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Numerical Analysis of Masonry Structures by Finite Discrete Element Model

Željana Nikolić[*], **Hrvoje Smoljanović**[*], **Nikolina Živaljić**[*]

[*]*University of Split, Faculty of Civil Engineering, Architecture and Geodesy, Matice Hrvatske 15, Split, Croatia*
E-mails: {hrvoje.smoljanovic, zeljana.nikolic, nikolina.zivaljic}@gradst.hr

Abstract. This paper presents numerical model based on the finite discrete element method for the analysis and prediction of the collapse of masonry structures with mortar joints and dry stone masonry structures. The model consists of a numerical model in a finite element, contact interaction algorithm which simulates the interaction between stone blocks in dry joint and numerical model in an interface element which simulate the behavior of the mortar joints and unit-mortar interface.

The verification of the model was performed on examples by comparing it with the results obtained with the known numerical and experimental results from literature.

1 Introduction

Building construction using dry stone walls or clay bricks which are held together by mortar is one of the oldest building techniques which are still in use today. Masonry has a long worldwide tradition of use in construction due to its simplicity. In spite of the simplicity that is manifested during the construction of masonry structures, understanding and describing mechanical behavior of those structures represents a true challenge due to the nature of masonry structure which, due to the presence of joints among units, shows a complex and particular nonlinear behavior.

Most of the models for simulation of the behavior of masonry structures are based on the finite element method. The analysis of masonry by finite element method is usually based on the modelling of the material as a fictitious homogeneous orthotropic continuum. Strong discontinuities between different units of the masonry can be simulated by joint interface elements.

Another approach for modelling of the cracking in these materials is discrete element method. The behavior of masonry is based on the idealization of the material as a discontinuum where joints are modelled as contact surfaces between different units. In recent times an increasing number of models attempted to combine the advantages of finite and discrete element methods. These methods are designed to handle contact situations in which transition from continua to discontinua can appear.

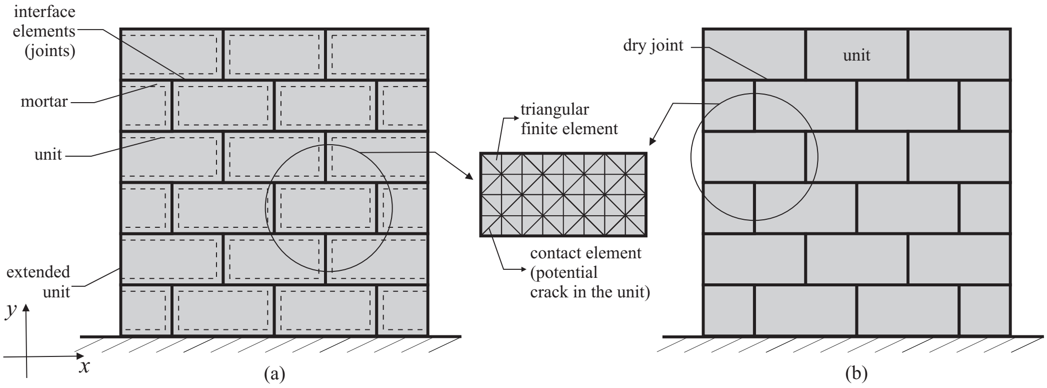
The finite-discrete element method (FEM/DEM), subject of this paper, was firstly developed for the simulation of fracturing problems considering deformable particles that may split and separate during the analysis. Within the framework of this method the discrete elements are discretized by constant strain triangular finite elements. Material non-linearity including fracture and fragmentation of discrete elements is considered through contact elements [1] which are implemented within a finite element mesh. The interaction between discrete elements is considered through the contact interaction algorithm based on the principle of potential contact forces [2] and the Coulomb-type law for friction [3]. The method uses an explicit numerical integration of the equation of motion.

This paper presents numerical model based on FEM/DEM which can capture the main features related to the behavior of dry stone masonry structure and masonry structures with mortar joints. The model consists of a numerical model in a finite element which simulates the behavior of units, contact interaction algorithm which simulates the interaction between stone blocks in dry joint and material model in an interface element which simulate the behavior of the mortar joints and unit-mortar interface [4-6].

The application of the model was performed on examples by comparing it with the results obtained with the known numerical and experimental results from literature.

2 Numerical Model

In this numerical model masonry structure is considered as assemblage of discrete elements as it is shown in figure 1.



(a) (b)

Figure1: Discretization of masonry structure: (a) mortar masonry; (b) dry stone masonry

Each discrete element, which can split and separate during the analysis, is discretized by constant triangular finite elements. Material non-linearity, fracture and fragmentation are considered through the contact elements which are implemented within the finite element mesh of each block.

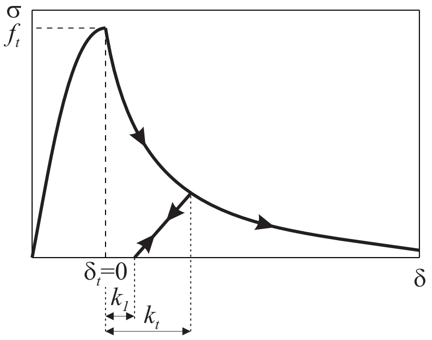


Figure2: Material model in contact element

Material model in contact element is shown in figure 2 where the ratio of k_1/k_t necessary for describing cyclic loading is adopted in amount of 0.73 [7].

Contact interaction between units in dry stone masonry structures is considered through the contact interaction algorithm based on the principle of potential contact forces [2] which include the Coulomb-type law for friction [3], while the mortar joints and unit-mortar interface are modelled by interface elements [5, 6].

2.1 Numerical model in finite element

Due to the geometrical arrangement of units and mortar, the constitutive behaviour of masonry is highly anisotropic, even if the properties of these constituents are isotropic. Oriented voids in perforated unit elements also contribute to anisotropic behaviour of masonry structures whose material axis, in most cases, coincides with horizontal and vertical directions.

Unlike concrete structures in which the collapse usually appears due to cracking of material in tension or shear, in masonry structures, besides these two failure mechanism, the material often crushes in compression. This failure mechanism may be especially important in masonry structures built with perforated bricks.

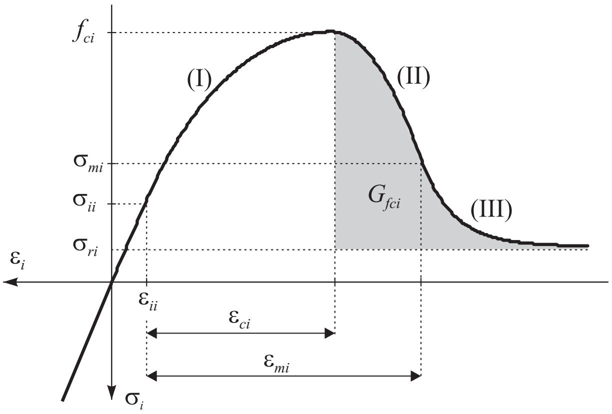


Figure3: Hardening/softening law for compression

In this paper, the orthotropic constitutive material behaviour with hardening/softening law for compression is considered in finite elements [5-6]. Elliptical hardening followed by parabolic/exponential softening law in compression defined by four inelastic parameters (σ_i, ε_i), (f_c, ε_c), (σ_m, ε_m), (σ_r, ε_r), shown in figure 3, is considered for both material axes, with different compressive fracture energies and different compressive strengths. A redefined compressive fracture G_{fci} corresponds only to the local contribution of σ_i-ε_i diagram, where subscript *i* refers to the material axes which correspond to global axes x and y.

Cyclic behaviour is adopted as t is shown in figure 4, where the ratio of k_1/k_c is adopted in amount of 0.935.

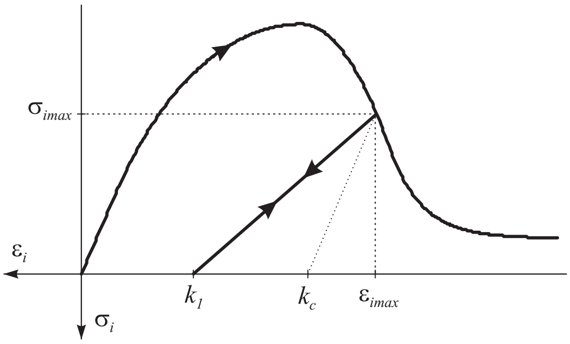


Figure4: Cyclic behavior in compression

2.2 Numerical model in interface element

The numerical model in the interface element simulates the behavior of the mortar joints and unit-mortar interface taking into account cracking of joints in tension and sliding along the bed or head joints in shear. Since the contact elements describe discontinuity in a displacement field after reaching ultimate tension or shear strength, their behavior is described in terms of the relation between stress and relative displacement based on a single and smeared crack model.

Since the experimental research conducted by Plujm [8] has shown that increasing the pre-compression stresses level in the contact between the block and the mortar causes increasing fracture energy in shear, in this numerical model, this phenomenon is taken into account according to the following relation:

$$G_f^{II} = G_{f0}^{II} - 106.31\sigma \quad \text{(N/m)} \tag{1}$$

where G^{II}_{f0} is the value of the fracture energy in shear in the case when the normal pre-compression stress is equal to zero, while σ is pre-compression stress in MPa.

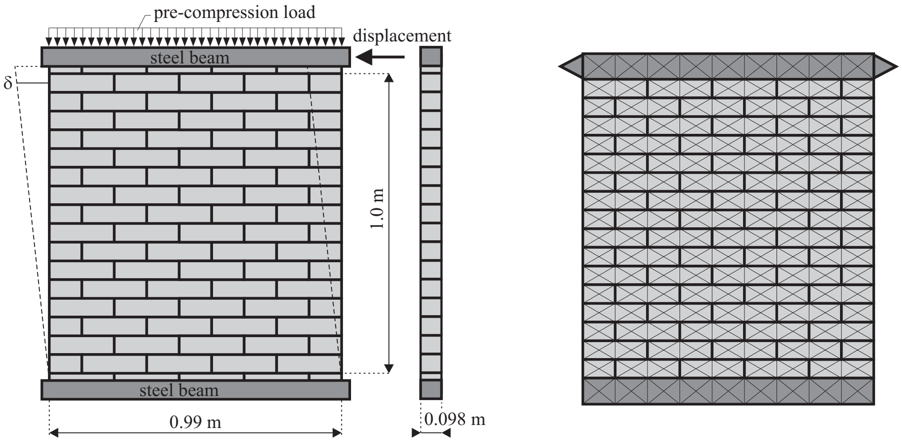
3 Numerical examples

3.1 Masonry shear walls exposed to a monotonically increasing load

In this example, the ability of the numerical model to reproduce the main features that characterize the behaviour of masonry shear walls under monotonic increasing loading is performed by comparing experimental and numerical results. The numerical analyses were performed on several shear walls which Raijmakers and Vermeltfoort [9] analysed within the CUR project. The walls analyzed in this example, correspond to the samples J4D, J5D and J7D.

Geometrical characteristics of the walls are shown in figure 4. The walls have a width/height ratio of one with dimensions 990 x 1000 mm2 and consisted of 18 rows of blocks, of which only 16 were active, while the remaining two were clamped in the steel beams (figure 4a). The walls were made of solid clay bricks with dimensions 210 x 98 x 50 mm and 10 mm thick mortar. Discretization of the structure with finite element mesh used in the numerical analysis is shown in figure 4b.

Different vertical pre-compression stresses were applied to the walls: 0.3 MPa for walls J4D and J5D and 2.12 MPa for wall J7D. After applying the vertical stress, the walls were exposed to the horizontal load, which is achieved through the controlled displacement of steel beam at the top of the walls. The loading rate was twenty micro millimetre per second. During the application of the horizontal displacement, vertical movements of the top and bottom of the steel beams were prevented.



(a) (b)

Figure5: Masonry shear wall: (a) geometry; (b) finite element mesh

Mechanical characteristics of materials used in numerical analysis are based on data taken from the literature [9] and shown in table 1. When more than one value is given in the same column, it means that the different parameters are used for the walls with an initial vertical load of 0.3 and 2.12 MPa respectively.

Table 1: Mechanical characteristics of materials

Unit					
E_x (MPa)	E_y (MPa)	v_{xy}	$f_{cx} = f_{cy}$ (MPa)	f_t (MPa)	f_s (MPa)
7 520	3 960	0.09	10.5	2.0	2.8
G_f^I (N/m)	G_f^{II} (N/m)	G_{fcx} (N/m)	G_{fcy} (N/m)	$\mu_0 = \mu_r$	
80	500	20 000	15 000	1.0	
Interface element					
f_t (MPa)	f_s (MPa)	G_f^I (N/m)	G_f^{II} (N/m)	$\mu_0 = \mu_r$	
0.16	0.224	18	50	0.75	

In the performed analysis, the horizontal displacement of the top of the wall in dependence of the horizontal force was measured. In order to show influence of new non-linear material model in finite element to the accuracy of the results, numerical analyses were performed with non-linear material model in finite element presented in this paper and linear elastic numerical model in finite element.

The comparison of numerical results obtained in this paper with experimental results [9] and numerical results obtained by Lorenço [10] are shown in figure 5.

The numerical results of Lourenço were obtained by a numerical model based on the finite element method in which the units were discretized with continuum elements while the joints were discretized with interface elements. A composite interface model, which includes softening for tension, shear and compression, was based on the modern plasticity concept.

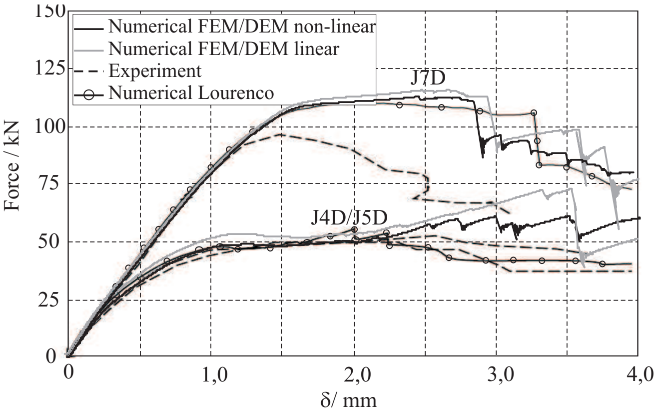


Figure6: Comparison of experimental and numerical displacements top of the wall

Good agreement can be observed between numerical results obtained in this paper by the non-linear model based on FEM/DEM with experimental and numerical results obtained by Lourenço [10] for walls J4D and J5D.

In the case of wall J7D, the numerical results obtained by non-linear FEM/DEM model do not show a significant difference compared to the numerical results obtained by Lourenço [10]. On the other hand, all the numerical results provide approximately 15% higher ultimate load for wall J7D compared to those obtained by the experiment. It is possible that the cause was some kind of local weakening that appeared in the experiment which was not taken into account as part of the mechanical characteristics of the materials shown in Tab. 3. It can be also seen that numerical results obtained with linear material model in finite element provide approximately 25% higher ultimate load for wall J4D/J5D compared to those obtained by non-linear numerical model. For wall J7D there is no significant difference between numerical results obtained by linear and non-linear model since the collapse appeared due to exceeding shear strength of wall.

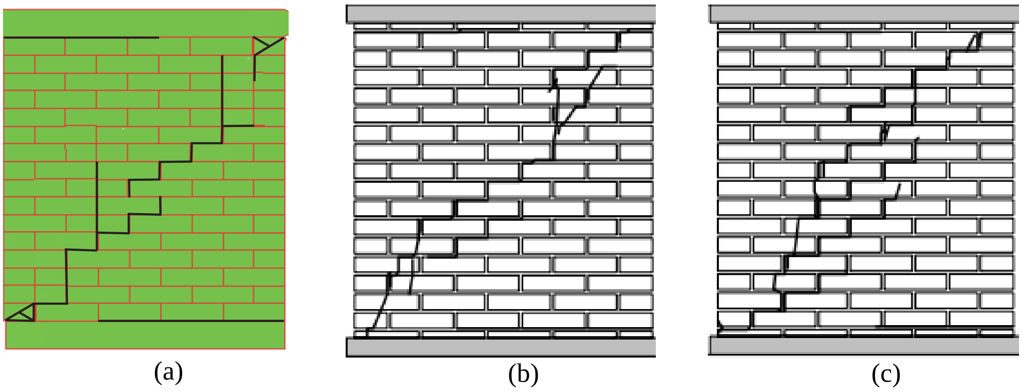

(a) (b) (c)

Figure7: Crack pattern in walls: (a) this work, (b) experiment [10] J4D, (c) experiment [10] J5D

Failure patterns, just before the complete breakdown, obtained from experiments (wall J4D/J5D) and numerical analyses where compared in figure 6. It can be seen that experimental and numerical crack patterns are similar. At the early loading stage the horizontal tensile cracks develop at the bottom and top of the wall, but diagonal stepped crack with cracks in the units lead to the collapse.

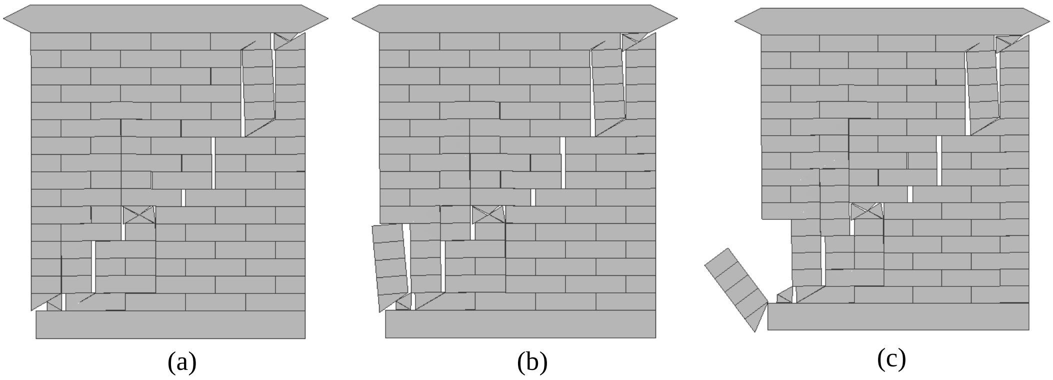


Figure8: Collapse mechanism of the wall at displacement: (a) δ=18 mm, (b) δ=21 mm and (c) δ=24 mm

Figure 7 shows the behavior of the wall after collapse. This example highlights the ability of a combined FEM/DEM in simulating the behaviour of the structure after reaching ultimate load which can be important in analysing progressive collapse of structure.

3.2 *Dry stone wall exposed to monotonically increasing shear load*

This example demonstrates the ability of the numerical model to reproduce the main features that characterize the behavior of dry stone masonry shear wall under monotonic increasing shear load.

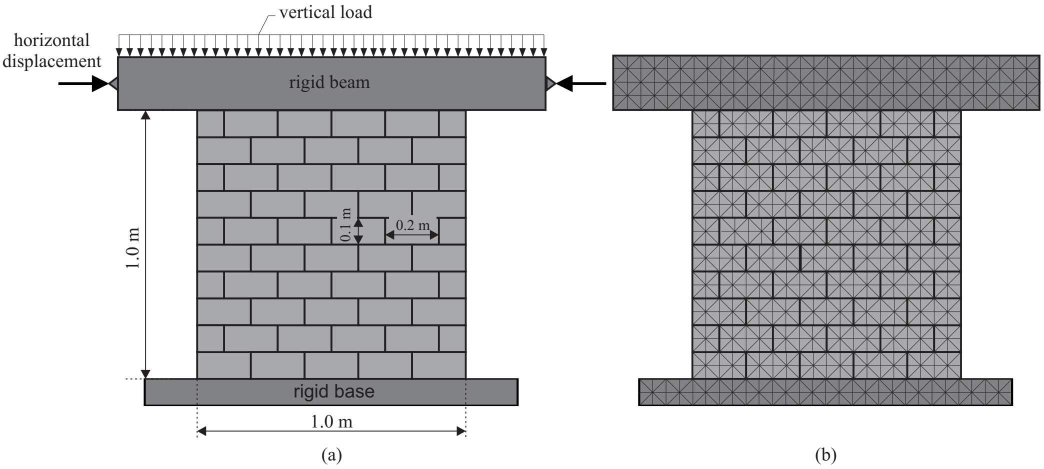


Figure9: Schematic view of a stone wall: (a) geometry and load; (b) discretization of the structure

For that purpose experimental program conducted by Oliveira [11] was chosen in order to compare experimental results and numerical results obtained by FEM/DEM method.

The experimental program consisted of a series of quasi-static monotonic tests conducted on a small stone wall sample whose geometry is shown in figure 9a. The wall discretization used in numerical analysis is presented in figure 9b.

The wall consisted of stone blocks of regular dimensions. Average values of mechanical characteristics of granite used in the experiment are taken from literature and are presented in table 2.

Table 2: Mechanical characteristics of stone used in numerical analysis [11]

Modulus of elasticity (MPa)	Tensile strength (MPa)	Compressive strength (MPa)	Fracture energy (N/m)
15500	3.7	57.0	110

The shear behavior of walls was analyzed for vertical longitudinal force of 100 kN, which corresponds to the pre-compression stress of 0.5 MPa. The coefficient of friction μ=0.62 between stone blocks was obtained by experiment.

Comparison of experimental results, numerical results obtained by Lourenco and Rots [12], and numerical results obtained by presented numerical model is shown in figure 10. The numerical model developed by Lourenco and Rots is based on the finite element method containing also contact elements whose constitutive law of behavior is based on the theory of plasticity.

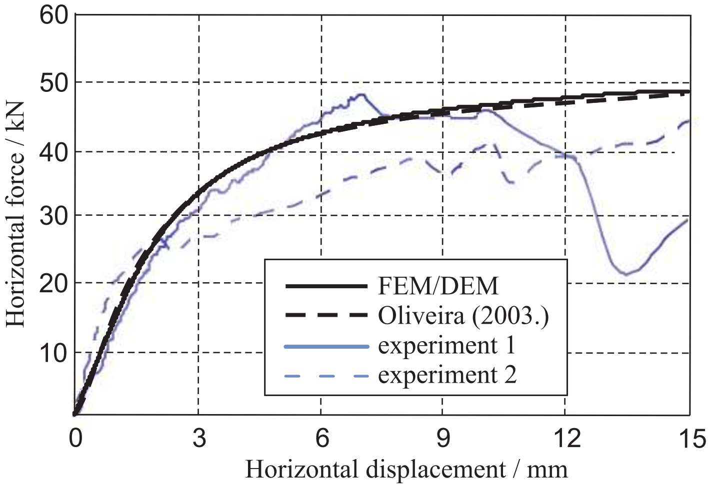
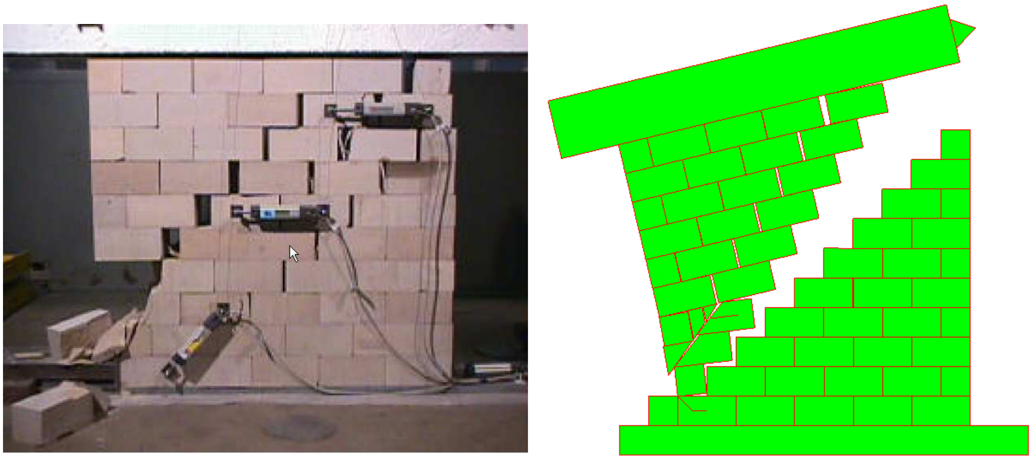


Figure10: Comparison of numerical and experimental results

A good correspondence can be observed between results obtained by model based on the FEM/DEM method, and numerical results obtained by Oliveira [11]. The correspondence between numerical and experimental results can be considered satisfactory because the experiment was conducted with the dry wall made of natural stone. In such walls, the influence of irregularities between blocks has a considerable effect on wall behavior, and this effect is very hard to model by numerical procedure. The comparison of failure mode obtained by numerical model and physical experiment is presented in figure11. It can be observed that failure of wall appears by rotation of the wall accomplished by cracking of stone blocks, and FEM/DEM model reproduce these effects very good.



(a) (b)

Figure11: Failure mechanism of wall: (a) experimental, (b) numerical

4 Conclusion

This paper presents numerical model for analysis and prediction of the collapse of dry stone masonry structures and masonry structures with mortar joints. The presented numerical model is based on finite discrete element method and considers a numerical model in a finite element which simulates the behavior of units, contact interaction algorithm which simulates the interaction between stone blocks in dry joint and a material model in an interface element for simulating the behavior of the mortar joints and unit-mortar interface.

Material model in finite element takes into account orthotropic and cyclic behavior, failure and softening. The interaction between stone blocks in dry joint is considered through the contact interaction algorithm based on the principle of potential contact forces and the Coulomb-type law for friction, while numerical model in contact element take into account possibility of failure and softening behavior in tension and shear, increasing of fracture energy in shear due to increasing pre compression stress, decreasing friction coefficient due to increasing shear displacement as well as cyclic behavior in interface element.

The performance of the presented numerical model was investigated on two masonry walls with mortar joints and one dry stone masonry shear wall. The numerical results show that the presented model is able to capture the main features that characterize the behavior of masonry shear walls through the whole range of loading history.

The advantage of the presented model is its ability to simulate the behavior of the masonry structure through the entire failure mechanism from the continuum to the discontinuum which has been recognized as vital in modelling the collapse of masonry structures due to hazardous loading conditions such as intensive seismic excitation, explosions, missile impact, etc.

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Acknowledgements

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