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FINITE-DISCRETE NUMERICAL MODELLING OF REINFORCED CONCRETE STRUCTURES

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Abstract. *Crack opening is one of the dominant causes of nonlinearity in brittle and quasi-brittle structures which leads to localized failure and stands out as a serious challenge in numerical modelling. Therefore, a realistic modelling of crack initiation and propagation is one of the key factors that affect the reliability of the model for analysing the structures, especially those subjected to earthquakes. The largest number of structures in the world's most populated seismic areas is made of materials such as reinforced concrete, stone and masonry, in which localized failure dominantly influences the structural collapse. Sophisticated numerical models based on time dependent and incremental dynamic analysis can play an important role in simulating the behaviour of such structures. In this work 2D and 3D numerical modelling for analysis of reinforced concrete structures under static and dynamic load is presented. The model uses the finite-discrete element method; thus taking into account the discontinuous nature of the concrete at the failure stages. This approach considers the structure as a deformable continuum until the opening of the cracks. The transition from the continuum to the discontinuum is a result of the crack initiations which are modelled through the contact elements implemented between finite elements. The interaction between discrete elements is considered through the contact interaction algorithm based on the principle of potential contact forces which are computed using penalty function method and the Coulomb friction law. The non-linear behaviour of the concrete in tension and shear takes into account strain-hardening, strain-softening and cyclic behaviour during dynamic loading. The model for reinforcing bars is based on an approximation of the experimental curves for the bar strain in the crack. The developed numerical model includes interaction effects between reinforcement and concrete as well as cyclic behaviour of concrete and steel during dynamic loading. Thus, it is possible to describe the crack initiation and propagation, collapse mechanism, energy dissipation by non-linear effects, inertial effects and contact interaction which are very important in the analysis of RC structures, especially those subjected to seismic load.*

1 INTRODUCTION

The development of the numerical models for simulation of the response of reinforced concrete structures under static and dynamic load, taking into consideration non-linear behavior of the concrete and reinforcement, makes possible the results of high accuracy. One of the main causes of concrete non-linear behavior is cracking. A reliable model for simulation of the opening and closing the cracks is especially important in the structures under dynamic load. The most of the models described in literature is based on finite element method where the cracking is described with smeared crack models or with embedded models where the cracks are modelled as discontinuity within the elements by enriching interpolation function. In contrast to the finite elements, in discrete models cracks are simulated as discontinuities of displacement between two elements.

A numerical model for analysis of reinforced concrete structures under static and cyclic load developed in this work is based on combined finite-discrete element method [1]. Transition from continua to discontinua in this method occurs through fracture and fragmentation processes. A typical combined finite discrete element method based simulation may start with a few discrete elements and finished with very large number of discrete elements. Fracture occurs through alteration, damage, yielding or failure of micro structural elements of the material.

There has been a number of fracture models proposed in the context of both discrete element methods and combined finite discrete element method. Some of the models are based on a global approach applied to each individual body, while others used a local smeared crack approach or local single-crack approach. In this work a model for plane and spatial crack initiation and crack propagation in concrete is used [2]. The model combines standard finite element formulation for the hardening part of the constitutive law with the single-crack model for the softening part of stress-strain curve. Finite elements are used to model behavior of the material up to the ultimate tensile strength while a discrete crack model is used for modelling of the crack opening and separation along edges of finite elements.

In this work cracking of the concrete in plane and space are enabled by a combined single and smeared crack model. An embedded model of reinforcing bars [3, 4] is implemented in finite-discrete element code [1]. The concrete and reinforcing bars are analyzed separately, but they are connected by the relation between the size of the concrete crack and strain of the reinforcing bar [5, 6]. Cyclic behavior of the steel during the cyclic load is modelled with improved Kato's model [7].

2 MODELLING OF THE REINFORCED CONCRETE STRUCTURE

The concrete structure in plane is discretized on triangular finite elements, while in the space it is discretized by tetrahedron finite elements. Reinforcing bars are modelled with linear one-dimensional elements which can be placed in arbitrary position inside the concrete finite elements. The model of the reinforced concrete structure with the embedded reinforcing bars in 2D and 3D concrete structure is presented in Fig. 1.

The structure behaves as continuum until opening of the crack. The deformation of the finite triangular element influence to the deformation of the reinforcing bars. When the crack in concrete appears, joint element in concrete as well as joint element in reinforcing bars is occurred. The concrete and reinforcing bars are analyzed separately, but they are connected by the relationship between the size of the concrete crack and strain of the reinforcing bar.

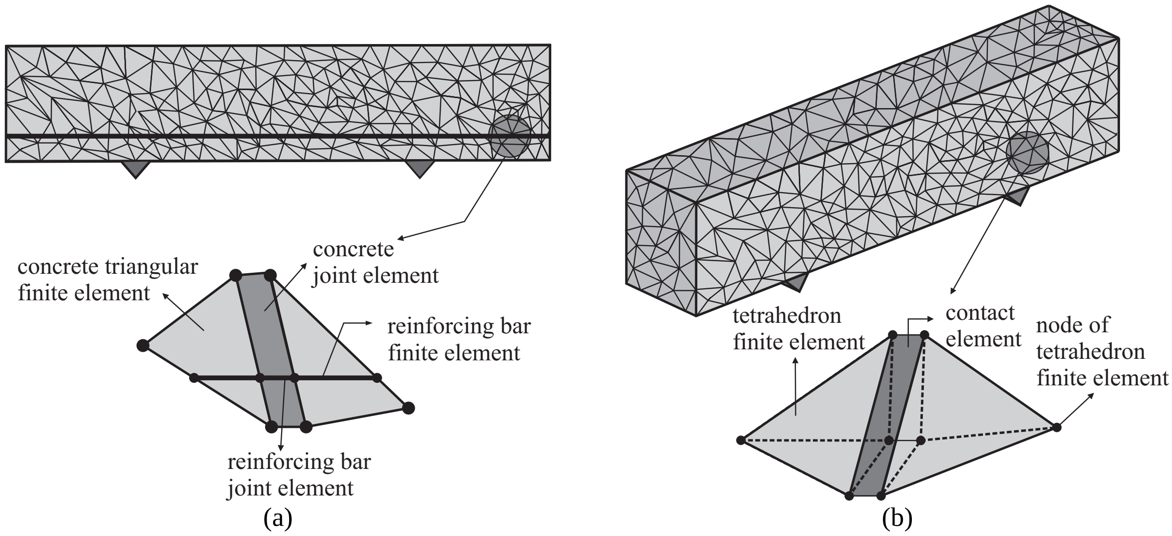


Figure 1: Discretization of reinforced concrete structure: (a) 2D model; (b) 3D model.

3 NON-LINEAR MATERIAL MODEL IN JOINT ELEMENTS

3.1 Concrete model in joint element

The concrete model is based on crack initiation and crack propagation in tension and shear [2]. It is developed on the basis of experimental stress-strain curves for concrete in tension.

The area under the stress-strain curve consists of two parts (Fig. 2), part for modelling of the concrete behavior up to the crack opening [2] and part which represents strain softening after the tensile strength is exceeded [8]. The assumption of the discrete crack model is that the cracks coincide with the finite element edges. The total number of nodes for each of the finite element meshes is doubled and the continuity between elements is realized through the penalty method [9]. Separation of the edges induces a bonding stress, which is a function of the size separation δ (Fig. 2).

The area under the stress-displacement curve represents the energy release rate $G_f = 2\gamma$, where γ is the surface energy, i.e. the energy needed to extend the crack surface by a unit area.

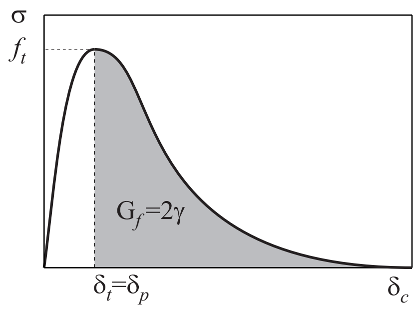


Figure 2: Strain softening stress-strain and stress-displacement curves.

No separation of the adjacent elements occurs before the tensile strength is reached, i.e. the edges of two adjacent elements are held together by normal and shear springs (Fig. 3). Procedure of the separation of the elements and complete relationship for the normal and shear bonding stress are given in [2].

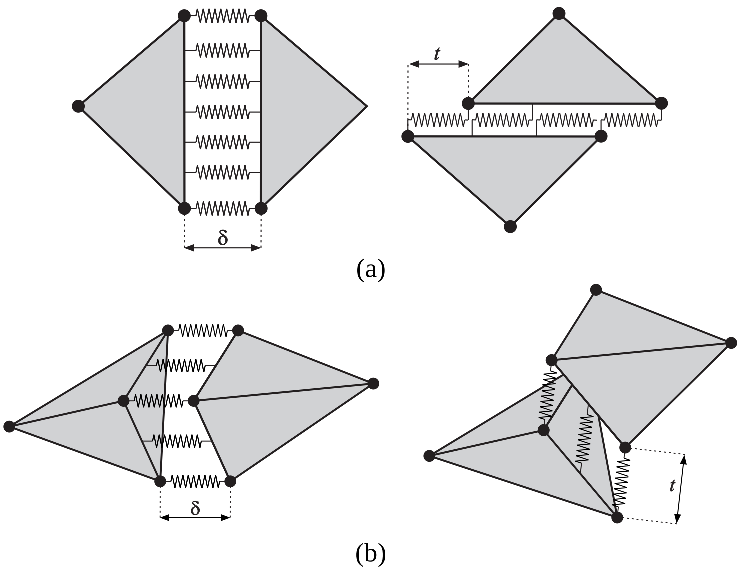


Figure 3: Normal and shear springs between the finite elements: (a) 2D model, (b) 3D model.

3.2 Steel material model in reinforcement joint element

In this work a model of the relationship between the concrete crack size and strain of the reinforcing bar developed by Shima [5] and Shin [6] is applied. The model is based on experimental strain-slip curves and represents well approximation of the behavior of reinforcing bar with the expressed plastic strain caused by cyclic loading.

The steel strain-slip relation before the yielding of reinforcing bar is given by expressions:

$$s = \varepsilon_s (6 + 3500 \varepsilon_s) \tag{1}$$

$$s = \left(\frac{S}{D}\right) \cdot K_{fc}, \quad K_{fc} = \left(\frac{f_c'}{20}\right)^{2/3} \tag{2}$$

where $s = s(\varepsilon_s)$ is normalized steel slip, D is bar diameter and f_c' is concrete strength.

Normalized slip in the post-yield range is given by expression:

$$s = s_{pl} + s_e \tag{3}$$

where s_e is slip in the elastic region and s_{pl} is slip in the yield region. A strain distribution along the reinforcing bar in the post-yield region is shown in Fig. 4, where ε_{se} a strain at the yield boundary point on the elastic region is and ε_{sp} is a strain at the yield boundary point on the yield region.

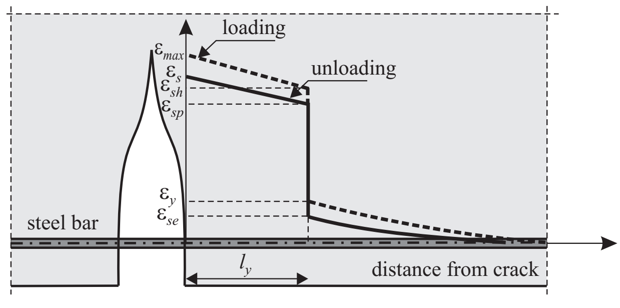


Figure 4: Strain distribution along the reinforcing bar in the post-yield range.

Normalized plastic steel slip in the yield region is given by an expression [5, 6]:

$$s_{pl} = \frac{(1+\beta)\varepsilon_s + \varepsilon_{sh} - \beta\varepsilon_{\max}}{\varepsilon_{\max} + \varepsilon_{sh}}(s_{\max} - s_y^*) \tag{4}$$

where $\beta = \sigma_{\max} / \sigma_y$ represents the gradient of the line shown in Figure 4, $\sigma_{\max}$ is the maximum stress in reinforcing bar under tensile loads, $s_{\max}$ is a function of $\varepsilon_{\max}$ and $s_y^* = \varepsilon_s(2+3500\varepsilon_s)$.

Non-linear material model for steel is based on experimental stress-strain curve and it is shown in Fig. 5a. Cyclic behavior of the steel during the cyclic load (Fig. 5b) is modelled with improved Kato's model [7].

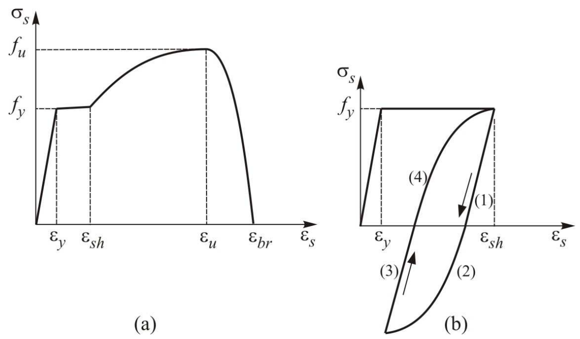


Figure 5: Material model for steel: (a) Stress-strain relation for steel, (b) Cyclic behavior of steel.

4 EXAMPLES

4.1 Sensitivity of the 3D model to the mesh refinement and penalty parameter

This example was chosen to analyse the sensitivity of the 3D numerical model to the mesh refinement and penalty parameter. Analysis was performed on concrete cantilever exposed to monotonic increasing tension load. Geometry and cross section of structure are shown in Fig. 6. Modulus of elasticity of concrete is E=30 GPa.

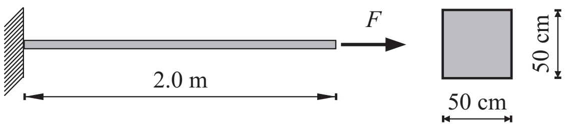
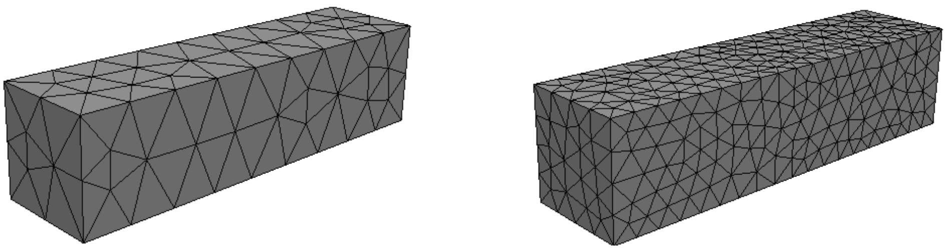


Figure 6: Geometry and load of structure.

The analyses were performed without and with contact elements using two different finite elements meshes refinement (mesh A, mesh B) as shown in Fig. 7.



(a) (b)

Figure 7: Discretization of structure: (a) mesh A, (a) mesh B.

Mesh A was comprised of 320, while mesh B was comprised of 3102 finite elements. In analyses performed with contact elements, penalty term p was 20 times higher than E_c.

Fig. 8 shows the comparison of the analytical and numerical results obtained by the presented model. It can be seen that for homogeneous state of stress the numerical 3D model is not sensitive to the mesh refinement regardless of the presence of the contact elements. It can be also seen when analyses were performed without contact elements, the numerical results correspond to analytical solution while the introduction of contact elements leads to relative error of numerical results in comparison to analytical solution.

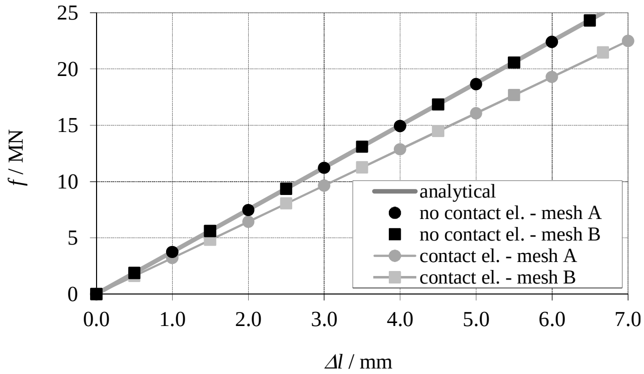


Figure 8: Force-displacement relations.

The sensitivity of the numerical model in linear elastic stage to the penalty parameter *p* was performed on concrete cantilever with finite element mesh shown in Fig. 7(a). The analyses were performed for three values of penalty parameter *p*. The comparison of the analytical and numerical results obtained by the presented model is shown in Fig. 9.

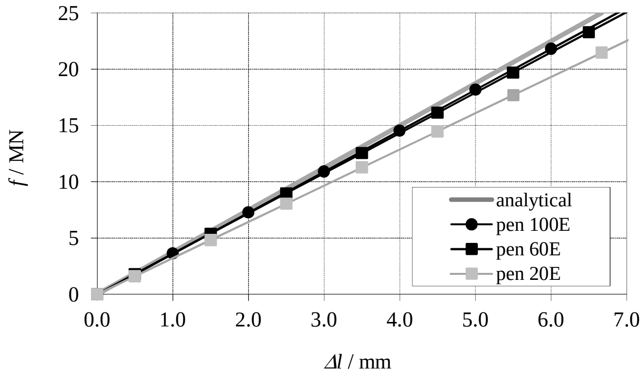


Figure 9: Force-displacement relations.

The influence of the penalty parameter *p* to the relative error is shown in table 1. It can be seen that for values of penalty term higher than 100*E* relative error is less than 2%. Similar conclusions have been obtained for 2D analysis [4].

Penalty parameter	Relative error
20Ec	14.1
60Ec	4.4
100Ec	1.9

Table 1: Influence of penalty parameter to the solution error.

4.2 Seismic analysis of coupled RC wall structure

The application of the presented finite-discrete numerical model of reinforced concrete structures was shown on five-storey reinforced concrete building with coupled wall system. The vertical load of the building consists of the own weight of structural elements, an additional dead load of 2.5 kN/m^2 and impose load of 4.0 kN/m^2 at floor slabs.

Material characteristics of concrete and steel used in a numerical analysis are shown in Table 1.

Concrete					
E_c / MPa	ν	f_t / MPa	f_c / MPa	G_f / N/m	ρ / kN/m^3
32800	0.2	3.80	38	150	25
Steel					
E_s / MPa	f_y / MPa	f_u / MPa	ε_{sh}	ε_u	
210000	500	600	0.02	0.1	

Table 2: Material characteristics of the RC wall.

The application of the presented model in incremental dynamic analysis [10] of RC structure was performed for the boundary coupled RC wall with geometry, reinforcement and discretisation shown in Fig. 10.

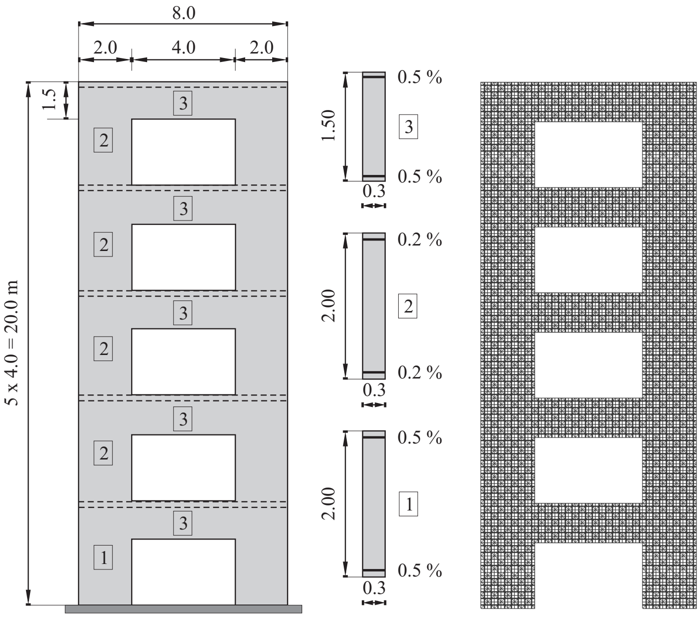


Figure 10: Coupled RC wall: (a) Geometry and reinforcement of the cross-sections, (b) Discretisation.

Seismic loading is represented by a time function of the horizontal ground acceleration recorded during real earthquakes. The earthquake accelerograms were taken from the European Strong-Motion Database [11]. The selected earthquakes are Petrovac (Montenegro)-1979, South Iceland (Iceland)-2000 and Campano Lucano (Italy)-1980, as shown in Fig. 11.

The observed five-storey RC wall structure is exposed to the horizontal ground acceleration of selected three earthquakes. The amplitudes were gradually increased until the collapse of the structure. The relation between the ratio of peak ground acceleration and gravity constant a and the ratio of maximum top displacement u and the height of structure H (u/H) obtained by incremental dynamic analysis is shown in Fig. 12.

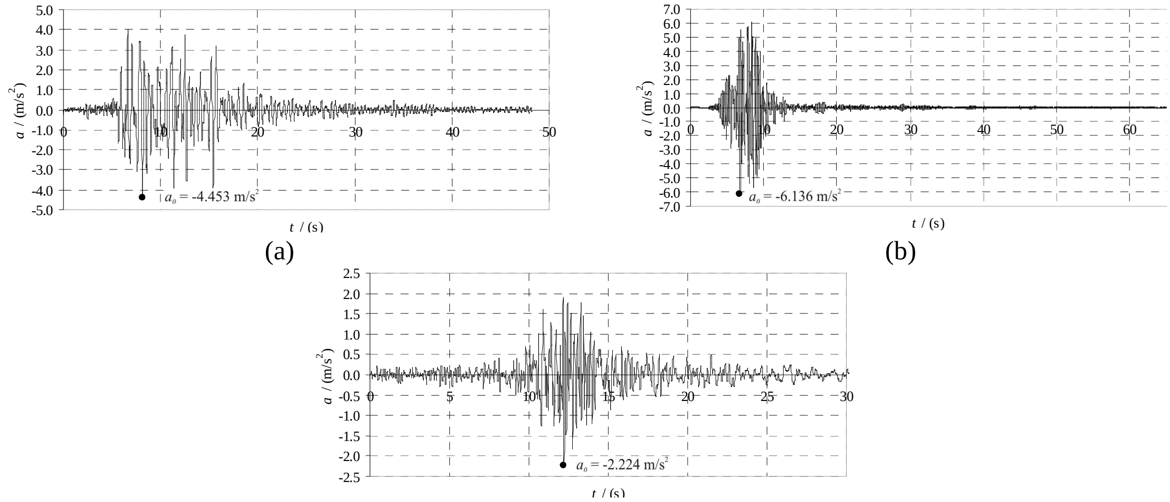


Figure 11: Accelerograms of three real earthquakes: (a) Petrovac (1979.), (b) South Iceland (2000.), (c) Campano Lucano (1980.)

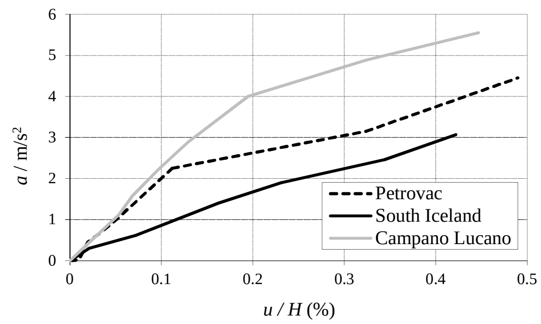


Figure 12: The ratio between the maximum roof displacement u and the height of structure H for coupled RC wall structure in dependence to the peak ground acceleration.

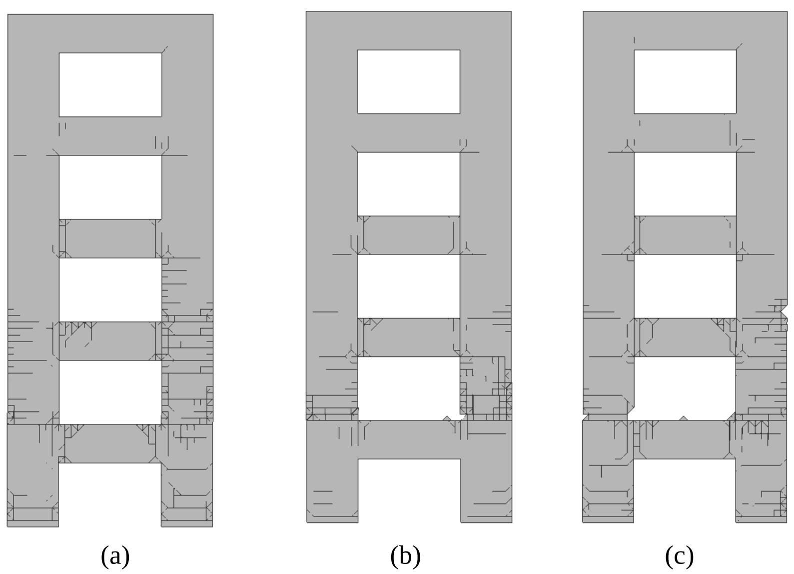


Figure 13: Crack pattern for the coupled RC wall structure exposed to the failure peak ground acceleration of earthquake: (a) Petrovac, (b) South Iceland, (c) Campano Lucano.

Finite-discrete numerical model based on discrete cracks presented in this paper can simulate opening and closing the cracks as well as realistic crack patterns for different intensity of accelerations. Crack pattern for the coupled RC wall structure exposed to the failure peak ground acceleration of three earthquakes is shown in Figure 13.

This example highlights the ability of the developed model based on discrete cracks for simulation of opening and closing the cracks during the earthquake loading as well as obtaining of the ultimate acceleration, which is very important in the estimation of the safety of RC structures under the seismic loading.

5 CONCLUSIONS

- This paper presents numerical model for analysis of reinforced concrete structures under static and dynamic load.
- Simple numerical model of reinforcement which is implemented in combined finite discrete element code is presented. Model based on approximation of the experimental curves for behaviour of the concrete and steel in crack, is primarily aimed for monitoring of the behaviour of reinforced concrete structure subjected to static and dynamic load up to failure.
- Cracking of the structure is enabled by discrete model of cracks where the cyclic behaviour of concrete and steel was successfully implemented.
- Interaction between reinforcement and concrete was not directly considered, but it was taken into consideration by defining the average spacing between the cracks.
- The model provides quite realistic description of cracking in the reinforced concrete structures under the seismic loading until the collapse.

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