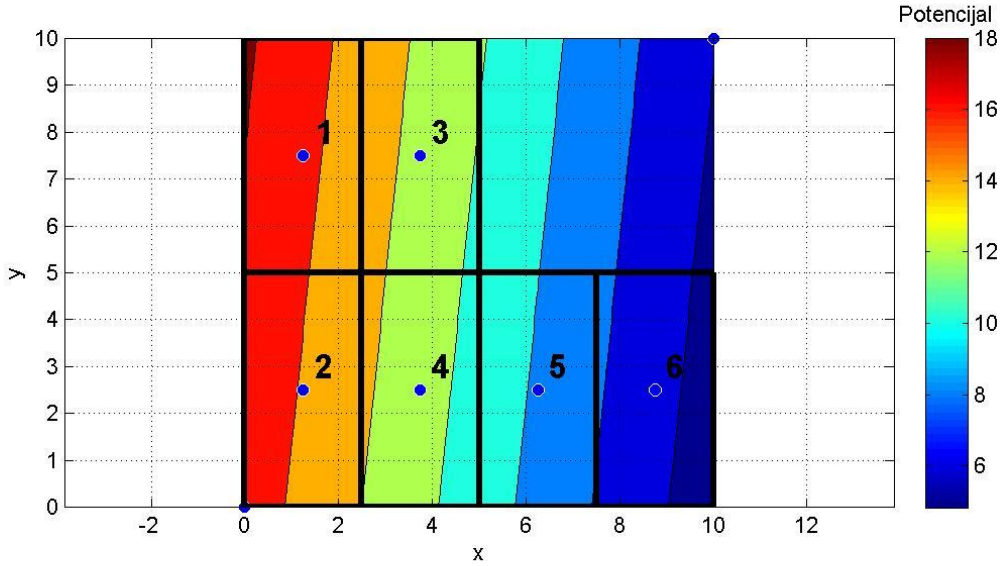
općenito:

$$q_{\text{stvarno,i/j}} = \frac{q_{Darcy}}{n} = -K_{i/j} \cdot \frac{h_j - h_i}{\Delta x \cdot n}\left(m/s\right)$$

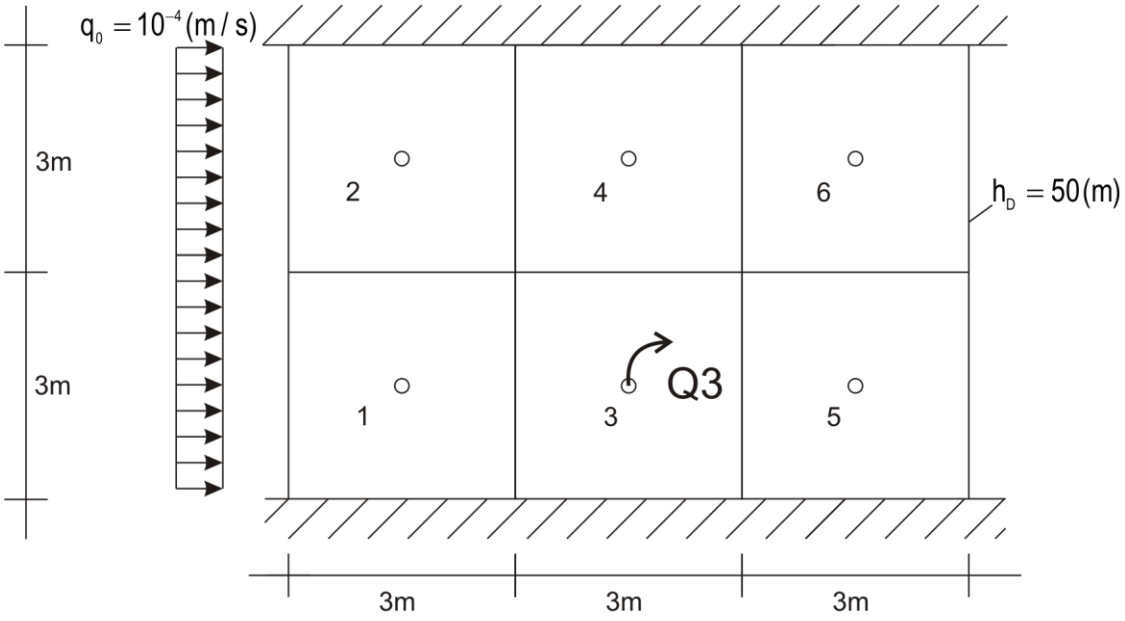
$$q_{01}^{\text{stvarna}} = q_1 / n = 1,6 \cdot 10^{-5} m/s$$
$$q_{02}^{\text{stvarna}} = q_2 / n = 10^{-4} m/s$$
$$q_{1/3}^{\text{stvarna}} = -K_A \cdot \frac{h_3 - h_1}{\Delta x \cdot n} = 1,6 \cdot 10^{-5} m/s$$
$$q_{3/4}^{\text{stvarna}} = -K_{3,4} \cdot \frac{h_4 - h_3}{\Delta y \cdot n} = 3,15 \cdot 10^{-5} m/s$$
$$q_{2/4}^{\text{stvarna}} = -K_A \cdot \frac{h_4 - h_2}{\Delta x \cdot n} = 10^{-4} m/s$$
$$q_{4/5}^{\text{stvarna}} = -K_B \cdot \frac{h_5 - h_4}{\Delta x \cdot n} = 1,16 \cdot 10^{-4} m/s$$
$$q_{5/6}^{\text{stvarna}} = -K_{5,6} \cdot \frac{h_6 - h_5}{\Delta x \cdot n} = 1,155 \cdot 10^{-4} m/s$$



Prikaz fitovanog polja potencijala:



2.PRIMJER:



1.varijanta $\begin{cases} q_0 = 10^{-4} m/s \\ Q_3 = -10\, m^3/dan \end{cases}$ 2.varijanta $\begin{cases} q_0 = 10^{-6} m/s \\ Q_3 = -0.01\ m^3/dan \end{cases}$

$K_1 = K_5 = K_6 = 10^{-6} m/s \rightarrow K_1$
$K_2 = K_3 = K_4 = 10^{-7} m/s \rightarrow K_2$
$h_D = 50\, m$
$n = 0{,}15$

$Q_3(1.\text{varijanta}) = -10/(24 \cdot 3600) = 1{,}16 \cdot 10^{-4} m^3/s$
$Q_3(1.\text{varijanta}) = -0{,}01/(24 \cdot 3600) = 1{,}16 \cdot 10^{-7} m^3/s$

Koeficijent propusnosti između blokova (na granicama) se dobiva kao harmonijska srednja vrijednost koeficijenata koji pripadaju graničnim blokovima:

$$K_{1/2} = \frac{2 \cdot K_1 \cdot K_2}{K_1 + K_2} = \frac{2 \cdot 10^{-6} \cdot 10^{-7}}{10^{-6} + 10^{-7}} = 1{,}82 \cdot 10^{-7} m/s \rightarrow K_A$$

- Rubni uvjeti:

Lijeva granica- dinamički(prirodni) rubni uvjet, zadan q!!! $Q_{01} = Q_{02} = \Delta y \cdot q_0$

Desna granica – kinematički(prisilni) rubni uvjet, zadani h!!! $h_5 = h_6 = 50\, m$

Donja i gornja granica – nepropusne!

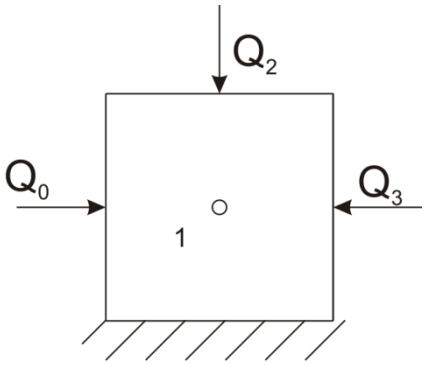
- <u>Ravnoteža protoka na blokovima u području:</u>

1.

$$\sum_{i=1}^{4} Q_i = 0$$

$$q_0 \cdot \Delta y + K_A \cdot \frac{h_2 - h_1}{\Delta y} \Delta x + K_A \cdot \frac{h_3 - h_1}{\Delta x} \Delta y = 0$$

$$-2K_A \cdot h_1 + K_A \cdot h_2 + K_A \cdot h_3 = -3q_0$$

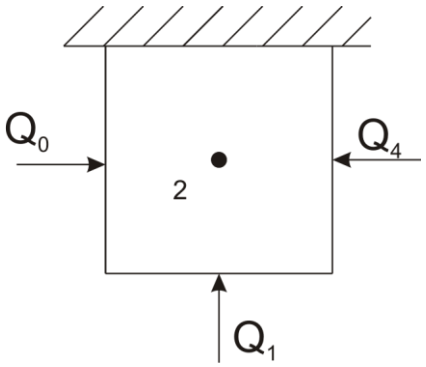


2.

$$\sum_{i=1}^{4} Q_i = 0$$

$$q_0 \cdot \Delta y + K_2 \cdot \frac{h_4 - h_2}{\Delta x} \Delta y + K_A \cdot \frac{h_1 - h_2}{\Delta y} \Delta x = 0$$

$$K_A \cdot h_1 - \left(K_2 + K_A\right) \cdot h_2 + K_2 \cdot h_4 = -3q_0$$

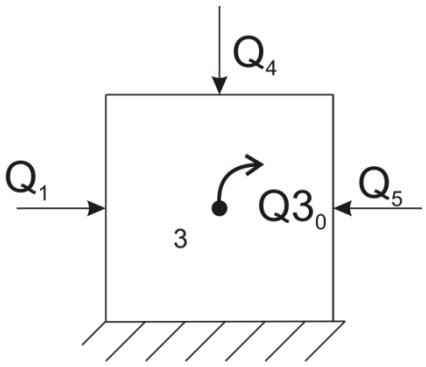


3.

$$\sum_{i=1}^{4} Q_i - Q3_0 = 0$$

$$K_A \cdot \frac{h_1 - h_3}{\Delta x} \Delta y + K_A \cdot \frac{h_5 - h_3}{\Delta x} \Delta y + K_2 \cdot \frac{h_4 - h_3}{\Delta y} \Delta x - Q3_0 = 0$$

$$K_A \cdot h_1 - \left(K_2 + 2K_A\right) \cdot h_3 + K_2 \cdot h_4 = -K_A \cdot h_5 + Q3_0$$

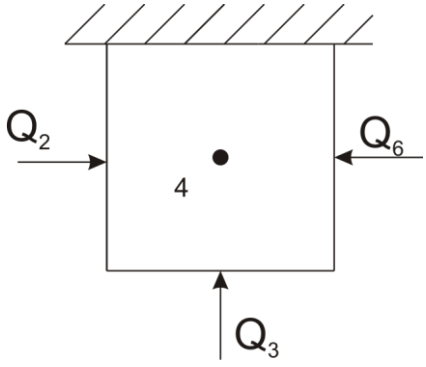


4.

$$\sum_{i=1}^{4} Q_i = 0$$

$$K_2 \cdot \frac{h_2 - h_4}{\Delta x} \Delta y + K_2 \cdot \frac{h_3 - h_4}{\Delta y} \Delta x + K_A \cdot \frac{h_6 - h_4}{\Delta x} \Delta y = 0$$

$$K_2 \cdot h_2 + K_2 \cdot h_3 - \left(2K_2 + K_A\right) \cdot h_4 = -K_A \cdot h_6$$



- Rješenje sustava:

1.varijanta

$$\begin{bmatrix} -3,64 & 1,82 & 1,82 & 0 \\ 1,82 & -2,82 & 0 & 1 \\ 1,82 & 0 & -4,64 & 1 \\ 0 & 1 & 1 & -3,82 \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} = \begin{bmatrix} -3000 \\ -3000 \\ 1069 \\ -91 \end{bmatrix}$$

2.varijanta

$$\begin{bmatrix} -3,64 & 1,82 & 1,82 & 0 \\ 1,82 & -2,82 & 0 & 1 \\ 1,82 & 0 & -4,64 & 1 \\ 0 & 1 & 1 & -3,82 \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} = \begin{bmatrix} -30 \\ -30 \\ -89,84 \\ -91 \end{bmatrix}$$

  - Napomena: obe strane sustava su podijeljene sa 10^{-7} radi jednostavnijeg proračuna
  - Sustav riješen u excelu/derive-u/matlabu...
  -
  - 1.varijanta: $\begin{cases} h_1 = 3388,2m \\ h_2 = 3735,1m \\ h_3 = 1393,1m \\ h_4 = 1366,3m \end{cases}$ 2.varijanta: $\begin{cases} h_1 = 86,7m \\ h_2 = 89,7m \\ h_3 = 67,4m \\ h_4 = 65m \end{cases}$

- Izračun protoka u blokovima sa zadanim prisilnim rubnim uvjetom:

5.

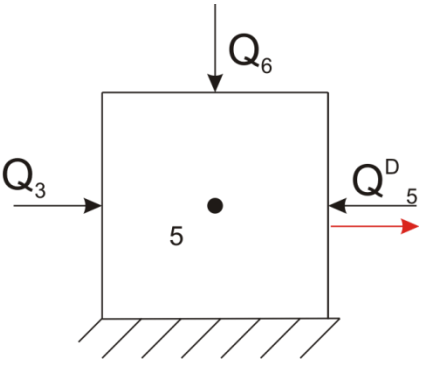
$$\sum_{i=1}^{4} Q_i = 0$$

$$K_A \cdot \frac{h_3 - h_5}{\Delta x} \Delta y + K_1 \cdot \frac{h_6 - h_5}{\Delta y} \Delta x + Q_5^D = 0$$

$$Q_5^D = -K_A \cdot \frac{h_3 - h_5}{\Delta x} \Delta y$$

$$Q_5^D(1.\text{varijanta}) = -2,4 \cdot 10^{-4} m^3/s *$$

$$Q_5^D(2.\text{varijanta}) = -3,17 \cdot 10^{-6} m^3/s *$$



6.

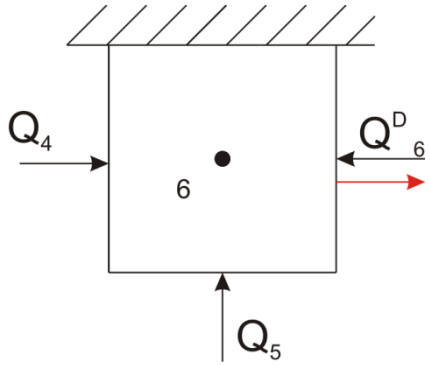
$$\sum_{i=1}^{4} Q_i = 0$$

$$K_A \cdot \frac{h_4 - h_6}{\Delta x}\Delta y + K_1 \cdot \frac{h_5 - h_6}{\Delta y}\Delta x + Q_5^D = 0$$

$$Q_6^D = -K_A \cdot \frac{h_4 - h_6}{\Delta x}\Delta y$$

$$Q_6^D(1.\text{varijanta}) = -2,4 \cdot 10^{-4}\, m^3/s *$$

$$Q_6^D(2.\text{varijanta}) = -2,73 \cdot 10^{-6}\, m^3/s *$$



*m^3/s jer se uzima i treća dimenzija Δz=1m

- <u>Provjera ravnoteže cijelog sustava:</u>

$$q_0 \cdot 2 \cdot \Delta y = Q_5^D + Q_6^D + Q3_0$$

$$1.\text{varijanta:} \begin{cases} 6 \cdot 10^{-4} = (2,4 + 2,4 + 1,16) \cdot 10^{-4} \\ 6 \cdot 10^{-4} \approx 5,96 \cdot 10^{-4} \end{cases}$$

$$2.\text{varijanta:} \begin{cases} 6 \cdot 10^{-6} = (3,17 + 2,73 + 0,016) \cdot 10^{-6} \\ 6 \cdot 10^{-6} \approx 6,016 \cdot 10^{-6} \end{cases}$$

- <u>Proračun stvarnih brzina na rubovima blokova:</u>

općenito:

$$q_{\text{stvarno,i/j}} = \frac{q_{Darcy}}{n} = -K_{i/j} \cdot \frac{h_j - h_i}{\Delta x \cdot n}(m/s)$$

1.varijanta:

$$q_{01}^{\text{stvarna}} = q_{02}^{\text{stvarna}} = q_0 / n = 4,44 \cdot 10^{-3}\, m/s$$

$$q_{1/2}^{\text{stvarna}} = -K_A \cdot \frac{h_2 - h_1}{\Delta y \cdot n} = -1,4 \cdot 10^{-4}\, m/s$$

$$q_{1/3}^{\text{stvarna}} = -K_A \cdot \frac{h_3 - h_1}{\Delta x \cdot n} = 8,08 \cdot 10^{-4}\, m/s$$

$$q_{2/4}^{\text{stvarna}} = -K_2 \cdot \frac{h_4 - h_2}{\Delta x \cdot n} = 4,5 \cdot 10^{-4}\, m/s$$

$$q_{4/3}^{\text{stvarna}} = -K_2 \cdot \frac{h_3 - h_4}{\Delta y \cdot n} = -5,96 \cdot 10^{-6}\, m/s$$

$$q_{4/6}^{\text{stvarna}} = -K_A \cdot \frac{h_6 - h_4}{\Delta x \cdot n} = 5,33 \cdot 10^{-4}\, m/s$$

$$q_{3/5}^{\text{stvarna}} = -K_A \cdot \frac{h_5 - h_3}{\Delta x \cdot n} = 5,43 \cdot 10^{-4}\, m/s$$

2.varijanta:

$$q_{01}^{\text{stvarna}} = q_{02} = q_0 / n = 6,67 \cdot 10^{-6}\, m/s$$

$$q_{1/2}^{\text{stvarna}} = -K_A \cdot \frac{h_2 - h_1}{\Delta y \cdot n} = -1,21 \cdot 10^{-6}\, m/s$$

$$q_{1/3}^{\text{stvarna}} = -K_A \cdot \frac{h_3 - h_1}{\Delta x \cdot n} = 7,81 \cdot 10^{-6}\, m/s$$

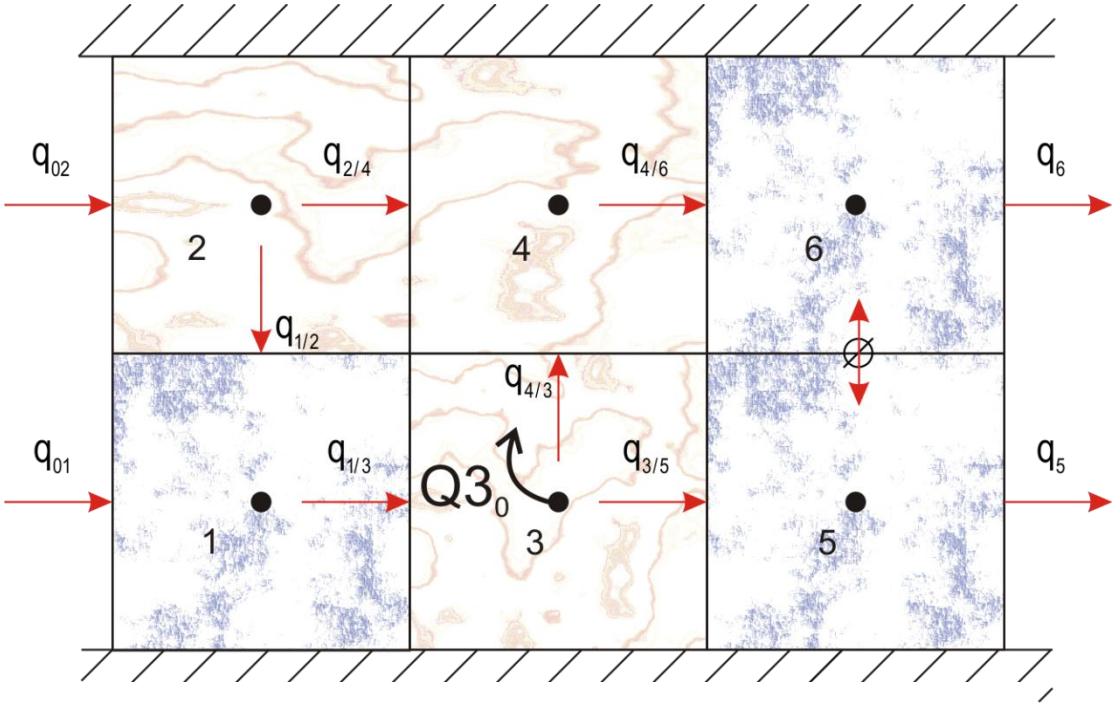
$$q_{2/4}^{\text{stvarna}} = -K_2 \cdot \frac{h_4 - h_2}{\Delta x \cdot n} = 5,5 \cdot 10^{-6}\, m/s$$

$$q_{4/3}^{\text{stvarna}} = -K_2 \cdot \frac{h_3 - h_4}{\Delta y \cdot n} = -5,33 \cdot 10^{-7}\, m/s$$

$$q_{4/6}^{\text{stvarna}} = -K_A \cdot \frac{h_6 - h_4}{\Delta x \cdot n} = 6,07 \cdot 10^{-6}\, m/s$$

$$q_{3/5}^{\text{stvarna}} = -K_A \cdot \frac{h_5 - h_3}{\Delta x \cdot n} = 7,04 \cdot 10^{-6}\, m/s$$

shematski prikaz izračunatih brzina na rubovima elementa:



Prikaz fitovanog polja potencijala za obe varijante: